QCD Simulations at Realistic Quark Masses: Probing the Chiral Limit

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– QCDSF Collaboration –

Special mention:

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Objective

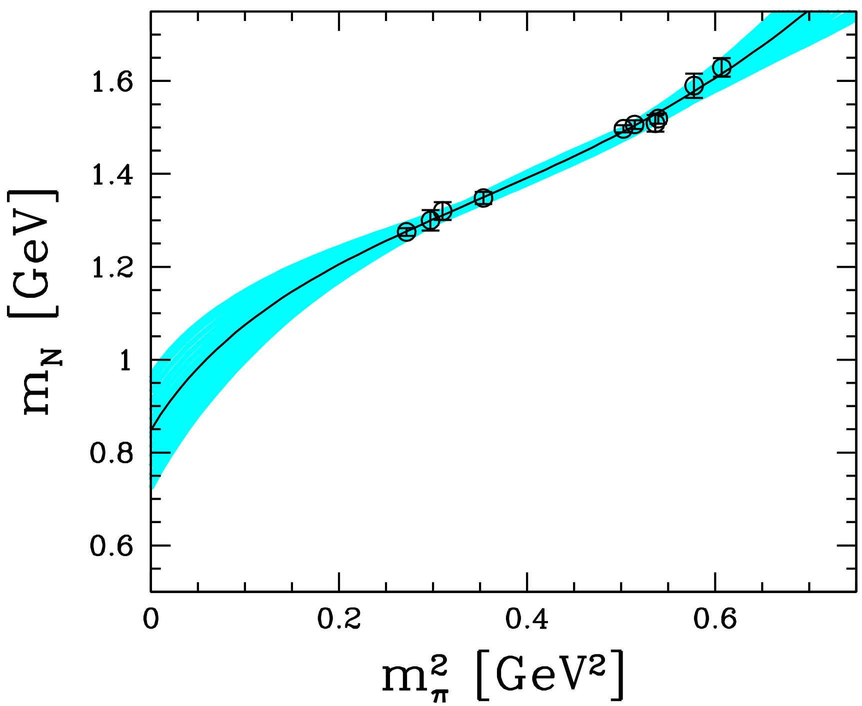
Solve QCD and probe the limits of the Standard Model $\cdots$

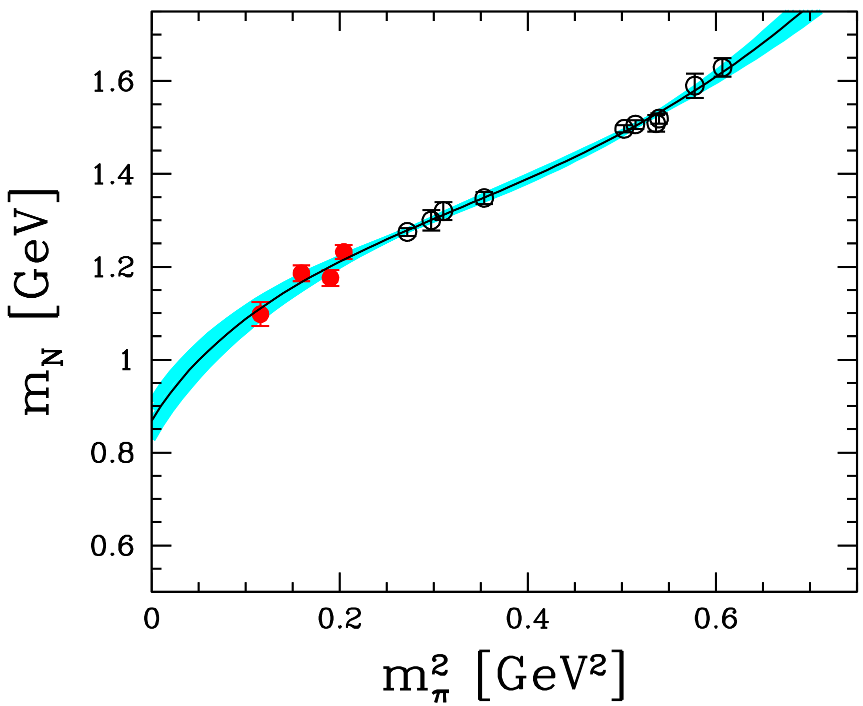
- Parameters of QCD
  - Λ_{QCD} resp. $\alpha_s(Q^2)$
  - Quark masses
  - θ angle
- QCD in the wider world
  - CKM matrix
- How does QCD work ?
  - Hadron structure
  - Spectroscopy
- Fundamental properties
  - χSB
  - Confinement

$\cdots$ in concert with Exp & Phen

Problem: Chiral Extrapolation

Recently





ChPT $O(p^4)$

Stat. error $\lesssim 5\%$

68.3% CL

Need to reduce (scale) error to a few %

Outline

Lattice Simulations

Pion Sector

Nucleon Sector

Miscellaneous

Conclusions & Outlook

Lattice Simulations

Action

$N_f = 2$

$$S = S_G + S_F$$

$$S_G = \beta \sum_{x,\mu<\nu} \left(1 - \frac{1}{3}\mathrm{Re}\,\mathrm{Tr}\, U_{\mu\nu}(x)\right)$$

$$S_F = \sum_x \Big\{ \bar\psi(x)\psi(x) - \kappa\, \bar\psi(x) U_\mu^\dagger(x-\hat\mu)[1+\gamma_\mu]\psi(x-\hat\mu) - \kappa\, \bar\psi(x) U_\mu(x)[1-\gamma_\mu]\psi(x+\hat\mu) - \frac{1}{2}\kappa\, c_{SW}\, g\, \bar\psi(x)\sigma_{\mu\nu}F_{\mu\nu}(x)\psi(x) \Big\}$$

