QCD in external fields

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Outline

- Introduction: QCD vacuum in a background field
- Numerical Experiments and Results
- Summary & Outlook

QCD VACUUM DYNAMICS AND CONFINEMENT

A conclusive explanation of confinement is still lacking

➡ It is important to explore any new path to get hints for understanding QCD vacuum and color confinement

An external field could be useful to probe the QCD dynamics

The lattice effective action

To investigate the vacuum structure of lattice gauge theories we introduced [Cea-Cosmai-Polosa, PLB392(1997)177; Cea-Cosmai, PRD60(1999)094506] a **lattice effective action for the external background field** $\vec{A}^{\rm ext}$

$$\Gamma[\vec{A}^{\rm ext}] = -\frac{1}{L_t} \ln \left\{ \frac{\mathcal{Z}[\vec{A}^{\rm ext}]}{\mathcal{Z}[0]} \right\}$$

$$\mathcal{Z}[\vec{A}^{\rm ext}] = \int_{U_k(\vec{x},x_t=0)=U_k^{\rm ext}(\vec{x})} \mathcal{D}U \, e^{-S_W}$$

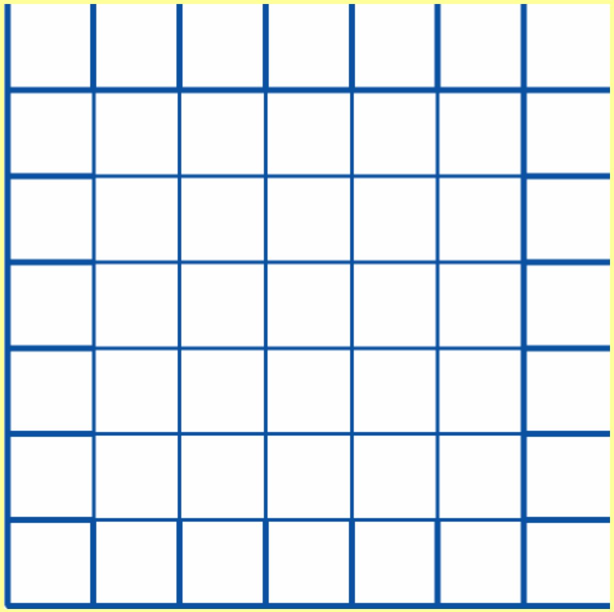
$$U_k(\vec{x}, x_t = 0) = U_k^{\rm ext}(\vec{x}) \, , \quad (k = 1,2,3) \, ,$$

<u>spatial lattice links belonging to a fixed time slice (and to spatial boundaries) are constrained</u>

vacuum energy in presence of the external field:

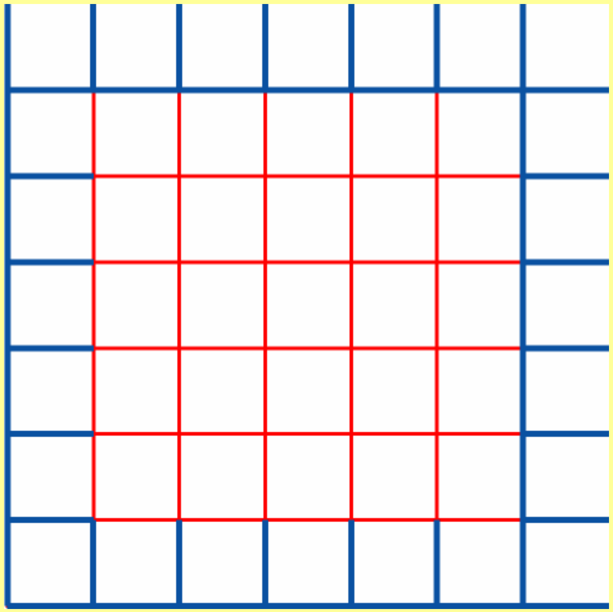
$$\Gamma[\vec{A}^{\rm ext}] \quad \longrightarrow \quad E_0[\vec{A}^{\rm ext}] - E_0[0]$$

t=0 slice



spatial links are **constrained**

other slices



spatial links exiting from sites belonging to the spatial boundary are **constrained**

temporal links are **not** constrained

The free energy functional

At **finite temperature** $T = 1/(aL_t)$ the relevant quantity is the **free energy functional** defined as

$$\mathcal{F}[\vec{A}^{\rm ext}] = -\frac{1}{L_t} \ln \left\{ \frac{\mathcal{Z}_T[\vec{A}^{\rm ext}]}{\mathcal{Z}_T[0]} \right\}$$

$$\begin{aligned}\mathcal{Z}_T\left[\vec{A}^{\rm ext}\right] &= \int_{U_k(L_t,\vec{x})=U_k(0,\vec{x})=U_k^{\rm ext}(\vec{x})} \mathcal{D}U\,\mathcal{D}\psi\,\mathcal{D}\bar{\psi} e^{-(S_W+S_F)} \\ &= \int_{U_k(L_t,\vec{x})=U_k(0,\vec{x})=U_k^{\rm ext}(\vec{x})} \mathcal{D}U e^{-S_W} \det M\,,\end{aligned}$$

S_W :Wilson action

S_F :fermionic action

M :fermionic matrix

thermal partition functional in presence of the the background field for a system in equilibrium at temperature T

NOTE THAT:

- **temporal links are <u>not</u> constrained**
- **fermionic fields are <u>not</u> constrained**

