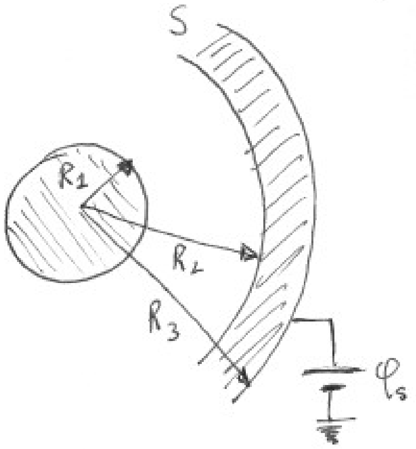
Risoluzione prova di esonero Fisica Generale II

Ing. dell'automazione + corso b Informatica - 7 maggio 2008

PROBLEMA 1



$R_1 = 2$ cm

$R_2 = 6$ cm

$R_3 = 9$ cm

$\varphi_{S'} = 100$ V

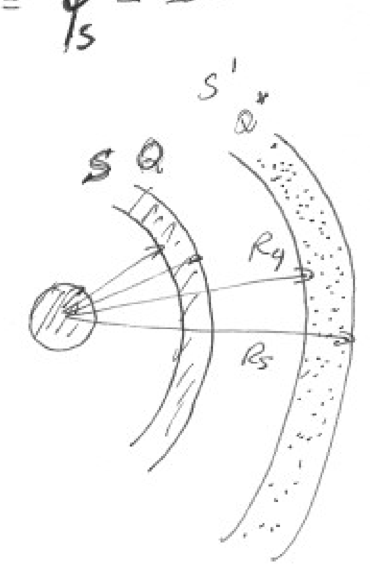
a) Sulla superficie di raggio R_3 avrò una carica Q tale che

$$\varphi_S = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R_3} \Rightarrow Q = 4\pi\varepsilon_0 \varphi_S R_3 = 1 \text{ nC}$$

$$\vec{E}(r) = \begin{cases} 0 & r < R_3 \\ \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} & r \geq R_3 \end{cases}$$

b) $\varphi_{sfera} = \varphi_S = 100$ V

c)



$R_4 = 12$ cm ; $R_5 = 14$ cm

$Q^* = 2$ pC

$$\rho = \frac{Q^*}{V} = \frac{Q^*}{\frac{4}{3}\pi(R_5^3 - R_4^3)}$$

$$\rho = 4.7 \cdot 10^{-10} \text{ C/m}^3$$

se $r > R_5$ $\quad \vec{E}(r) = \frac{1}{4\pi\varepsilon_0} \frac{Q+Q^*}{r^2} \hat{u}_r$

Se $R_3 \leq r \leq R_4$ $\quad \vec{E}(r) = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} \hat{u}_r$

se $R_4 < r < R_5$ $\quad E(r) = \frac{1}{4\pi\varepsilon_0} \frac{1}{r^2} Q_{INT} \hat{u}_r$ dove $Q_{INT} = \dots$

$$Q_{INT} = Q + \frac{4}{3}\pi\left(r^3 - R_4^3\right) \cdot \frac{Q^*}{\frac{4}{3}\pi\left(R_5^3 - R_4^3\right)} = Q + Q^* \frac{\left(r^3 - R_4^3\right)}{\left(R_5^3 - R_4^3\right)}$$

$$Q_{INT} = Q + \frac{\left(\frac{r}{R_4}\right)^3 - 1}{\left(\frac{R_5}{R_4}\right)^3 - 1} Q^*$$

$$\vec{E}(r) = \begin{cases} 0 & r < R_3 \\ \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} \hat{u}_r & R_3 \leq r \leq R_4 \\ \left[\frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} + \frac{1}{4\pi\varepsilon_0} \frac{Q^*}{\left[\left(\frac{R_5}{R_4}\right)^3 - 1\right]} \frac{\left(\frac{r}{R_4}\right)^3 - 1}{r^2}\right] \hat{u}_r & R_4 < r < R_5 \\ \frac{1}{4\pi\varepsilon_0} \frac{Q+Q^*}{r^2} \hat{u}_r & r > R_5 \end{cases}$$

N.B. $$\frac{1}{4\pi\varepsilon_0} \frac{Q^*}{\left[\left(\frac{R_5}{R_4}\right)^3 - 1\right]} \cdot \frac{\left(\frac{r}{R_4}\right)^3 - 1}{r^2} = \frac{1}{\frac{3 \cdot 4\pi\varepsilon_0}{3}} \frac{Q^*}{\frac{\cancel{1}}{R_4^3}\left(R_5^3 - R_4\right)^3} \frac{\frac{1}{R_4^3}\left(r^3 - R_4^3\right)}{r^2}$$