$\Updownarrow$

$$\boxed{\partial_\mu A_\mu^{\mathrm{imp}} = 2 m_q P}$$

Clover Fermions

Advantages

- Local
- Transfer matrix
- $O(a)$ improved
- Flavor symmetry

  Prerequisite to making contact with $SU(2)$ ChPT

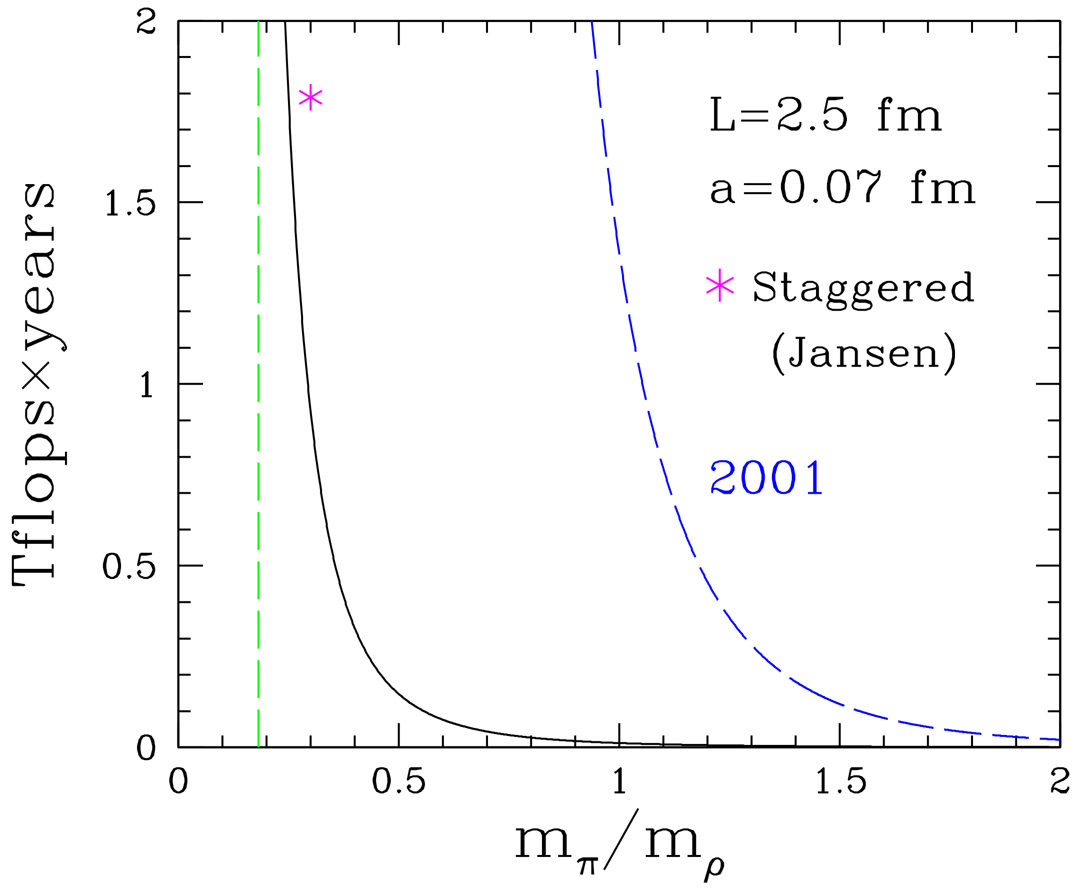
  – Finite size corrections

  – Chiral extrapolation

  – Determination of low-energy constants
- Fast to simulate

Cost of Simulation

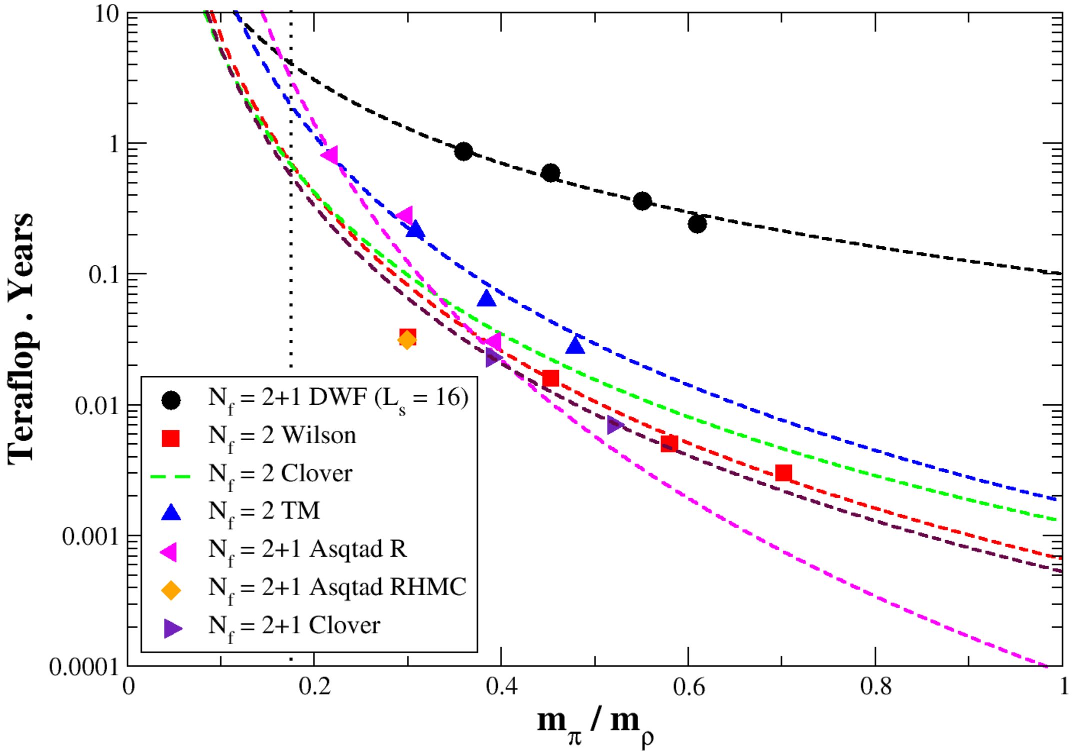
1000 Configurations



$\propto L^{4.8}\,(m_\pi/m_\rho)^{-3.6}\,(r_0/a)^{0.9}$

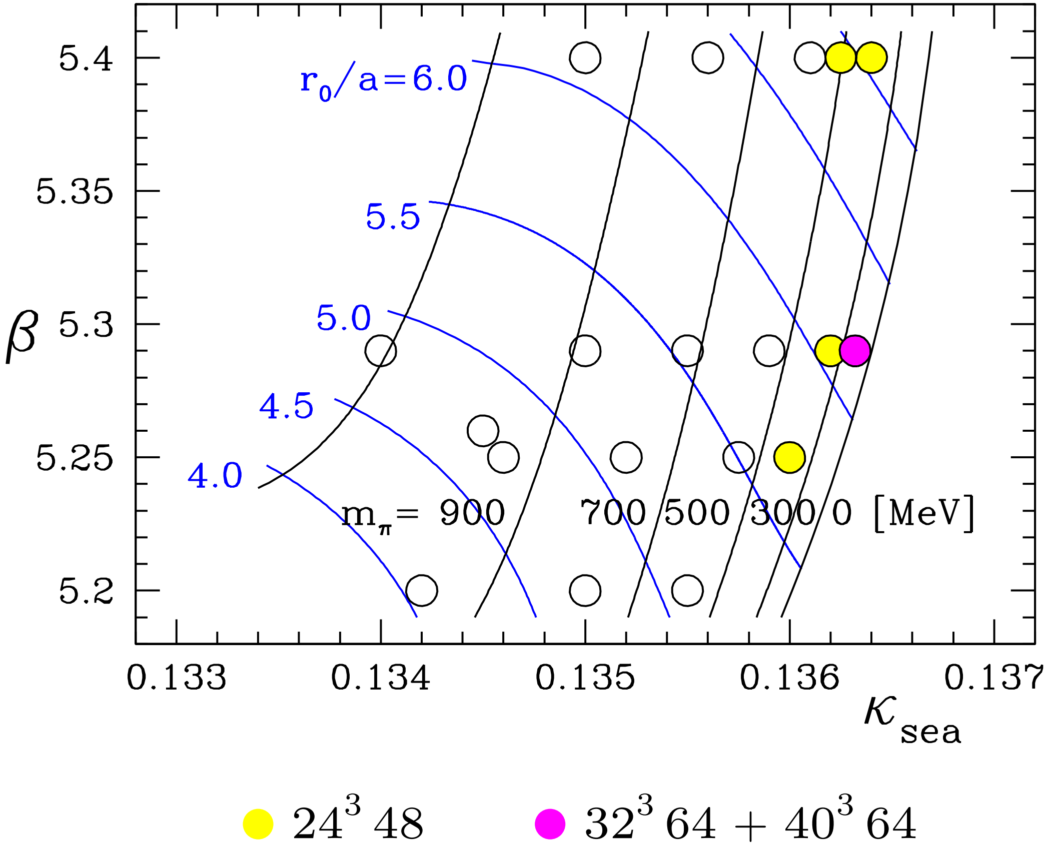
Hasenbusch, QCDSF, Lüscher, Urbach et al., $\cdots$