LATTICE SIMULATIONS

We can evaluate by numerical simulations the **derivative of the free energy functional** with respect to the gauge coupling

$$F'(\beta) = \frac{\partial \mathcal{F}(\beta)}{\partial \beta} = V \left[< U_{\mu\nu} >_{\vec{A}^{\text{ext}}=0} - < U_{\mu\nu} >_{\vec{A}^{\text{ext}}\neq 0} \right]$$

Using this method we have investigated
the response of the vacuum to external background fields:

- **abelian monopole field**
- **abelian vortex field**

[Cea-Cosmai, PRD62(2000)094510; JHEP11(2001)064]
[Cea-Cosmai-D'Elia, JHEP0402(2004)018]

- **constant abelian chromomagnetic field**

[Cea-Cosmai, PRD60(1999)094506; JHEP02(2003)031]

SU(3) at finite temperature in a constant abelian chromomagnetic field:

- quenched
- Nf=2

Abelian chromomagnetic field (SU(3))

$$\vec{A}_a^{\rm ext}(\vec{x}) = \vec{A}^{\rm ext}(\vec{x})\delta_{a,\tilde{a}}$$

$$A_k^{\rm ext}(\vec{x}) = \delta_{k,2} x_1 H$$

on the lattice →

$$U_1^{\rm ext}(\vec{x}) = U_3^{\rm ext}(\vec{x}) = 1\,,$$

$$U_2^{\rm ext}(\vec{x}) = \begin{bmatrix} \exp(i\frac{gHx_1}{2}) & 0 & 0 \\ 0 & \exp(-i\frac{gHx_1}{2}) & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

gauge potential for a **constant**
(in space and time)
abelian chromomagnetic field
(directed along spatial direction $\hat{3}$
and direction $\tilde{a}$ in the color space)

SU(3) constrained lattice links $(\tilde{a} = 3)$

(spatial lattice links belonging to a fixed time slice (and to spatial boundaries) are constrained)

Since our lattice has the topology of a torus:

$$a^2 \frac{gH}{2} = \frac{2\pi}{L_1} n_{\rm ext} \quad , n_{\rm ext} \;\; \text{integer}\,.$$

field strength is quantized

For a constant abelian background field the relevant quantity is the **density of the free energy**

$$f[\vec{A}^{\rm ext}] = \frac{1}{V} F[\vec{A}^{\rm ext}] \qquad V = L_s^3$$

The numerical (Monte Carlo) evaluation

of the derivative with respect to the gauge coupling β

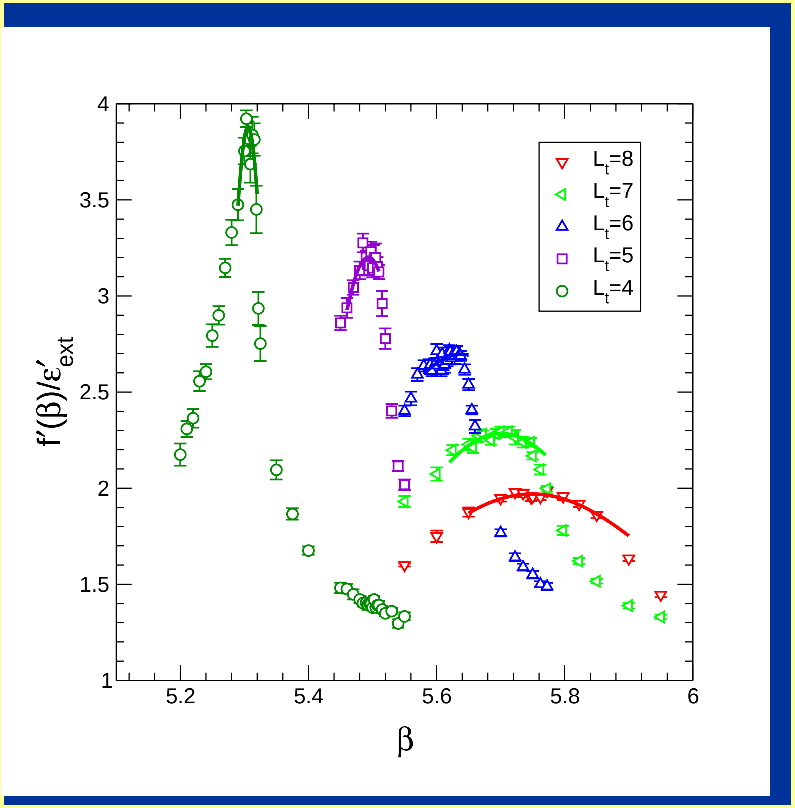
$$f'[\vec{A}^{\rm ext}] = \left\langle \frac{1}{\Omega} \sum_{x,\mu<\nu} \frac{1}{3} \,\mathrm{Re}\,\mathrm{Tr}\, U_{\mu\nu}(x) \right\rangle_0 - \left\langle \frac{1}{\Omega} \sum_{x,\mu<\nu} \frac{1}{3} \,\mathrm{Re}\,\mathrm{Tr}\, U_{\mu\nu}(x) \right\rangle_{\vec{A}^{\rm ext}}$$

