Nucleon structure from lattice QCD

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QCDSF Collaboration

apeNEXT: Computational Challenges and First Physics Results,
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Outline

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Introduction

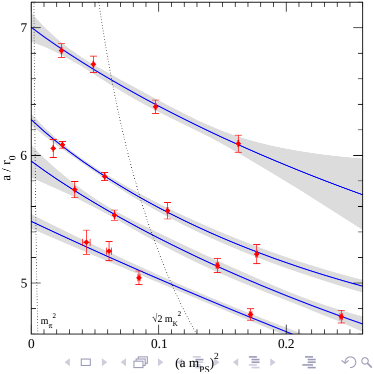
- Lattice in principle allows for ab initio calculation of
  - Moments of nucleon structure functions
  - Nucleon form factors (electromagnetic FF, GFF)
- Good control of several **systematic errors** related to extrapolations needed:
  - **Infinite volume limit** $V \to \infty$
  - **Continuum limit** $a \to 0$
  - **Chiral limit** $m_\mathrm{q} \to 0$
- Calculations now feasible for 2-3 flavours of dynamical fermions thanks to
  - Improved **algorithms** (e.g. mass preconditioning, multiple timescales)
  - New **capability computers** (e.g. BG/L, apeNEXT)
- This talk: selected results from QCDSF collaboration on nucleon structure

Simulation parameters

Configurations with $N_f = 2$ O(a)-improved dynamical quarks generated by UKQCD+QCDSF+DIK.

$$m_{\mathrm{PS,sea}} = 340, \ldots, 1170\ \mathrm{MeV} \qquad a = 0.07, \ldots, 0.11\ \mathrm{fm}$$
$$m_{\mathrm{PS,val}} = 340, \ldots, 1240\ \mathrm{MeV} \qquad V = 1.4, \ldots, 2.6\ \mathrm{fm}$$

- Simulations much closer to the physical quark mass
- Reasonably small lattice spacing
- Volumes at lower limit



Momenta and polarisations

- 3 initial state momentum:

$$\frac{L}{2\pi}\vec{p} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

- 3 choices for polarisations:

$$\begin{aligned} \Gamma &= (1+\gamma_4)/2 \\ \Gamma &= (1+\gamma_4)\mathrm{i}\gamma_5\gamma_1/2 \\ \Gamma &= (1+\gamma_4)\mathrm{i}\gamma_5\gamma_2/2 \end{aligned}$$

- 17 different choices of momentum transfer $\vec{q} = \vec{p}\,' - \vec{p}$
- Configurations at distance 5-10 trajectories, 16-4 different sources
- Use binning to eliminate auto-correlations

Calculation of matrix elements

- Matrix elements are determined from ratios:

$$R(t,\tau,\vec{p}\,',\vec{p}\,) = \frac{C_3(t,\tau,\vec{p}\,',\vec{p}\,)}{C_2(t,\vec{p}\,')} \times \left[\frac{C_2(\tau,\vec{p}\,')C_2(t,\vec{p}\,')C_2(t-\tau,\vec{p}\,)}{C_2(\tau,\vec{p}\,)C_2(t,\vec{p}\,)C_2(t-\tau,\vec{p}\,')}\right]^{1/2}$$

  where

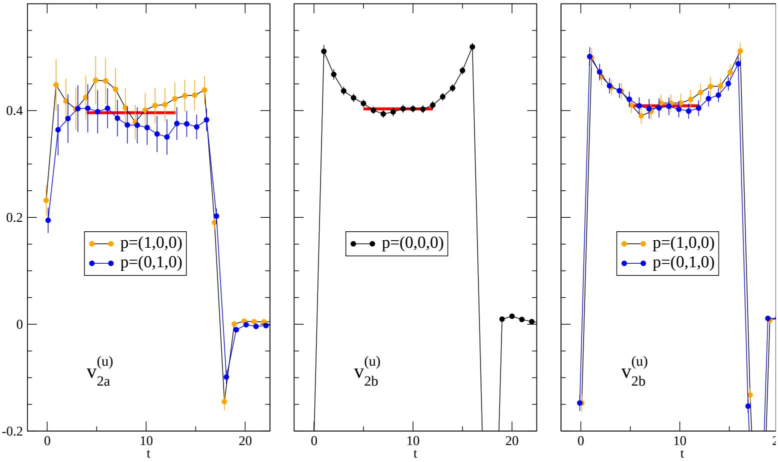
$$C_2(t,\vec{p}\,) = \sum_{\alpha\beta} \Gamma_{\beta\alpha} \langle B_\alpha(t,\vec{p}\,)\bar{B}_\beta(0,\vec{p}\,)\rangle$$

  and

$$C_3(t,\tau,\vec{p}\,',\vec{p}\,) = \sum_{\alpha\beta} \Gamma_{\beta\alpha} \langle B_\alpha(t,\vec{p}\,')\mathcal{O}(\tau)\bar{B}_\beta(0,\vec{p}\,)\rangle$$