Compared to · · ·



Clark

Parameters $N_f = 2$

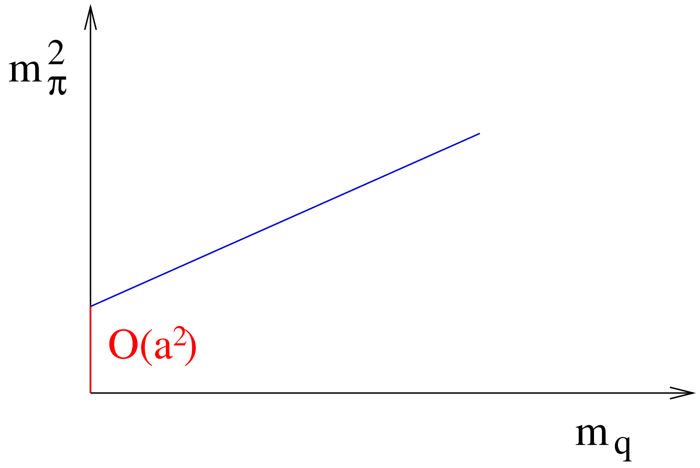


$\leftarrow a = 0.065$ fm

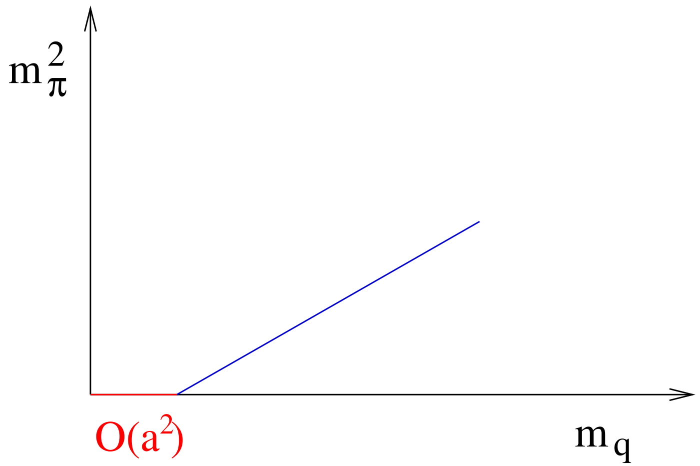
$\leftarrow a = 0.077$ fm

For gauge field sampling we use 'ordinary' HMC algorithm with Hasenbusch integration + 3 time scales

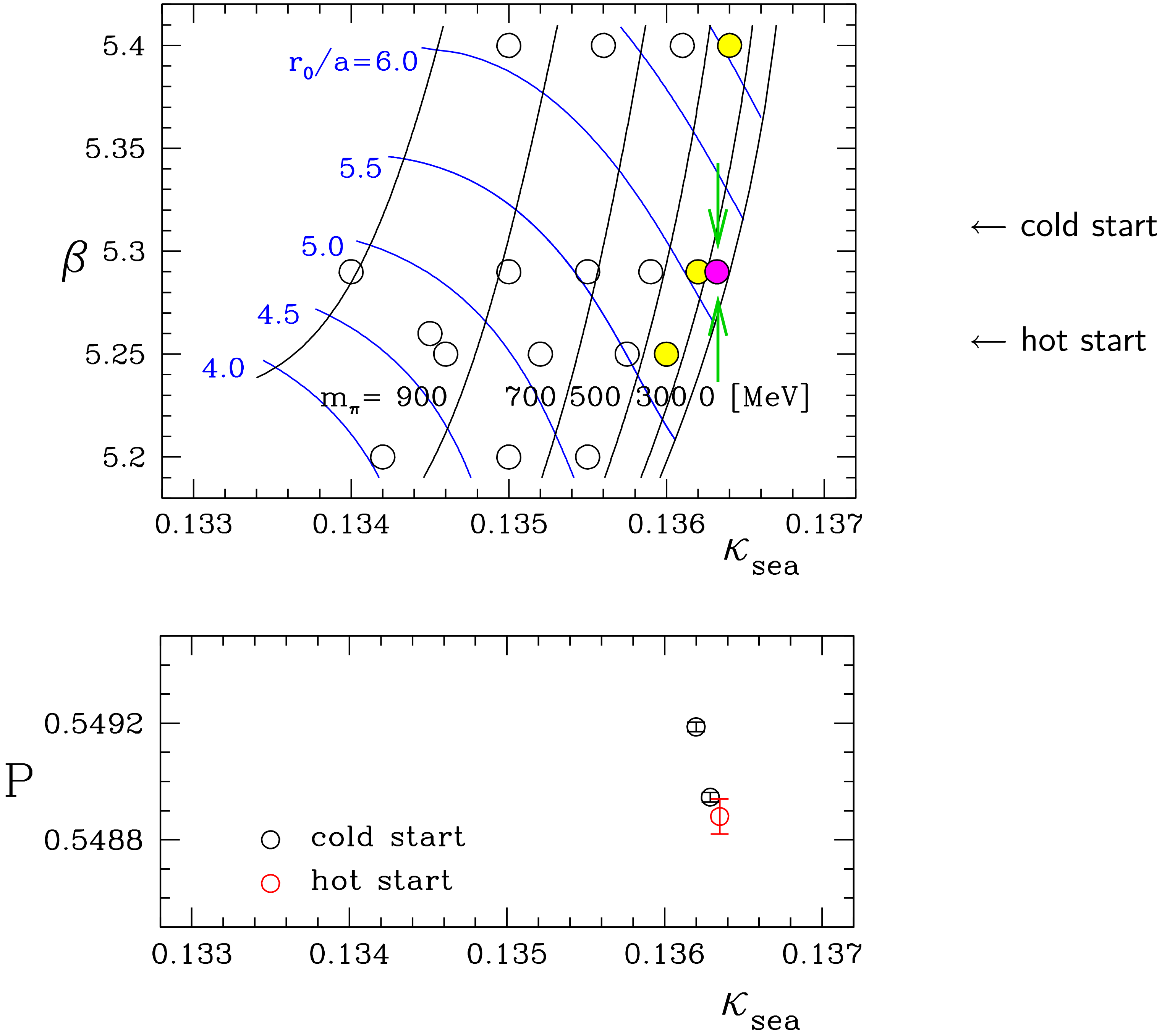
Obstructions ?



1^{st} order transition

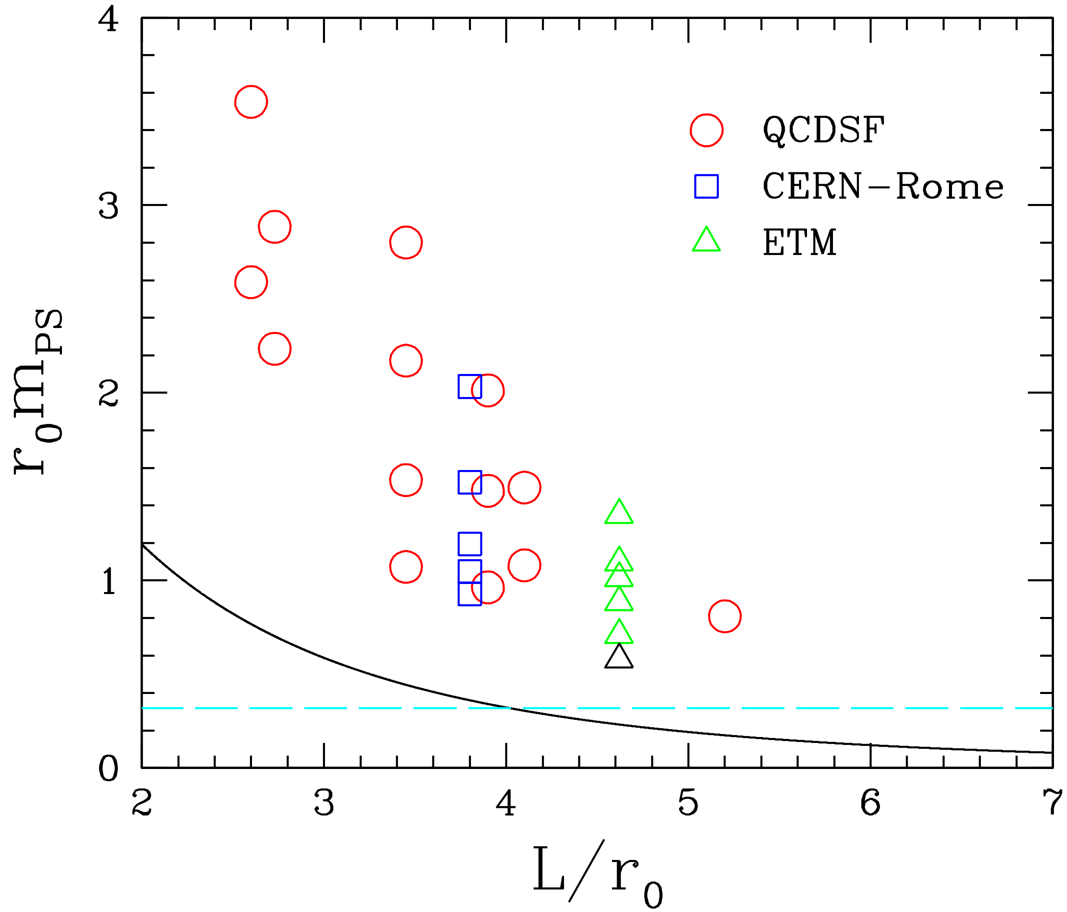


Aoki phase



r0/a=6.0
5.5
5.0
4.5
4.0
m_π= 900 700 500 300 0 [MeV]
β
κ_sea
← cold start
← hot start
P
cold start
hot start