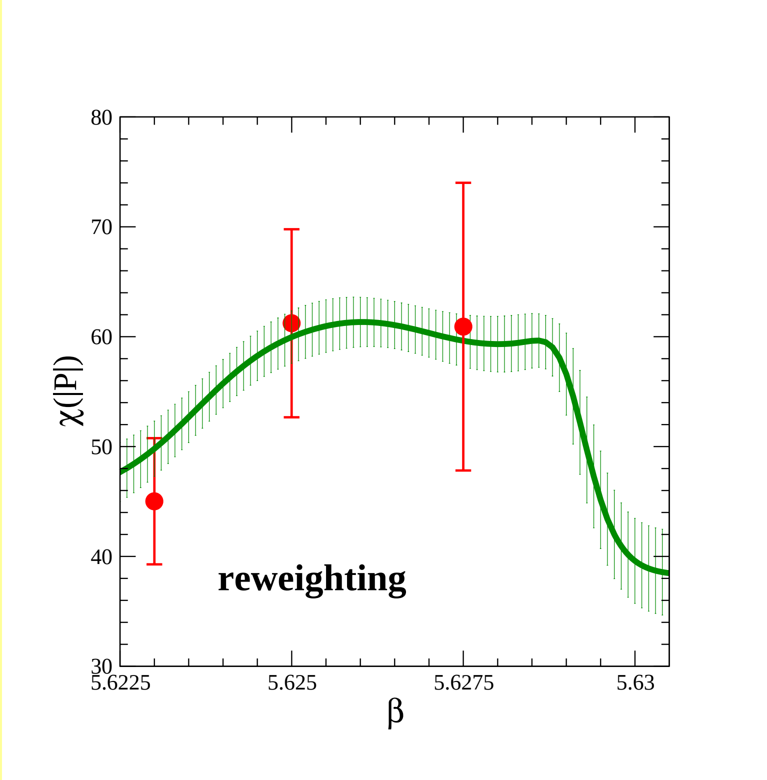
The free energy density may be eventually obtained by a numerical integration

$f[\vec{A}^{\rm ext}] = 0$ at $\beta = 0$

$$f[\vec{A}^{\rm ext}] = \int_0^{\beta} f'[\vec{A}^{\rm ext}] \, d\beta' \, .$$

SU(3) in (3+1) dimensions





$$\frac{f'(\beta, L_t)}{\varepsilon'_{\rm ext}} = \frac{a_1(L_t)}{a_2(L_t)[\beta - \beta^*(L_t)]^2 + 1}$$

$$\varepsilon'_{\rm ext} = \frac{2}{3}\left[1 - \cos\left(\frac{gH}{2}\right)\right] = \frac{2}{3}\left[1 - \cos\left(\frac{2\pi}{L_1} n_{\rm ext}\right)\right]$$

$$\chi(|P|) = < |P|^2 > - < |P| >^2$$

$$P = \frac{1}{V_s} \sum_{\vec{x}} \frac{1}{3} {\rm Tr} \prod_{x_4=1}^{L_t} U_4(x_4, \vec{x}) \, .$$

$f'(\beta)$ **peak**	$\beta_c = 5.6272(69)$
reweighting	$\beta_c = 5.6266(12)$

deconfinement temperature

versus

the strength of the external abelian chromomagnetic field

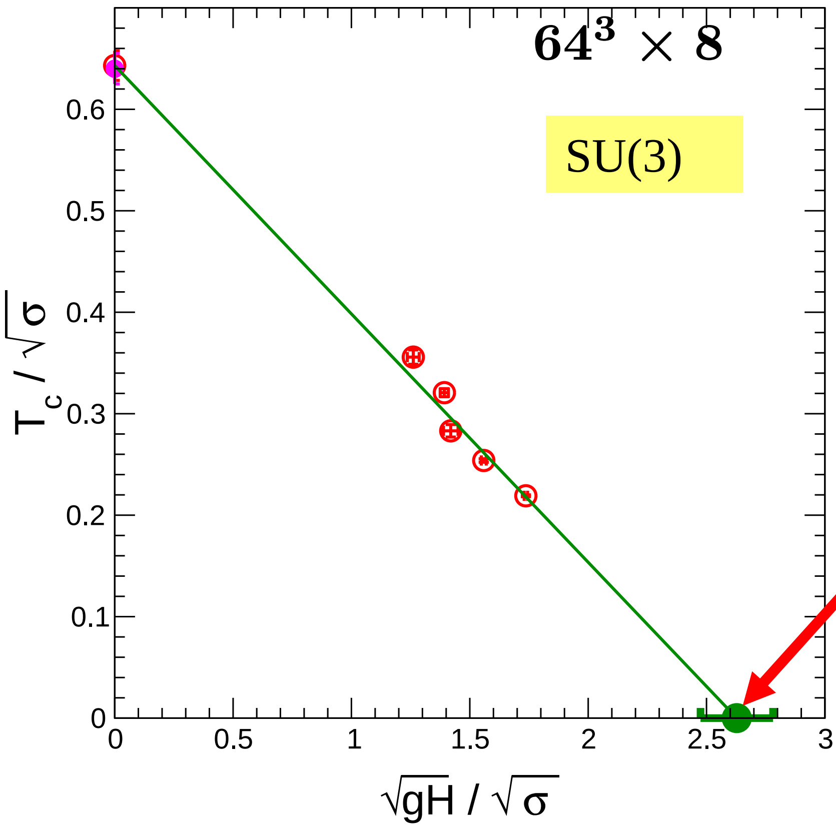
deconfinement temperature

$$\frac{T_c(gH)}{\sqrt{\sigma}} = \frac{1}{L_t\sqrt{\sigma(\beta_c(n_{\rm ext})}}$$

field strength

$$\frac{\sqrt{gH}}{\sqrt{\sigma}} = \sqrt{\frac{4\pi n_{\rm ext}}{L_x\sigma(\beta_c(n_{\rm ext}))}}$$

in units of the string tension:



$$\frac{T_c}{\sqrt{\sigma}} = \alpha\frac{\sqrt{gH}}{\sqrt{\sigma}} + \frac{T_c(0)}{\sqrt{\sigma}}$$

$$\frac{T_c(0)}{\sqrt{\sigma}} = 0.643(15)$$

$$\alpha = -0.245(9)$$

linear parameterization

critical temperature (at zero external field):

$$T_c/\sqrt{\sigma} = 0.640(15)$$

[Teper, hep-th/9812187]

