

## Constraining $\nu$ SMEFT coefficients: The case of the extra $U(1)'$

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We study the constraints on low-energy coefficients of the  $\nu$ SMEFT generalization of the Standard Model effective theory in the simple case of a  $U(1)'$  enlargement of the Standard Model gauge group. In particular, we analyze the constraints imposed by the requirement that the extended theory remains free of gauge anomalies. We present the cases of explicit realizations, showing the obtained correlations among the coefficients of  $d = 6$  operators.

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### I. INTRODUCTION

The search for physics beyond the Standard Model (SM) is justified by several motivations. There are conceptual issues and cosmological observations suggesting the existence of a more fundamental theory beyond SM. Tensions between SM predictions and experimental results, in particular in the flavor sector, reinforce such a widespread conviction. However, direct searches at colliders have not produced evidence of new particles and/or mediators of new interactions yet; hence the alternative way to gain evidence of physics beyond the Standard Model (BSM) is investigating virtual effects of possible new heavy degrees of freedom, as done in flavor physics [1].

In this framework, two approaches can be followed towards BSM. The first one consists in formulating a specific extended theory and deriving predictions to be contrasted with experiment for a validation or a discrimination with respect to different new physics (NP) scenarios. The second approach consists in extending the SM at the electroweak (EW) scale in the most general way compatible with the SM gauge symmetry, investigating the constraints imposed by the experiments on the resulting generalization.

A remarkable example of the second approach is the Standard Model effective field theory (SMEFT) [2–5], widely used in the quest for BSM physics. The SM is

considered as an effective field theory describing physics at and below the EW scale. At higher scales a new gauge theory (the UV completion) should exist, with a gauge group extending the SM one and undergoing spontaneous symmetry breaking (SSB) to it. If  $\Lambda$  is the NP scale, then at the EW scale the SMEFT Lagrangian consists of an expansion in the parameter  $1/\Lambda$ . The first term of the expansion is the SM Lagrangian density containing operators of canonical dimension up to  $d = 4$ . Subsequent terms are suppressed by powers of  $1/\Lambda$  and comprise operators of increasing dimension:

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}}^{(4)} + \mathcal{L}^{(5)} + \mathcal{L}^{(6)} + \dots \quad (1)$$

The apex ( $d$ ) indicates the canonical dimension of the operators entering in each term  $\mathcal{L}^{(d)}$  written as

$$\mathcal{L}^{(d)} = \sum_i \frac{C_i}{\Lambda^{d-4}} \mathcal{O}_i^{(d)}, \quad (2)$$

with dimensionless Wilson coefficients  $C_i$ . The operators are constructed in terms of the SM fields and satisfy the SM gauge symmetry. SM accidental symmetries are allowed to be violated: for example, baryon and lepton number violating operators are included in (2), namely odd-dimension operators violating  $B$  and/or  $L$  conservation [6]. The operators contain no reference to the field content of the UV theory. However, their coefficients depend of the details of such a theory, i.e., the couplings and masses of the new particles that, supposed to be  $M \simeq \mathcal{O}(\Lambda)$ , are integrated out in the EFT Lagrangian at the EW scale. A few assumptions concern the UV theory. It should contain only particles with spin  $J \leq 1$ ; new vector fields could be either gauge fields (massless before SSB in the UV theory) or massive Proca fields; new fermions can be introduced provided that they are vectorlike with respect to the SM gauge group, to

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maintain the SM free of gauge anomalies. Even though in the construction of the SMEFT operators the latter requirement is taken into account, in more general frameworks it can be relaxed, provided that together with new fermions (not necessarily vectorlike), other contributions are added that maintain the SM gauge group anomaly free [7].

One can use the construction in two ways. Choosing the UV completion, the Wilson coefficients of the SMEFT operators can be determined through matching and running procedure [8–10]. On the other hand, without assumptions on the UV completion, the coefficients are treated as parameters. These two steps are complementary to each other. Having gained model independent information on the coefficients in the effective theory, it is possible to contrast them with the features required in a specific scenario in order to validate or discard it.

The phenomenological evidence that neutrinos have nonvanishing mass induces to consider the  $\nu$ SMEFT extension of SMEFT, which comprises three right-handed sterile neutrino fields in the sub-TeV mass range [11–18]. The inclusion does not invalidate the requirement that the SM gauge group is free of gauge anomalies. In the extension,  $\mathcal{L}^{(5)}$  consists of three operators, while only the Weinberg operator appears at this order in the absence of  $\nu_R$  [19]. The choice of the  $d = 6$  operators is not unique, and different bases have been proposed, i.e., complete sets of independent, nonredundant operators.<sup>1</sup> A popular basis is the Warsaw one [3]. In each basis the operators are collected in classes according to their field content.