- Non-perturbative renormalisation using, e.g., Rome-Southampton method (RI′-MOM scheme)

Example: v_{2a} vs. v_{2b}



$(\beta, \kappa_{\text{sea}}) = (5.4, 0.13560)$, V=$24^3 \times 48$

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- Form factor of the proton axial current:

$$\langle p', s' | A_\mu^{u-d} | p, s \rangle = \bar{u}(p', s') \left[\gamma_\mu \gamma_5 G_A(q^2) + \gamma_5 \frac{q_\mu}{2m_N} G_P(q^2) \right] u(p, s)$$

- Axial coupling constant: $g_A = G_A(0)$
- Parton model interpretation: Fraction of the nucleon spin carried by the quarks Δq

$$\langle p, s | A_\mu^{u-d} | p, s \rangle = 2 g_A s_\mu = 2(\Delta u - \Delta d) s_\mu$$

- Precise experimental value for g_A

Chiral effective field theory

[QCDSF 2006]

- Evaluation of matrix element of isovector axial current within a low energy effective theory of QCD feasible:

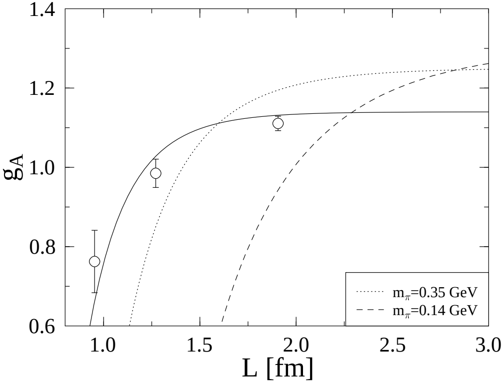
$$g_A(m_{\mathrm{PS}})$$

- Calculation for finite spatial cubic box of length L:

$$g_A(m_{\mathrm{PS}}, L) = g_A(m_{\mathrm{PS}}) + \Delta g_A(m_{\mathrm{PS}}, L)$$

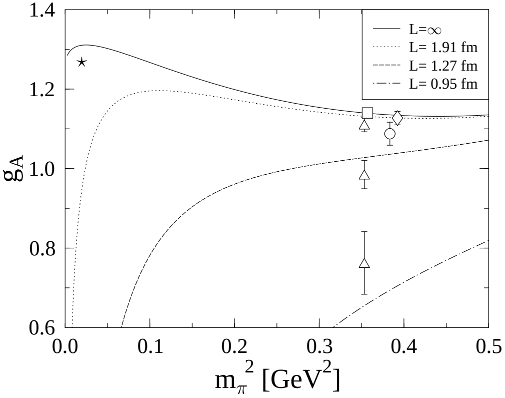
- Lattice results not sufficient to fix all parameters $\rightarrow$ phenomenological input required:
  - Pion decay constant
  - $N\Delta$ mass splitting Δ_0 and axial coupling c_A
  - Coupling $B^r_{20}(\lambda)$

Exploring finite size effects



- Results for g_A at $\beta = 5.29$, $\kappa = 0.1359 \rightarrow m_{\mathrm{PS}} \simeq 0.6\ \mathrm{GeV}$

Chiral extrapolation



- Consistent description of lattice and experimental results
- Results for small quark masses ($m_{\mathrm{PS}}^2 < 0.1\ \mathrm{GeV}^2$) and large volumes ($L > 2\ \mathrm{fm}$) needed

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$$\int_0^1 dx\, x^{n-2}\, F^{\mathrm{u-d}}(x, Q^2) = f\, E^{\mathcal{S}}_{F;v_n}\left(\frac{M^2}{Q^2}, g^{\mathcal{S}}(M)\right) v_n{}^{\mathcal{S}}(g^{\mathcal{S}}(M))$$