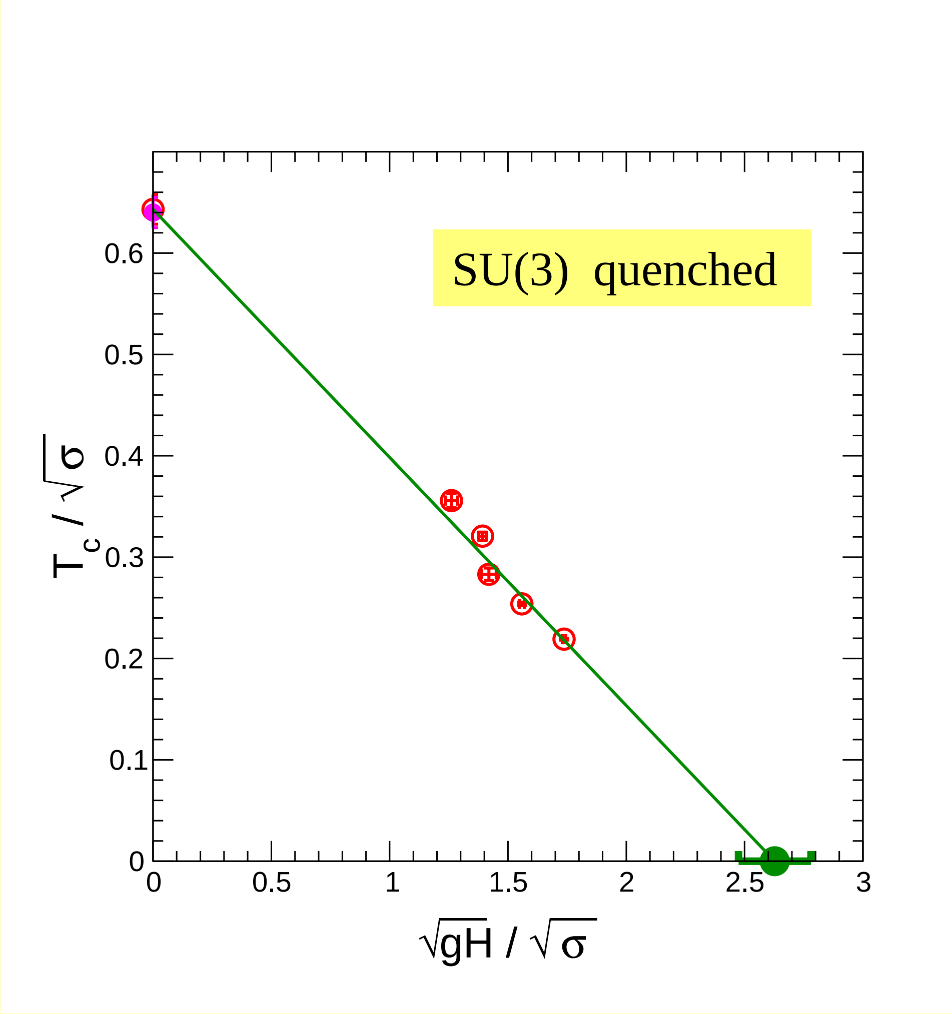
critical field :

$$\frac{\sqrt{gH_c}}{\sqrt{\sigma}} = 2.63 \pm 0.15$$

$$\sqrt{gH_c} = (1.104 \pm 0.063)\,\mathrm{GeV} \sim 6.26(2) \times 10^{19}\,\mathrm{Gauss}$$

the deconfinement temperature depends
on the strength of an external abelian chromomagnetic field.

the deconfinement temperature decreases when the strength of the applied field is increased and eventually goes to zero



(compact quark stars: P.Cea, JCAP03(2004)011)

We performed (*) the same analysis in case of:

- **Non Abelian gauge theories:**
  - **different number of colors (N=2, N=3)**
  - **different space-time dimensions (3+1 dim, 2+1 dim)**

- **Abelian gauge theory**
  - **U(1) in 4 dim**
  - **U(1) in (2+1) dim**

(*) [Cea-Cosmai, JHEP08(2005)079]

Our numerical results can be summarized as follows:

SU(3), SU(2) (3+1) dim (2+1) dim	The deconfinement temperature **depends** on the strength of the constant chromomagnetic background field
U(1) 4 dim U(1) 2+1 dim	No evidence for a dependence of the critical coupling from the strength of the external magnetic field

(*)

(*) No evidence in the case of an Abelian monopole background field [Cea-Cosmai-D'Elia JHEP02(2004)018]