$$= \frac{\rho}{3\varepsilon_0}\left(r - \frac{R_4^3}{r^2}\right)$$

$$\varphi_S - \varphi_\infty = \int_S^\infty E\,dr = \underbrace{\int_{R_3}^{R_4} E\,dr}_{(1)} + \underbrace{\int_{R_4}^{R_5} E\,dr}_{(2)} + \underbrace{\int_{R_5}^{\infty} E\,dr}_{(3)}$$

$$\int_{R_3}^{R_4} E\,dr = \int_{R_3}^{R_4} \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\,dr = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R_3} - \frac{1}{4\pi\varepsilon_0}\frac{Q}{R_4} \qquad (1)$$

$$\int_{R_4}^{R_5} E\,dr = \int_{R_4}^{R_5} \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\,dr + \int_{R_4}^{R_5} \frac{\rho}{3\varepsilon_0}\left(r - \frac{R_4^3}{r^2}\right)dr \qquad (2)$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Q}{R_4} - \frac{1}{4\pi\varepsilon_0}\frac{Q}{R_5} + \frac{\rho}{3\varepsilon_0}\left[\frac{r^2}{2} + \frac{R_4^3}{r}\right]_{R_4}^{R_5} =$$

$$= \frac{1}{4\pi\varepsilon_0} Q\left(\frac{1}{R_4} - \frac{1}{R_5}\right) + \frac{\rho}{3\varepsilon_0}\left(\frac{R_5^2}{2} + \frac{R_4^3}{R_5} - \frac{R_4^2}{2} - R_4^2\right) =$$

$$= \frac{1}{4\pi\varepsilon_0} Q\left(\frac{1}{R_4} - \frac{1}{R_5}\right) + \frac{\rho}{3\varepsilon_0}\left(\frac{R_5^2}{2} + \frac{R_4^3}{R_5} - \frac{3}{2}R_4^2\right)$$

Infine $\displaystyle\int_{R_5}^{\infty} E\,dr = \int_{R_5}^{\infty} \frac{1}{4\pi\varepsilon_0}\frac{Q+Q^*}{r^2}\,dr = \qquad (3)$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Q}{R_5} + \frac{1}{4\pi\varepsilon_0}\frac{Q^*}{R_5}$$

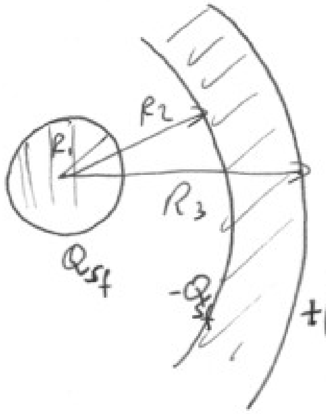
Mettendo insieme i tre pezzi

$$\varphi_S = \frac{1}{4\pi\varepsilon_0} Q\left(\frac{1}{R_3} - \cancel{\frac{1}{R_4}} + \cancel{\frac{1}{R_4}} - \cancel{\frac{1}{R_5}} + \cancel{\frac{1}{R_5}}\right) + \frac{\rho}{3\varepsilon_0}\left(\frac{R_5^2}{2} + \frac{R_4^3}{R_5} - \frac{3}{2}R_4^2\right)$$
$$+ \frac{1}{4\pi\varepsilon_0}\frac{Q^*}{R_5}$$

$$\varphi_S = \underbrace{\frac{1}{4\pi\varepsilon_0}\frac{Q}{R_3}}_{100\ V} + \underbrace{\frac{\rho}{3\varepsilon_0}\left(\frac{R_5^2}{2} + \frac{R_4^3}{R_5} - \frac{3}{2}R_4^2\right)}_{\sim 0.01\ V} + \underbrace{\frac{1}{4\pi\varepsilon_0}\frac{Q^*}{R_5}}_{0.13\ V} = \underline{100.14\ V}$$

$\rho = 4.7 \cdot 10^{-10}\ \frac{C}{m^3}$

d) Elimino il guscio isolante S' e carico la sfera metallica



$R_1 = 2$ cm

$R_2 = 6$ cm

$R_3 = 9$ cm

(1 μC era già presente in precedenza)

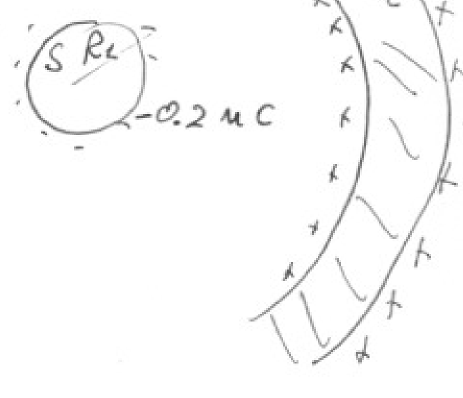
$$\varphi_P(R_P = 8\text{ cm}) = \varphi(R_2) = \varphi(R_3)$$

$$\varphi(R_3) = \frac{1}{4\pi\varepsilon_0}\frac{Q_{TOT}}{R_3} = 80\,V \Rightarrow$$

$$\Rightarrow Q_{TOT} = (80\,V)\cdot 4\pi\varepsilon_0 R_3 = 8.0\cdot 10^{-10}\,C = \underline{0.8\,\mu C}$$