- Matrix elements $\mathbf{v_n}$ can be measured on the lattice

$$\langle N(\vec{p})| \left[\mathcal{O}_q^{\{\mu_1\cdots\mu_n\}} - \mathrm{Tr}\right] |N(\vec{p})\rangle^{\mathcal{S}} := 2 v_n^{(q)\mathcal{S}}\ [p^{\mu_1}\cdots p^{\mu_n} - \mathrm{Tr}]$$

- Results have to be renormalised using a scheme (e.g. $\overline{\mathrm{MS}}$) and scale (e.g. 2 GeV).

Chiral extrapolation

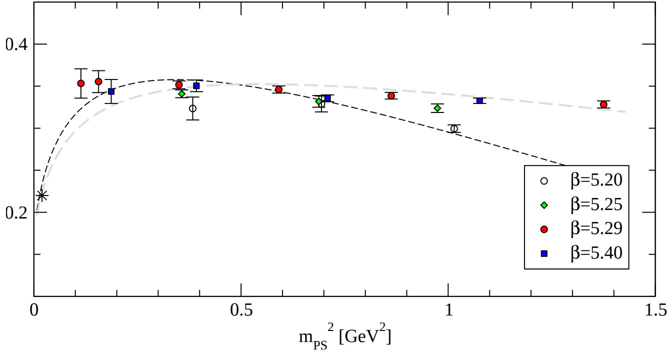
Fit function model taking 'pion cloud' around nucleon into account:

[Thomas et al., Detmold et al.]

$$\begin{aligned} v_n^{\mathrm{RGI}}(r_0 m_{\mathrm{PS}}) &= F^{v_n}(r_0 m_{\mathrm{PS}}) \\ F^{v_n}(x) &= v_n^{\mathrm{RGI}} \left(1 - C x^2 \ln \frac{x^2}{x^2 + (r_0 \Lambda_\chi)^2}\right) + a_n x^2 \end{aligned}$$

Lattice results for v_{2b}^{RGI}

- Fit to $\beta = 5.29$ data, $\Lambda_\chi = 500$ MeV, experimental value fixed



- Fit ansatz does not describe data very well
- No indication found for strong FSE

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Introduction

- Standard decomposition of nucleon electromagnetic matrix elements:

$$\langle p', s' | J^\mu | p, s \rangle = \overline{u}(p', s') \left[\gamma_\mu \mathrm{F_1(q^2)} + i\sigma^{\mu\nu} \frac{q_\nu}{2M_N} \mathrm{F_2(q^2)} \right] u(p, s)$$

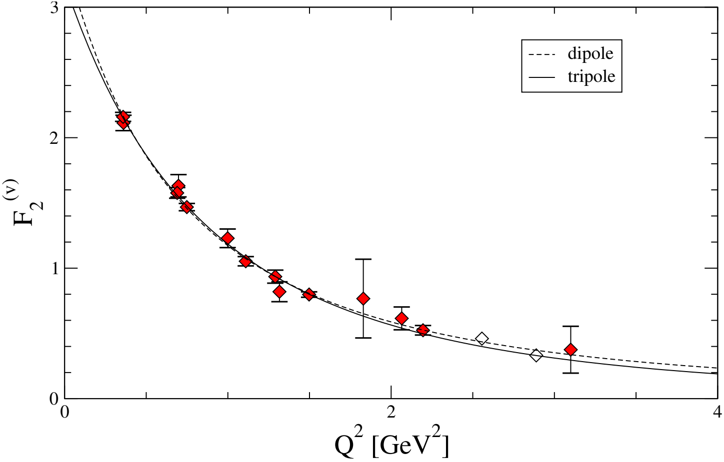
- We will consider, e.g.