Landscape

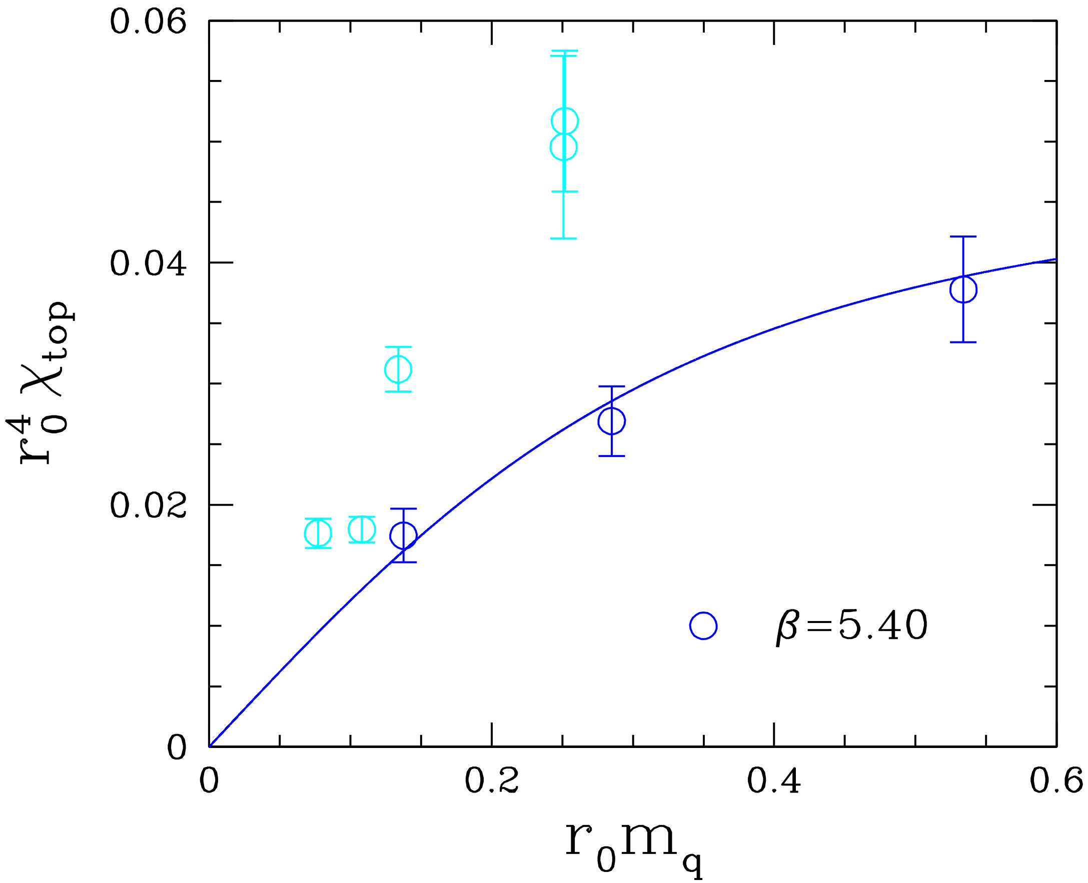


Minimal pion mass : $$m_\pi(L) = \frac{3}{2f_0^2L^3}\left(1+\frac{2}{4\pi f_0^2L^2}\,2.837\right)^{-1}$$

Leutwyler

Hasenfratz & Niedermayer

$$\chi_{\rm top} \equiv \frac{\langle Q^2 \rangle}{V} = \frac{\Sigma\, m_q}{2}$$

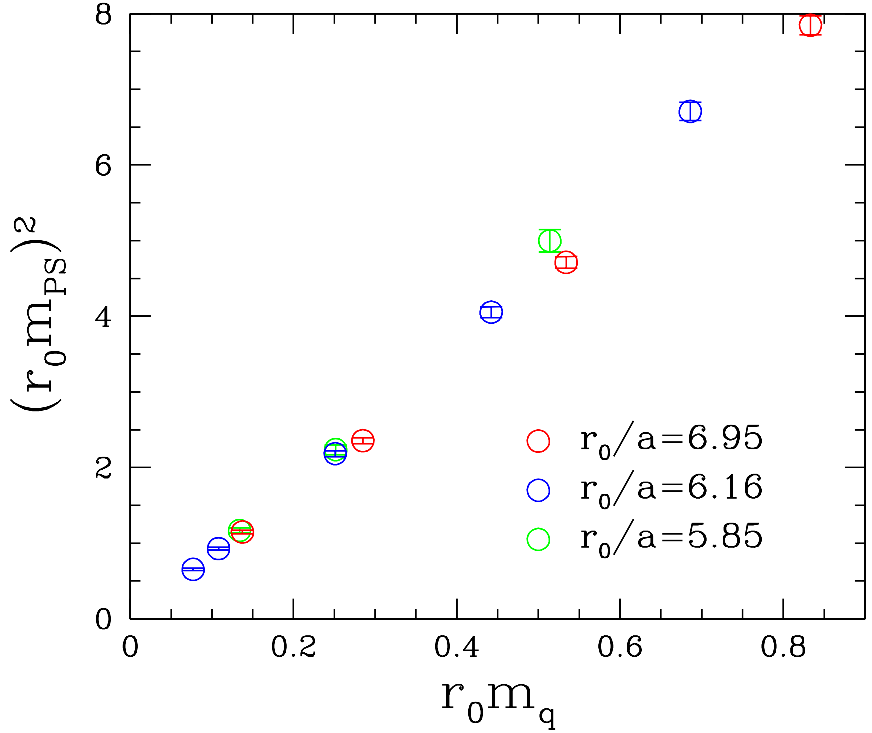


$$\left(\frac{1}{\chi_{\rm top}}\right)^2 = \left(\frac{2}{\Sigma\, m_q}\right)^2 + \left(\frac{1}{\chi_{\rm top}^\infty}\right)^2$$

Pion Sector

Pion Mass

Raw data



↑

NPRen

No 1st order phase transition or Aoki phase !