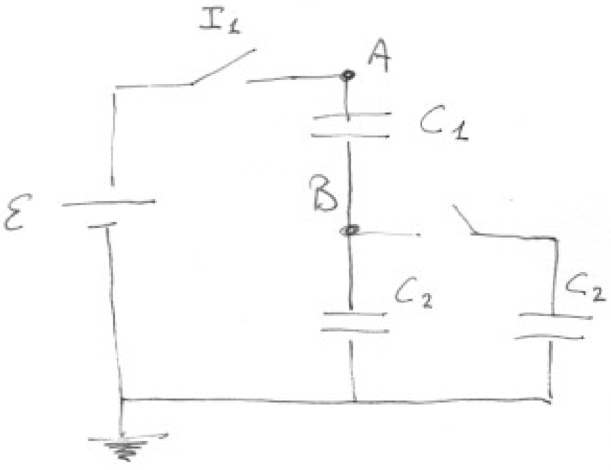
Quindi $Q_{sf} + 1\,\mu C = 0.8\,\mu C \Rightarrow Q_{sf} = -0.2\,\mu C$

allora la situazione è la seguente



$+0.8\,\mu C$ $(1\,\mu C - 0.2\,\mu C)$

PROBLEMA 2



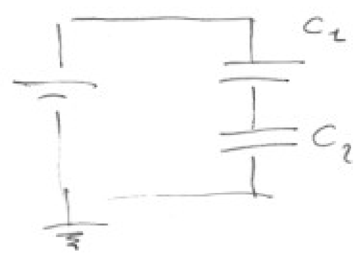
C_1 condensatore cilindrico
$(R_1 = 1\ cm\ ;\ R_2 = 2\ cm\ ;$
$L = 30\ cm)$

$C_2 = 8.85\ pF$

$\mathcal{E} = 100\ V$

a) $$C_1 = 2\pi\varepsilon_0 \frac{L}{\ln\frac{R_2}{R_1}} = 2.41\cdot 10^{-11}\ F = 24.1\ pF$$

b)



$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = \frac{24.1\ pF \cdot 8.85\ \cancel{pF}}{(24.1 + 8.85)\ \cancel{pF}} =$$

$$C_{eq} = 6.47\ pF$$

$$q = C_{eq}\cdot \mathcal{E} = 6.47\ pF \cdot 100\ V = 647\ pC$$

$\varphi_A = 100\ V$

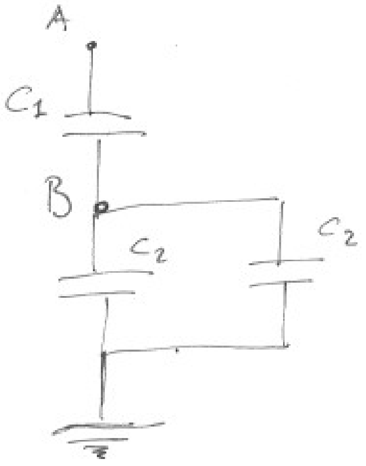
$$\varphi_{C_1} = \frac{q}{C_1} = \frac{647\ pC}{24.1\ pF} = 26.85\ V$$

$$\varphi_{C_2} = \mathcal{E} - \varphi_{C_1} = 73.15\ V$$

$\varphi_B = 73.15\ V$

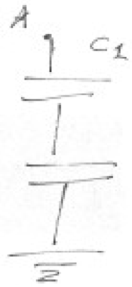
Successivamente si apre I_1 e si chiude I_2

c)



$q_1 = 647\ pC \Rightarrow \varphi_{C_1} = 26.85\ V$

$\Longrightarrow$



$C'_{eq} = C_2 + C_2 = 2C_2$

la carica presente su C'_{eq} è sempre $647\ pC \Rightarrow$

$$\Rightarrow \varphi_{C'_{eq}} = \frac{q}{C'_{eq}} = \frac{647\ pC}{2 \cdot 8.85\ pF} = 36.55\ V$$

$$\Rightarrow \underline{\varphi_B} = \underline{\underline{36.55\ V}}$$

$$\underline{\underline{\varphi_A}} = (36.55\ V + 26.85\ V) = \underline{\underline{63.4\ V}}$$

d) $C''_{eq} = (\text{Serie di } C_1 \text{ e } C'_{eq}) = \frac{C_1 C'_{eq}}{C_1 + C'_{eq}} = \frac{2C_1C_2}{C_1 + 2C_2} =$

$$= \frac{2 \cdot 8.85\ pF \cdot 24.1\ pF}{41.8\ pF} = 10.2\ pF$$

$$U = \frac{1}{2} \frac{q^2}{C''_{eq}} = \frac{1}{2} \frac{(647\ pC)^2}{10.2\ pF} = 2.05 \cdot 10^{-8}\ J =$$

$$= 20.5\ nJ$$