Proton form factors: $\frac{2}{3}\overline{u}\gamma^\mu u - \frac{1}{3}\overline{d}\gamma^\mu d$

Isovector form factors: $\overline{u}\gamma^\mu u - \overline{d}\gamma^\mu d$
$\rightarrow$ Disconnected terms cancel

- Experimental interest:
  - **q^2 scaling**
  - **Flavour dependence**

Typical result



- Difference of fits small wrt to statistical errors

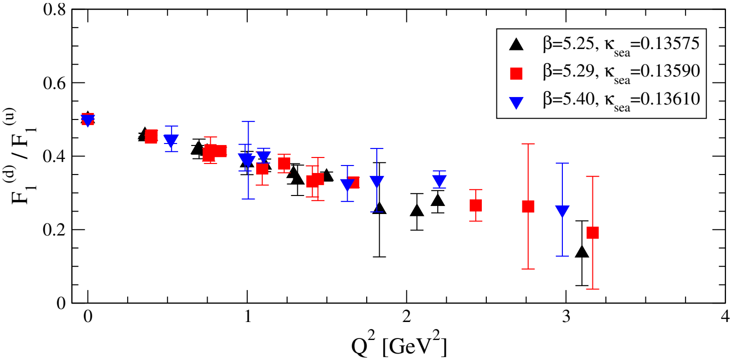
Parametrisation of lattice results

- Fit data to

$$F_i(q^2) = \frac{A_i}{(1 - q^2/M_i^2)^p}$$

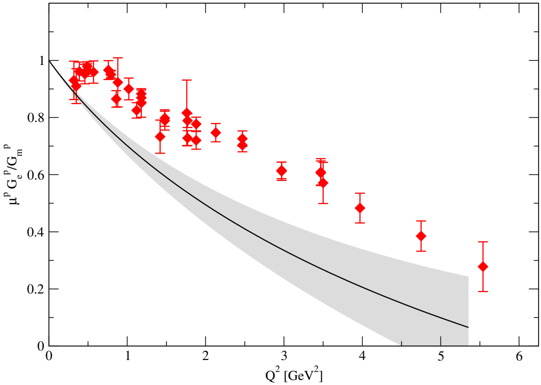
- Fit ansatz determines
  - $F_1^{(q)}(0)$, $F_2^{(q)}(0)$ ($q = u, d$)
  - Di-/tripole masses or, equivalently, the form factor radii
- Lattice data not precise enough to fix p
- Looking for minimal χ^2 suggests:
  - $p = 2$ for $F_1^{(u)}$
  - $p = 3$ for $F_1^{(d)}$, $F_2^{(u)}$, $F_2^{(d)}$

Scaling of $F_1^{(d)}/F_1^{(u)}$



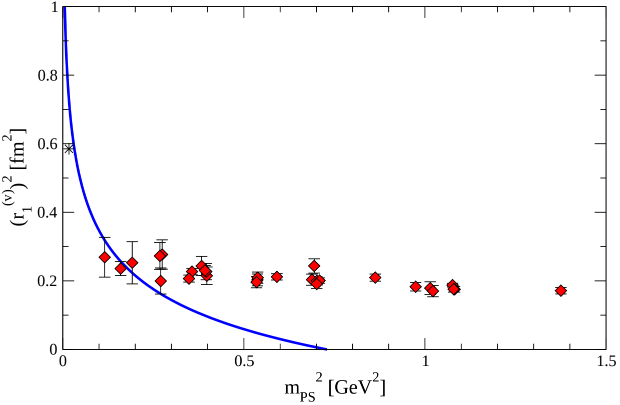
- Data suggests $p = 2$ and $p = 3$ to fit $F_1^{(u)}$ and $F_1^{(d)}$, respectively
- Flavour dependence also observed in fits to experimental data [Diehl et al., 2005]

Scaling of $\mu^{(p)} G_e^{(p)}(q^2)/G_m^{(p)}(q^2)$



- Ignore disconnected terms
- Naive extrapolation (linear in quark mass)
- **But:** Naive extrapolation is not correct