NLO

$$m_{PS}^2 = m_0^2 \left[1 + \frac{1}{2} x \, \hat{l}_3 + O(x^2) \right]$$

$$\frac{m_{PS} - m_{PS}(L)}{m_{PS}} = - \sum_{|\vec{n}| \neq 0} \frac{x}{2\lambda} \left[I^{(2)}_{m_{PS}}(\lambda) + x I^{(4)}_{m_{PS}}(\lambda) \right]$$

Colangelo, Dürr & Haefeli

$$m_0^2 = 2\Sigma m_q \,, \quad x = \frac{m_0^2}{16\pi^2 f_0^2} \,, \quad \lambda = m_{PS} |\vec{n}| L$$

$$\hat{l}_i = \ln \frac{\Lambda_i^2}{m_0^2}$$

$$I^{(2)}_{m_{PS}}(x) \;=\; -B^0(x)$$

$$I^{(4)}_{m_{PS}}(x) \;=\; \left(-\frac{55}{18} + 4\bar{l}_1 + \frac{8}{3}\bar{l}_2 - \frac{5}{2}\bar{l}_3 - 2\bar{l}_4\right) B^0(x)$$

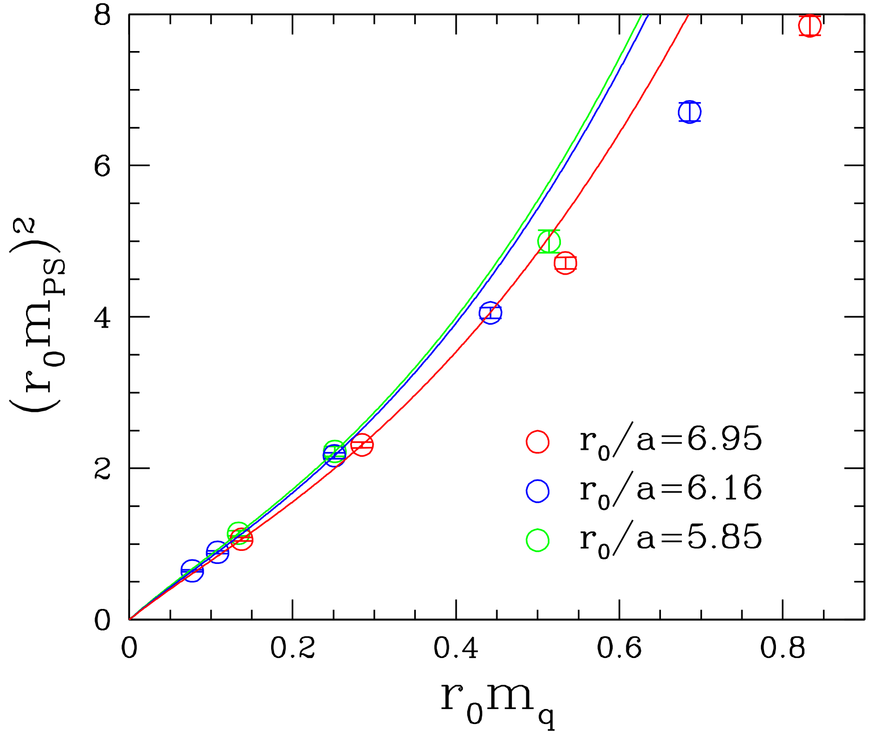
$$+ \;\left(\frac{112}{9} - \frac{8}{3}\bar{l}_1 - \frac{32}{3}\bar{l}_2\right) B^2(x) + S^{(4)}_{m_{PS}}(x)$$

$$S^{(4)}_{m_{PS}}(x) = \frac{13}{3} g_0 B^0(x) - \frac{1}{3}\left(40 g_0 + 32 g_1 + 26 g_2\right) B^2 + \cdots$$

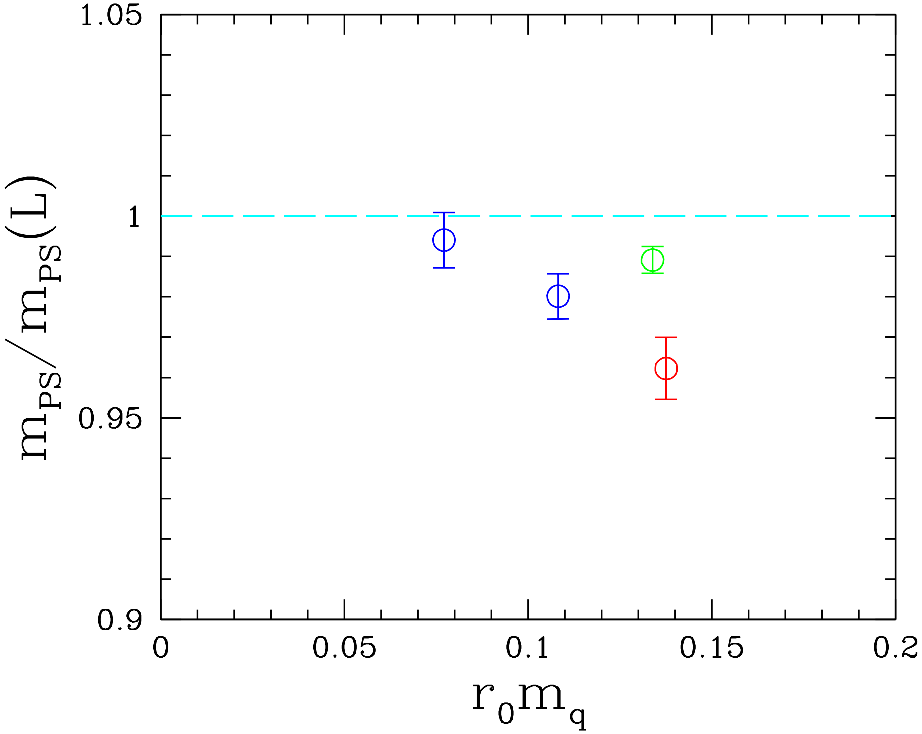
$$B^0(x) = 2K_1(x)\,, \quad B^2(x) = 2K_2(x)/x\,, \quad \bar{l}_i = \ln\frac{\Lambda_i^2}{m_{PS}^2}$$