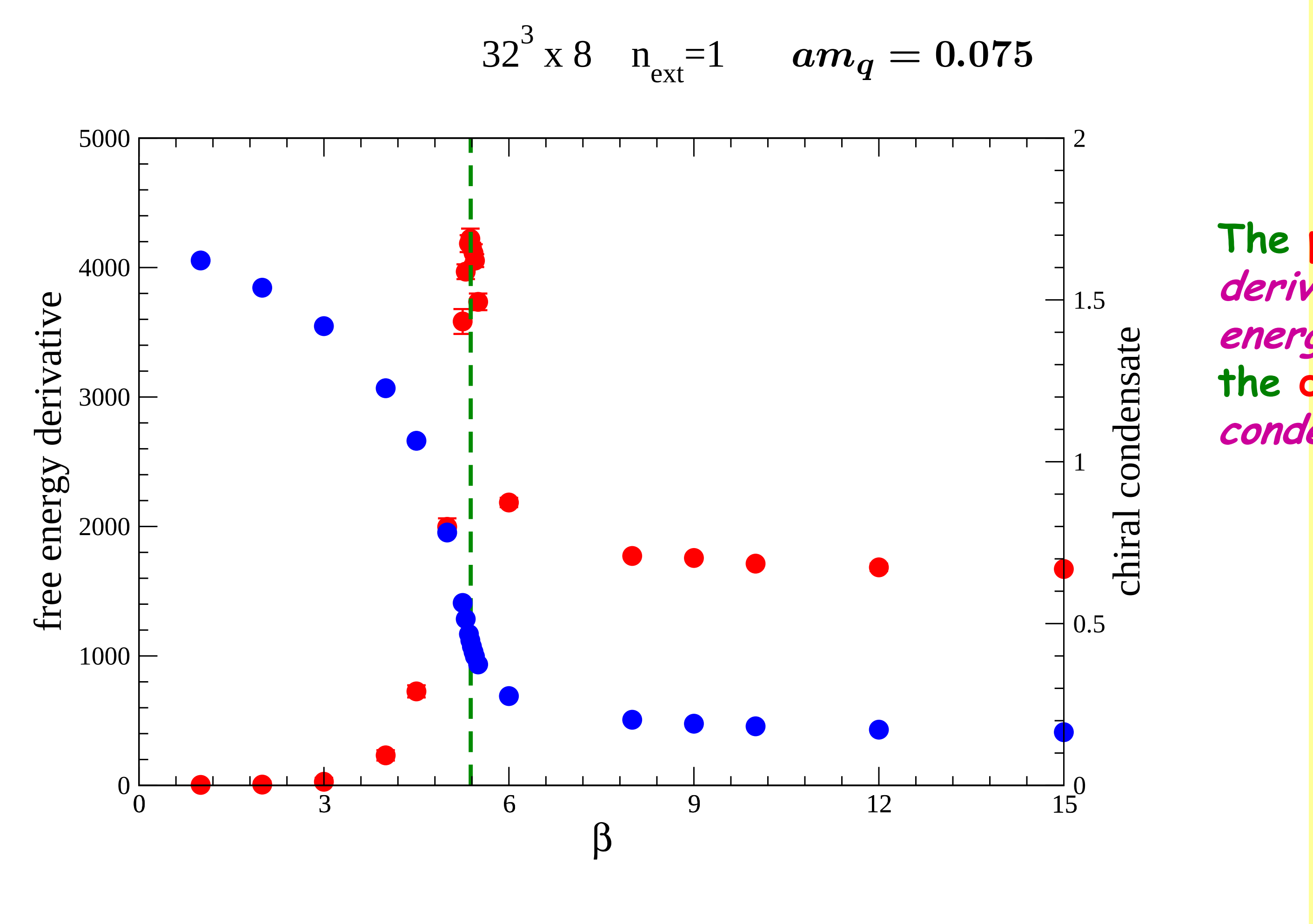
What happens including dynamical fermions ?

numerical simulations for
finite temperature N_f=2 QCD
in an external abelian chromomagnetic field.

simulations have been done using the computer facilities at
INFN apeNEXT Computing Center in Rome