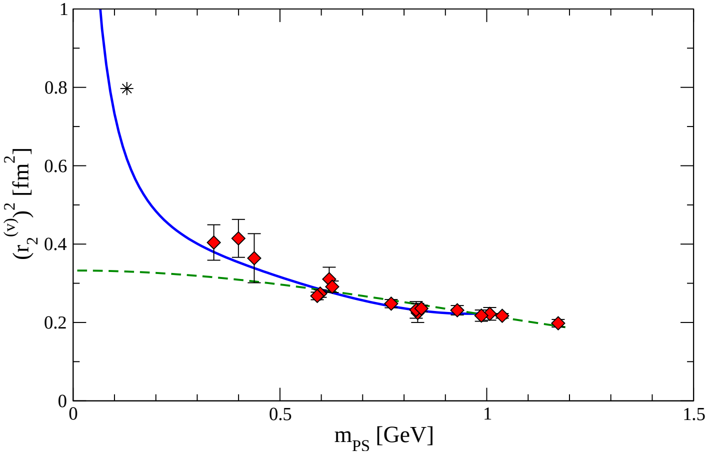
Dirac radius $[r_1^{(v)}]^2$



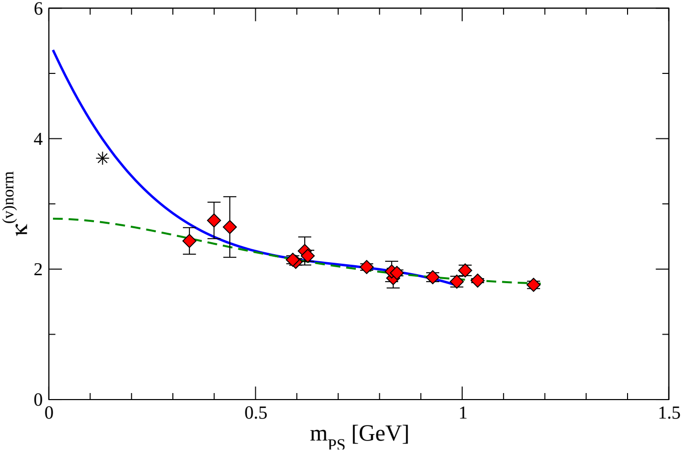
- Calculation of isovector radii within a low energy effective theory of QCD predicts strong quark mass dependence

Pauli radius $[r_2^{(v)}]^2$

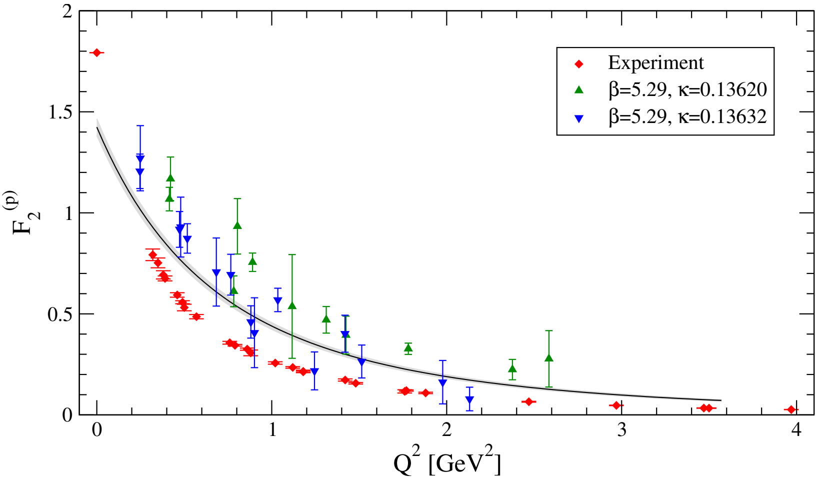
- Joined fit to $[r_2^{(v)}]^2$ and $\kappa^{(v)}$:



Anomalous magnetic moment $\kappa^{(v)}$



$F_2^{(p)}$: Lattice vs. Experiment



- Better agreement for $m_{\rm PS} \simeq 300$ MeV?

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Introduction

- Lattice methods allow for calculation of moments of the quark density $\rho(x, b_\perp, s_\perp, S_\perp)$

$$\rho^n(b_\perp, s_\perp, S_\perp) = \int_{-1}^{1} dx\, x^{n-1} \rho(x, b_\perp, s_\perp, S_\perp) =$$

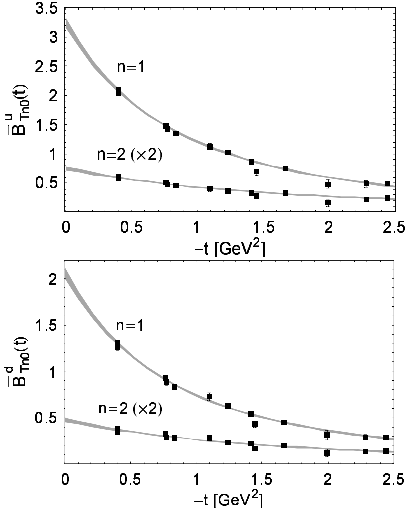
$$\frac{1}{2}\Big\{ A_{n0}(b_\perp^2) + s_\perp^i S_\perp^i \left(A_{Tn0}(b_\perp^2) - \frac{1}{4m^2} \Delta_{b_\perp} \widetilde{A}_{Tn0}(b_\perp^2) \right)$$