$\Lambda_i\,,\;\; g_i$ from hep-lat/05030142

FS corrected



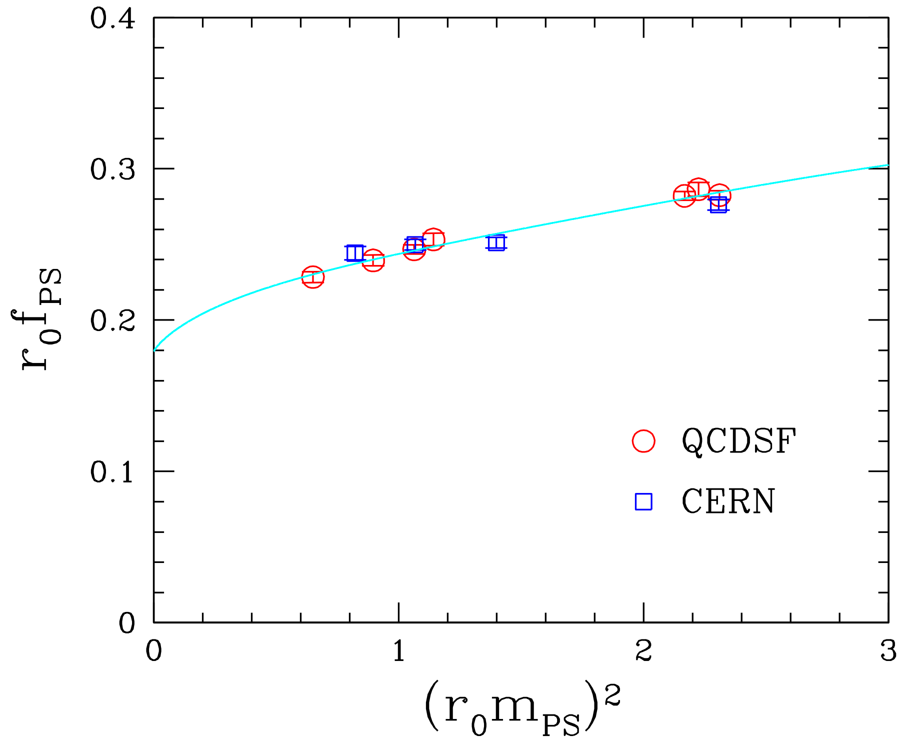
Corrections



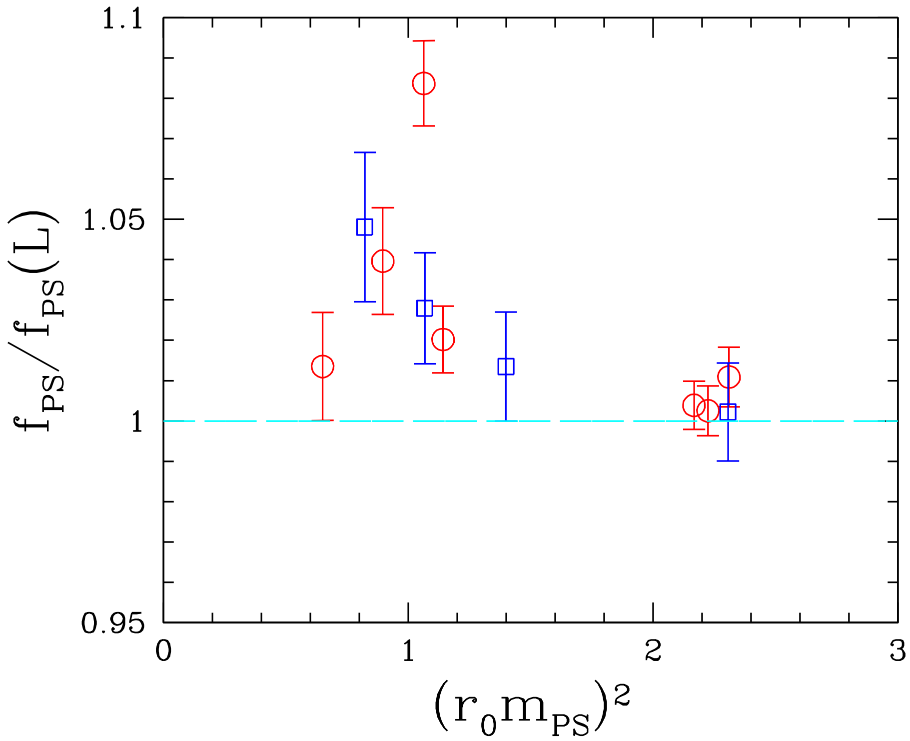
$r_0 f_0 = 0.179(2)\,, \quad r_0 \Lambda_3 = 1.82(7)$

Pion Decay Constant

FS corrected



Corrections



$$f_{PS} = f_0 \left[1 + x\,\hat{l}_4 + O(x^2)\right]$$

$$\frac{f_{PS} - f_{PS}(L)}{f_{PS}} = \sum_{|\vec{n}|\neq 0} \frac{x}{\lambda} \left[I^{(2)}_{f_{PS}}(\lambda) + x I^{(4)}_{f_{PS}}(\lambda)\right]$$

$r_0 f_0 = 0.179(2)$ $\quad r_0 \Lambda_4 = 3.32(6)$

$f_{PS} \leftarrow$ NPRen

$$I^{(2)}_{f_{PS}}(x) = -2B^0(x)$$

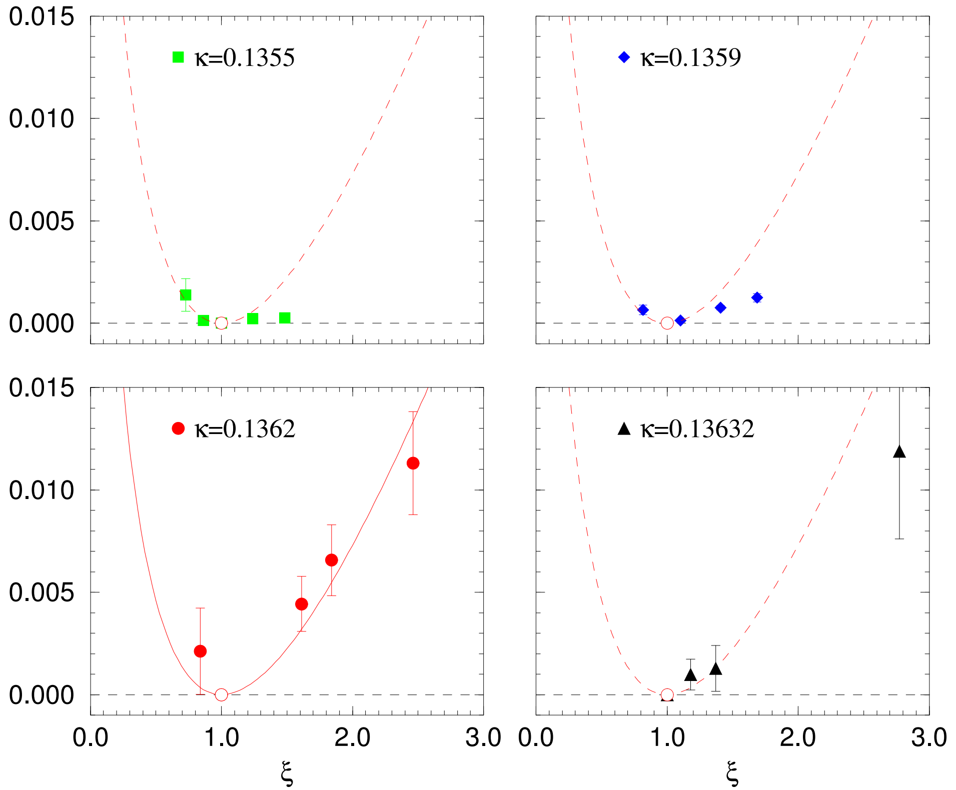
$$I^{(4)}_{f_{PS}}(x) = \left(-\frac{7}{9} + 2\bar{l}_1 + \frac{4}{3}\bar{l}_2 - 3\bar{l}_4\right) B^0(x) + \left(\frac{112}{9} - \frac{8}{3}\bar{l}_1 - \frac{32}{3}\bar{l}_2\right) B^2(x) + S^{(4)}_{f_{PS}}(x)$$

$$S^{(4)}_{f_{PS}}(x) = \frac{1}{6}\left(8g_0 - 13g_1\right) B^0(x) - \frac{1}{3}\left(40g_0 - 12g_1 - 8g_2 - 13g_3\right) B^2 + \cdots$$

Colangelo, Dürr & Haefeli

Partially Quenched

$$m_{PS} \equiv m_{PS}^{SS} \to m_{PS}^{AB}\,, \quad f_{PS} \equiv f_{PS}^{SS} \to f_{PS}^{AB}\,, \qquad A,B \in \{V,S|V \neq S\}$$



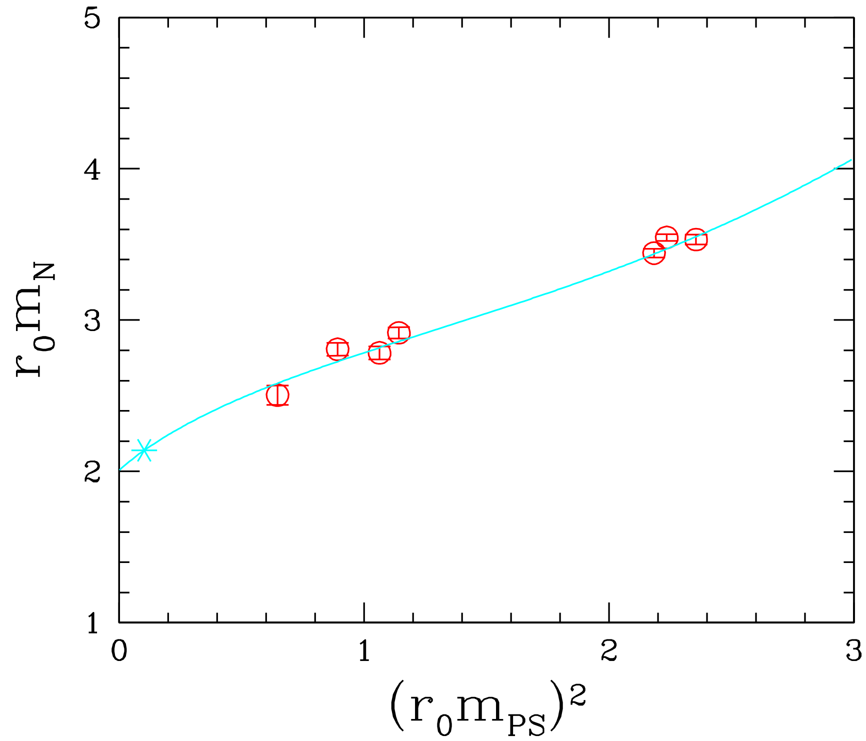
$$\hat{R} \equiv \frac{R}{m_{PS}^{SS\;2}} = \frac{f_{PS}^{VS}}{m_{PS}^{SS\;2}\sqrt{f_{PS}^{VV} f_{PS}^{SS}}} = -\frac{1}{8(4\pi r_0 f_0)^2}\left(\ln\frac{m_{PS}^{VV\;2}}{m_{PS}^{SS\;2}} - \frac{m_{PS}^{VV\;2}}{m_{PS}^{SS\;2}} + 1\right)\,, \quad \xi = \frac{m_{PS}^{VV\;2}}{m_{PS}^{SS\;2}}$$