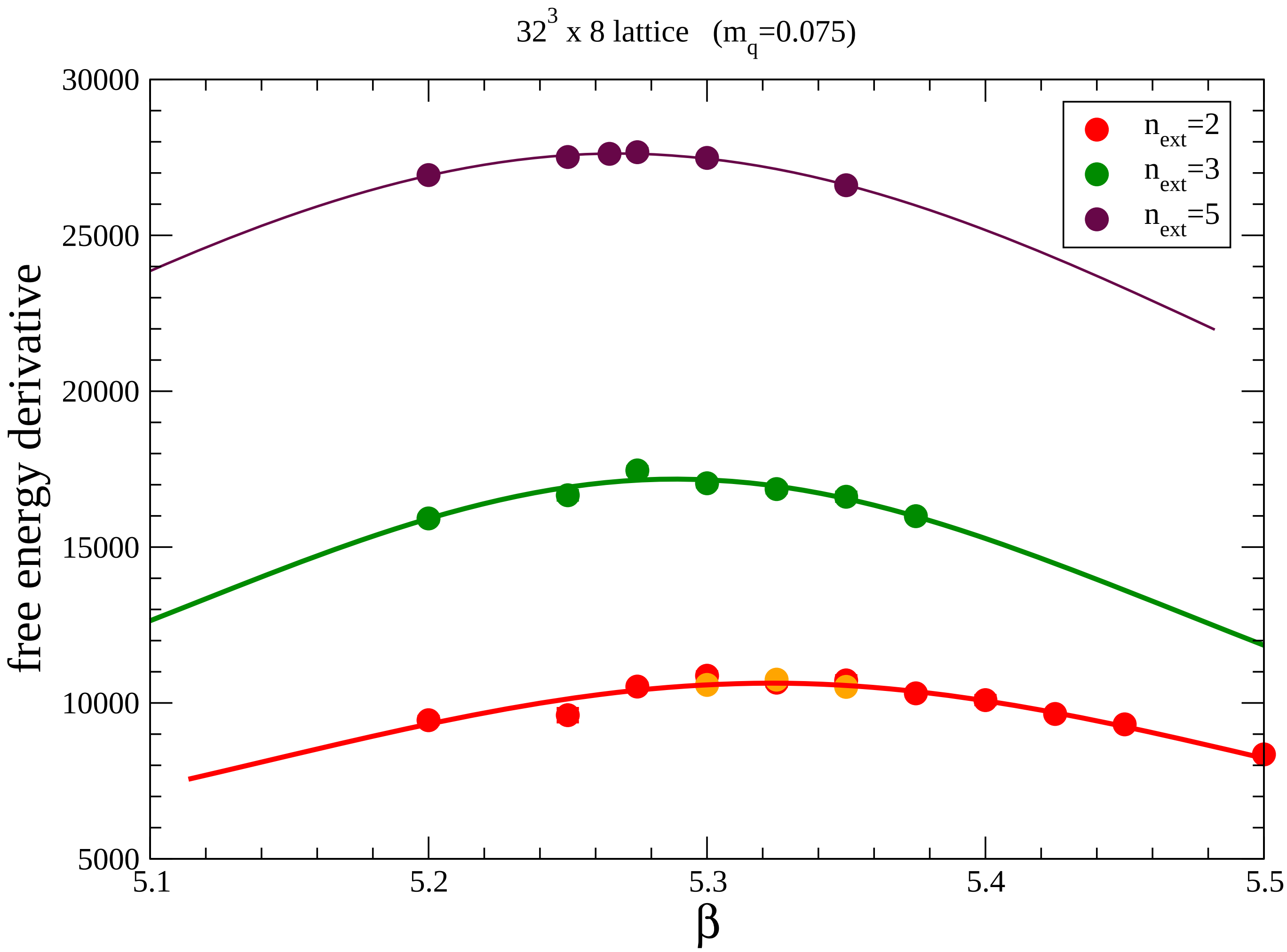
SU(3) Nf=2: numerical results

- The *derivative of the free energy* and the *chiral condensate* at fixed external field strength



The **peak** in the *derivative of the free energy* correlates to the **drop** in the *chiral condensate*

- The ***peak of the free energy*** by varying the strength of the external field