$$+ \frac{b_\perp^j \epsilon^{ji}}{m} \left(S_\perp^i B'_{n0}(b_\perp^2) + s_\perp^i \overline{B}'_{Tn0}(b_\perp^2) \right)$$

$$+ s_\perp^i (2 b_\perp^i b_\perp^j - b_\perp^2 \delta^{ij}) S_\perp^j \frac{1}{m^2} \widetilde{A}''_{Tn0}(b_\perp^2) \Big\}$$

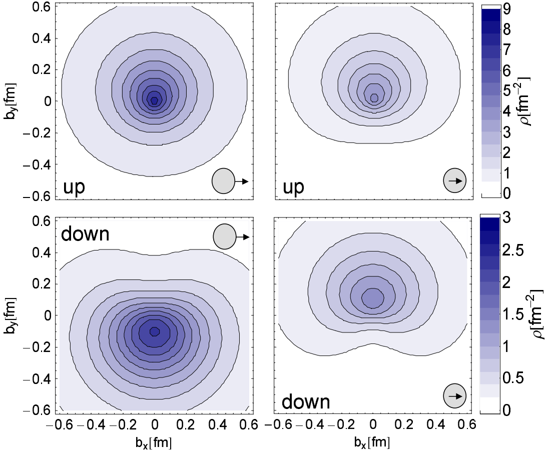
  - momentum fraction x
  - transverse spin $s_\perp$
  - distance $b_\perp$ from the center-of-momentum
  - transverse spin $S_\perp$

Generalized form factor $\overline{B}_{T(n=1,2)0}(t)$



Quark densities in nucleon

- Parametrize lattice data using p-pole fits
- Fourier transformation to impact parameter $b_\perp$ space



Content

APE and the QCDSF physics program

APE machines at NIC/DESY (Zeuthen)

APE100	1994-2004
APEmille	2000-....
apeNEXT	2005-....

Selected bibliography:

- 1994: Towards a Lattice Calculation of the Nucleon Structure Functions
- 1997: Pion and Rho Structure Functions from Lattice QCD
- 1999: A Lattice Determination of Light Quark Masses
- 2000: A lattice calculation of the nucleon's spin-dependent structure function g_2 revisited
- 2001: Determination of $\Lambda_{\overline{MS}}$ from quenched and $N_f = 2$ dynamical QCD
- 2002: A lattice study of the spin structure of the Lambda hyperon
- 2003: Nucleon electromagnetic form factors on the lattice and in chiral effective field theory
- 2004: A lattice determination of moments of unpolarised nucleon structure functions using improved Wilson fermions
- 2005: Quark helicity flip generalized parton distributions from two-flavor lattice QCD
- 2006: Moments of pseudoscalar meson distribution amplitudes from the lattice

QCDSF program was only possible with APE

International Lattice Datagrid

- Huge efforts required to push towards $m_q \to 0$, $V \to \infty$, $a \to 0$
  - Collaborations start to become larger
  - Different sources for compute power used

  ☞ Important to improve on data sharing
- A number of relevant standards have been defined within ILDG
- Started to build-up a (continental) European infrastructure which starts to become heavily used:

SESAM/TχL/GRAL	O(50,000)
ETMC	O(25,000)
QCDSF	O(20,000)
total	O(95,000)

- MWWG goal: Interoperability for download operations by LAT'07

Conclusions and outlook

- Lattice continues to be interesting method to explore hadron structure:
  - Axial coupling g_A, moments of unpolarised structure functions
  - Electro-magnetic form factors and GFF (transverse spin structure)
- Hadron structure is hot topic for various experimental programs (e.g. JLAB, GSI)
- Control on systematic errors improved but is often not sufficient
  - Possibly large **finite size effects**
  - Strong Quark mass dependence for $m_{\mathrm{PS}} \to m_\pi$
    ☞ **chiral extrapolations** critical
  - Discretisation effects seem to be relatively small
    ☞ **O(a) improvement program** successful
- Exploring the light quark mass region starts to become feasible
  ☞ **Exciting prospect**
- **But:** Computing power $O(100)$ TFlops (sustained) required