Sharpe

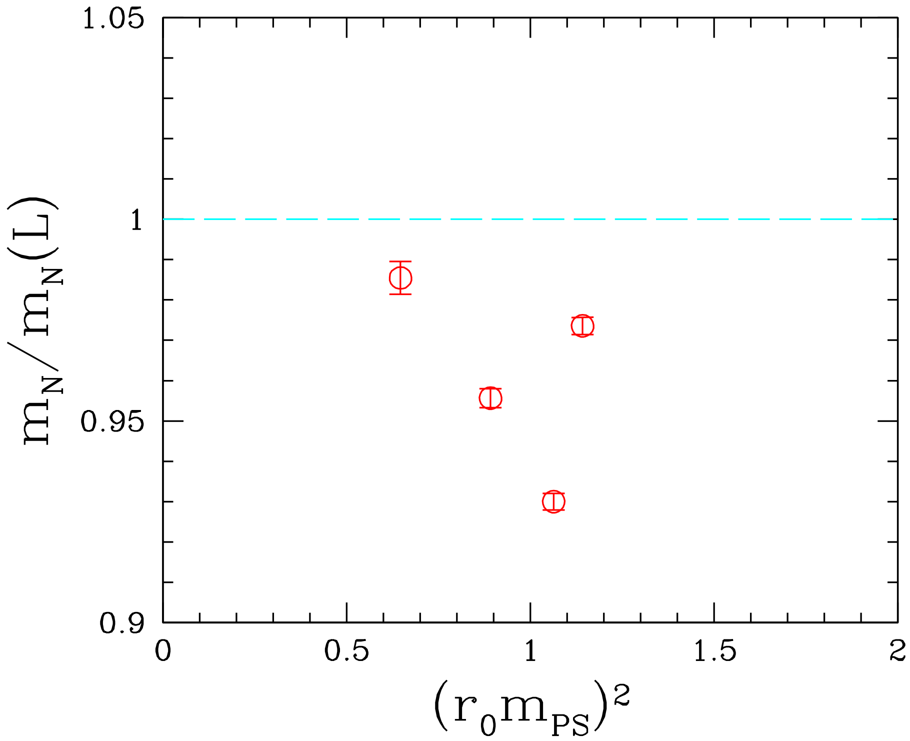
Nucleon Sector

Nucleon Mass

FS corrected



Corrections



$r_0 f_0 = 0.179(2)$ $\quad g_A^0 = 1.15$

$r_0 = 0.45(3)$ fm

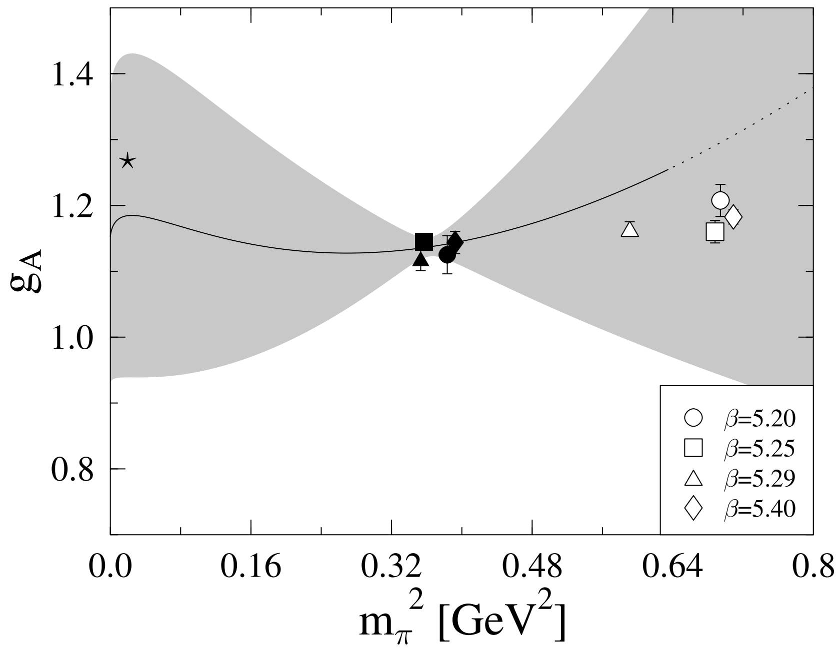
$r_0 m_0 = 2.00$

$c_1/r_0 = -0.43$

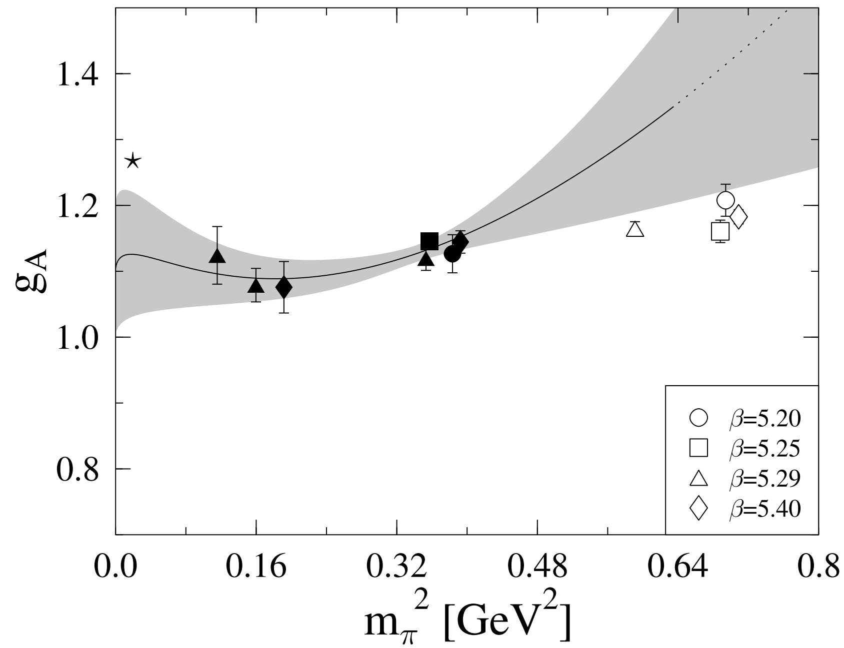
$$m_N = m_0 - 4c_1 m_{PS}^2 - \frac{3g_A^{0\,2}}{32\pi f_0^2} m_{PS}^3 + \Bigg[e_1(\mu) - \frac{3}{64\pi^2 f_0^2}\Big(\frac{g_A^{0\,2}}{m_0} - \frac{c_2}{2}\Big) - \frac{3g_A^{0\,2}}{32\pi^2 f_0^2}\Big(\frac{g_A^{0\,2}}{m_0} - 8c_1 + c_2 + 4c_3\Big)\ln\frac{m_{PS}}{\mu}\Bigg] m_{PS}^4 + \frac{3g_A^{0\,2}}{256\pi f_0^2 m_0^2} m_{PS}^5 + O(m_{PS}^6)$$

$$m_N - m_N(L) = -\frac{3g_A^{0\,2} m_0 m_{PS}^2}{16\pi^2 f_0^2} \sum_{|\vec{n}|\neq 0} \int_0^\infty dz K_0\left(\sqrt{m_0^2 z^2 + m_{PS}^2(1-z)}|\vec{n}|L\right) - \frac{3m_{PS}^4}{4\pi^2 f_0^2} \sum_{|\vec{n}|\neq 0} \left[(2c_1 - c_3)\frac{K_1(m_{PS}|\vec{n}|L)}{m_{PS}|\vec{n}|L} + c_2 \frac{K_2(m_{PS}|\vec{n}|L)}{(m_{PS}|\vec{n}|L)^2}\right] + O(m_{PS}^5)$$