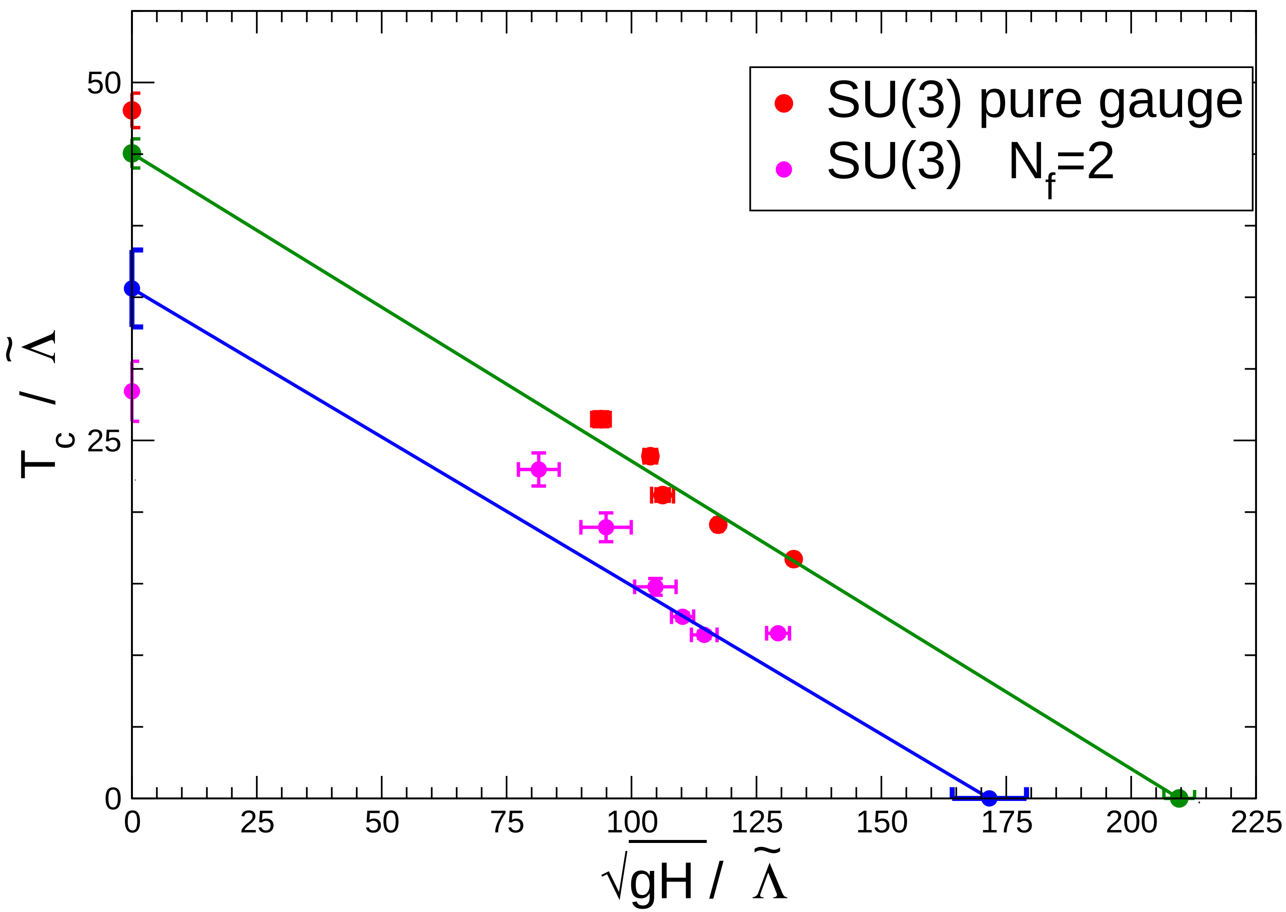


The peak position in the derivative of the free energy depends on the strength of the applied field

- The ***critical temperature*** versus the strength of the external field

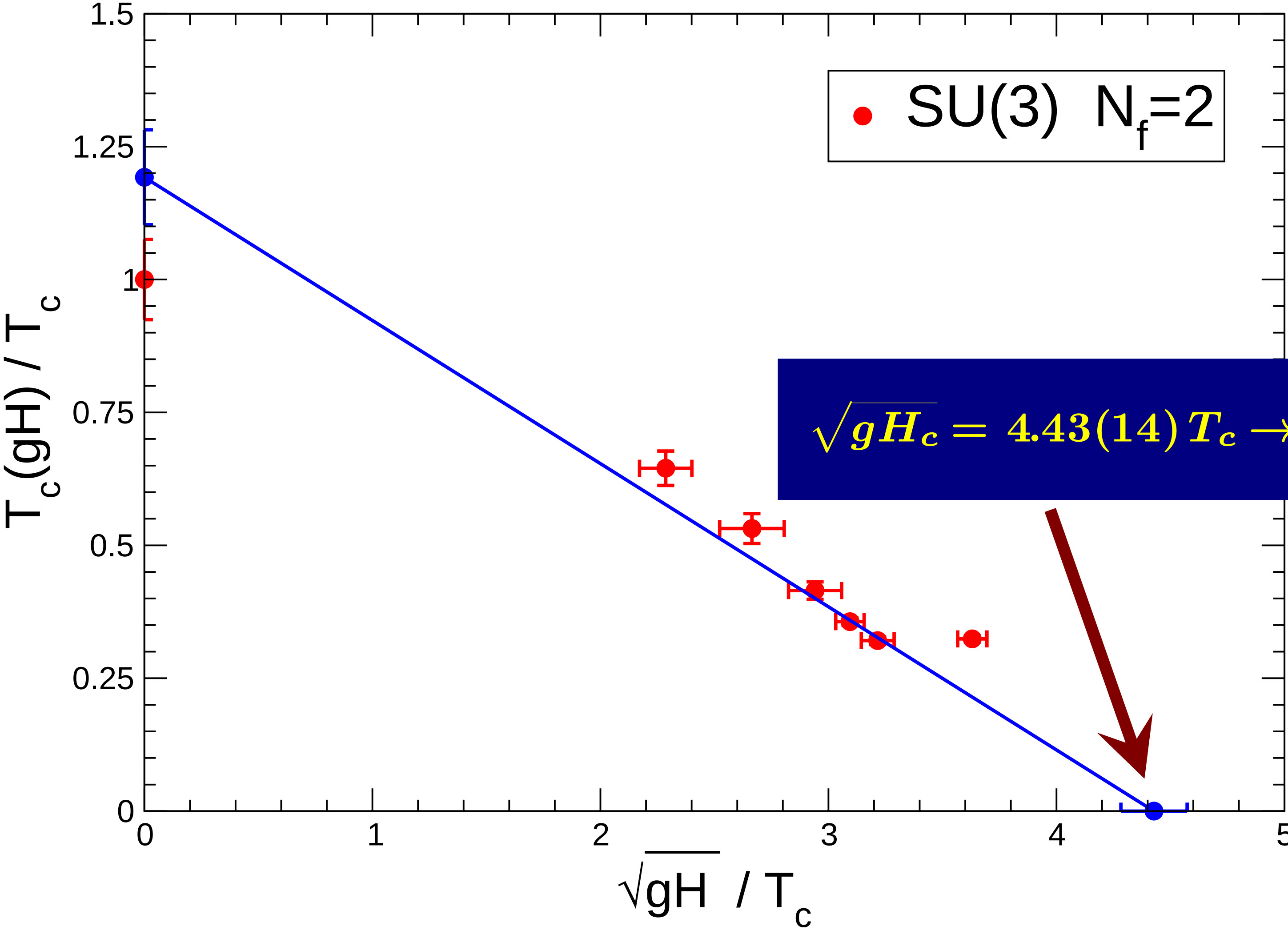
$$\widetilde{\Lambda} = \frac{1}{a} f(g^2)(1 + c_2 \hat{a}(g)^2 + c_4 \hat{a}(g)^4) \qquad \hat{a}(g)^2 \equiv \frac{f(g^2)}{f(g^2 = 1)}$$