Axial Coupling



Preliminary ↓

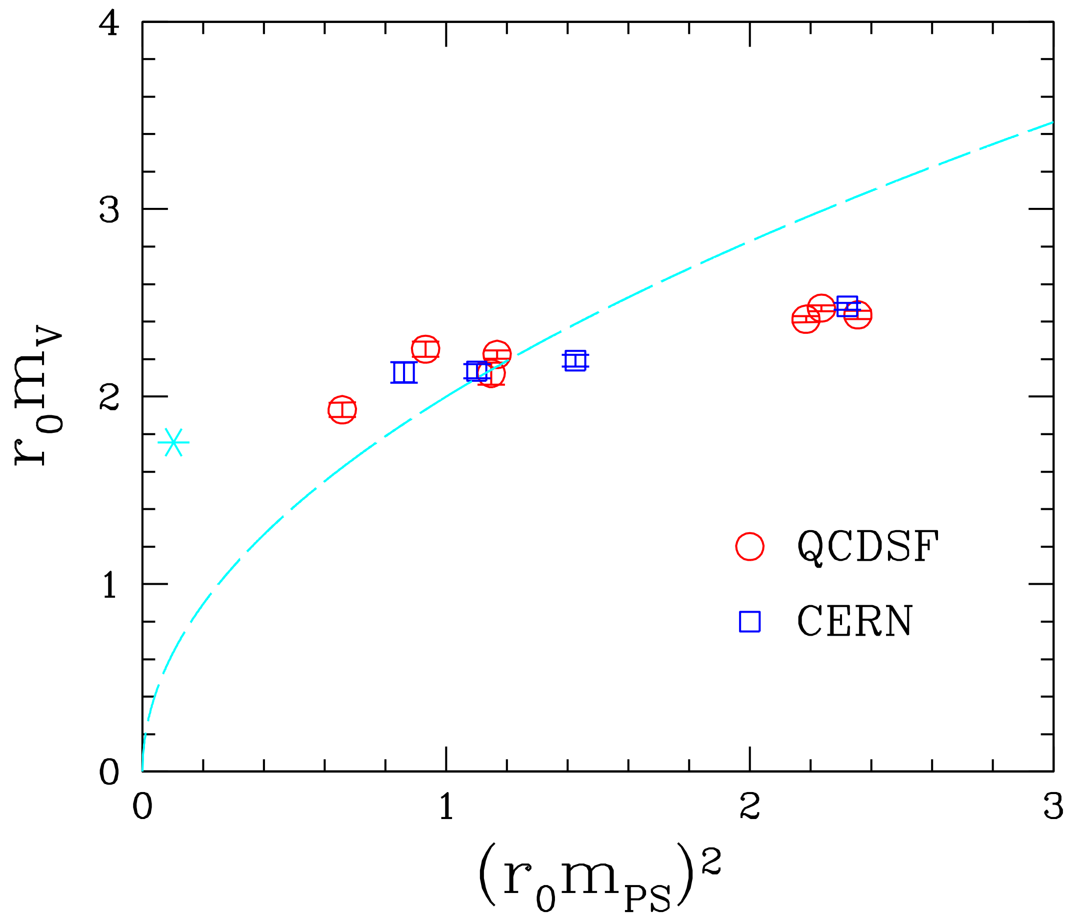


χPT $O(p^3)$

68.3% CL

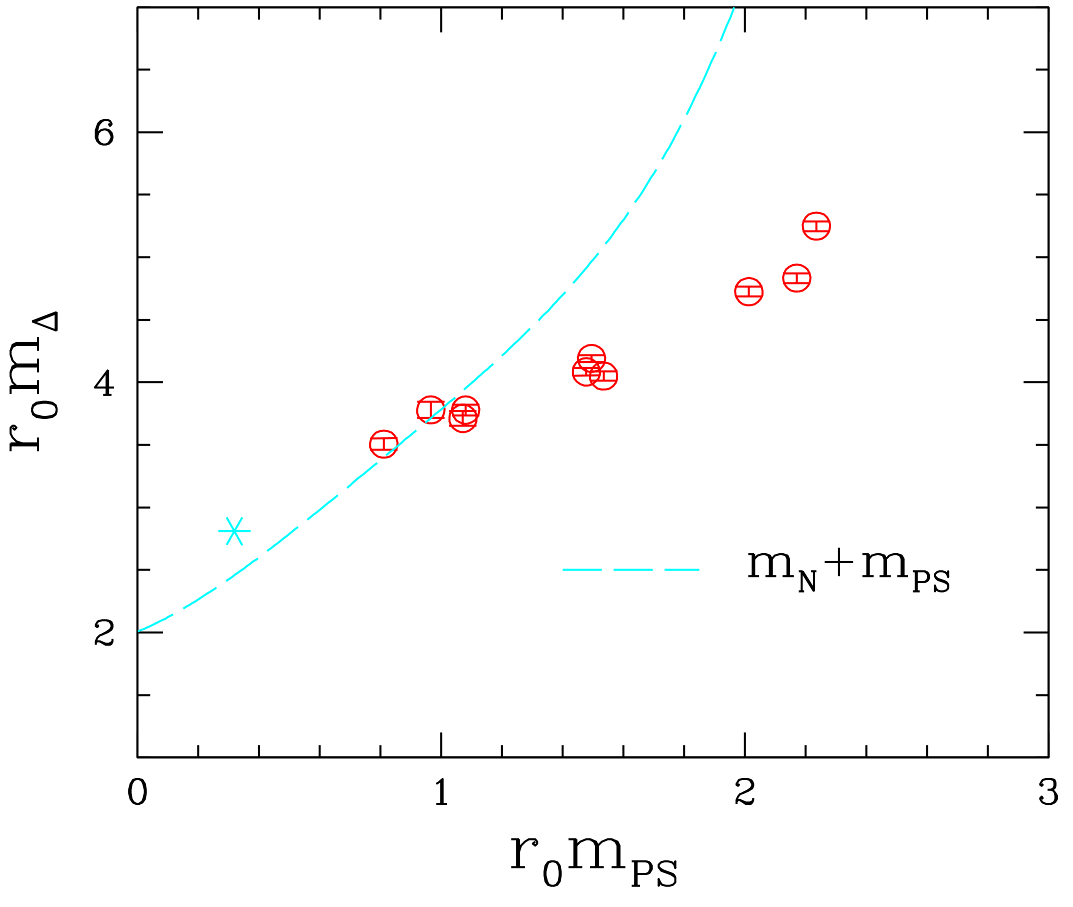
Miscellaneous

Rho Mass



Not FS corrected

Delta Mass



Not FS corrected

Conclusions & Outlook

- Simulations at pion masses of $O(300)$ MeV with Wilson-type fermions feasible now
  - Improvement of algorithms
  - Increase of computing power

- Extrapolation to chiral limit and infinite volume greatly improved

  FS corrections surprisingly well described by ChPT

- First meaningful lattice determination of low energy constants : Preliminary !

r_0	f_0	Λ_3	Λ_4
0.45(3) fm	79(5) MeV	0.80(5) GeV	1.46(10) GeV

- Major investment in FS corrections (including partially quenched data) and δ expansion needed