two-loop scaling function with N_f=2



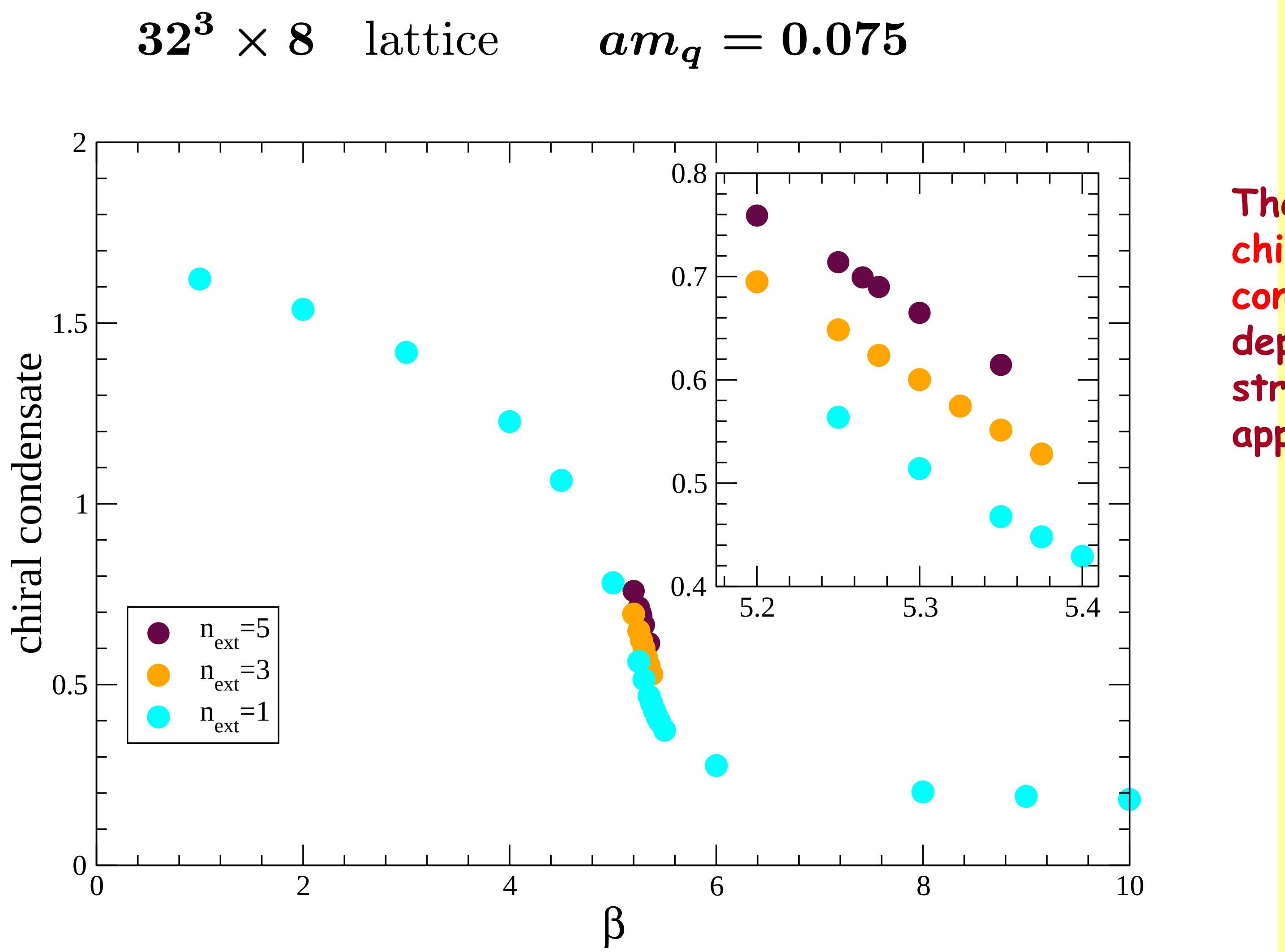
analysis using the scale introduced in
[C.Allton, hep-lat/9610016; Edwards-Heller-Klassen, NPB517(19980377]

- In order to reduce the effect of the scale Lambda previously introduced we can consider the ratios



$$\sqrt{gH_c} = 4.43(14)\, T_c \rightarrow 886(73)\ \mathrm{MeV}$$

- The **chiral condensate** by varying the strength of the external field



The value of the chiral condensate depends on the strength of the applied field

Summary

Gauge Theories

We probed the dynamics of U(1) , SU(2) , and SU(3) l.g.t.'s by means of an external constant abelian (chromo)magnetic field.

We find that (both in (2+1) and (3+1) dimensions):

- for non abelian gauge theories the deconfinement temperature depends on the strength of the chromomagnetic background field and there is a critical field gH_c such that for $gH > gH_c$ the gauge system is in the deconfined phase
- for abelian gauge theories the critical coupling does not depend on the strength of the external constant magnetic field

QCD with 2 dynamical flavors

- Evidence for dependence of Tc on the external field even in full QCD. Assuming a linear dependence on $\sqrt{gH}$ as in the quenched case:

$$\sqrt{gH_c}(N_f = 2) < \sqrt{gH_c}(\text{quenched}) \simeq 1.1\text{GeV}$$

- The chiral critical temperature seems to be consistent with the deconfinement temperature and both depend on the strength of the external chromomagnetic field

- The chiral condensate increases with the strength of the external chromomagnetic field

Outlook

- simulations with larger temporal sizes for a better control of systematic effects and a better estimate of the deconfinement temperature

- study of the effect of the background field on the EOS of QCD