In our study we focus on the UV completion represented by the simplest extension of the SM gauge group comprising a new  $U(1)'$  gauge group, featured by the gauge coupling  $g_Z$  [20–23].  $Z'$  is the corresponding gauge field and the  $z$  hypercharge is the quantum number associated to the new symmetry. Many NP models introduce such a mediator with specific  $z$ -hypercharge assignments. Experimental searches for  $Z'$  rely on the assumptions for the hypercharges, and produce exclusion plots in the plane of the  $Z'$  production cross section versus  $M_{Z'}$ .<sup>2</sup> The NP scale can be identified with  $M_{Z'}$  acquired after spontaneous breaking of the new symmetry. We do not need to specify how such SSB occurs, we only assume that it happens at a much higher scale than the SM Higgs vacuum expectation value. We neglect the mixing with other neutral gauge bosons.<sup>3</sup>

In the chosen extension we work out the coefficients of the  $\nu$ SMEFT operators of dimension up to  $d = 6$ , aiming at

<sup>1</sup>Sources of redundancies are, e.g., operators obtained one from the other after integration by parts and discarding a total derivative; operators that can be discarded using equations of motion; equivalent operators upon Fierz transformations (in the case of four-fermion operators).

<sup>2</sup>See, e.g., the review: B.A. Dobrescu and S. Willocq, “ $Z'$ -boson searches,” in [24].

<sup>3</sup>Mixing at tree-level vanishes in models where the SM Higgs is assumed to be singlet under  $U(1)'$ .

the relations among them.<sup>4</sup> While the gauge structure of the theory already imposes nontrivial relations among various coefficients, further relations can be established requiring that the extended gauge group is anomaly free. We obtain results holding for a generic  $U(1)'$  extension. We also consider specific cases: universal  $Z'$  couplings to the three generations or only to the third generation;  $Z'$  only coupled to left- or right-handed fermions; lepto- or hadrophobic  $Z'$ ; the  $z$ -hypercharge assignment of the ABCD model [26]. In all cases, we find that the number of independent coefficients is reduced and remarkable correlations can be established among them, which are peculiar of each extension. The experimental test of such correlations would shed light on the particular completion, providing the widest information using measurements.

The plan of the paper is as follows. After Sec. II with the notations, in Sec. III we list the  $\nu$ SMEFT operators generated at the EW scale when the UV theory contains the new gauge boson  $Z'$ . The impact of the new gauge boson on the SMEFT Lagrangian density is considered in Sec. IV, with the list of the operators obtained when the  $Z'$  field is integrated out, the expressions of their Wilson coefficients and the relations due to the gauge structure of the extension. In Sec. V we consider the relations that the fermion  $z$  hypercharges must satisfy to fulfil the requirement of gauge anomaly cancellation in the SM gauge group extension, and how such relations can be translated into analogous ones among the SMEFT coefficients. We then discuss the results for the selected  $z$ -hypercharge assignments. The last section comprises the conclusions.

## II. NOTATIONS, $Z'$ COUPLINGS TO FERMIONS AND TO THE HIGGS FIELD

The most general renormalizable UV Lagrangian terms involving the gauge boson  $Z'$  of a new  $U(1)'$  group can be written as

$$\mathcal{L}^{Z'} = \mathcal{L}_{\text{free}}^{Z'} + \mathcal{L}_{\text{int,fermions}}^{Z'} + \mathcal{L}_{\varphi}^{Z'}. \quad (3)$$

The first term in (3) reads

$$\mathcal{L}_{\text{free}}^{Z'} = -\frac{1}{4}Z'_{\mu\nu}Z'^{\mu\nu} + \frac{1}{2}M_{Z'}^2Z'_\mu Z'^\mu. \quad (4)$$

$Z'_\mu$  is the gauge boson field and  $Z'_{\mu\nu} = \partial_\mu Z'_\nu - \partial_\nu Z'_\mu$  is the field strength tensor.

The second term in (3) describes the  $Z'$  coupling to fermions. We denote by  $q_L^i$  and  $\ell_L^i$  the  $SU(2)_L$  left-handed quark and lepton doublets, respectively, with generation index  $i = \{1, 2, 3\}$ .  $u_R^i, d_R^i, \nu_R^i, e_R^i$  are right-handed singlets.

Before the electroweak SSB the  $Z'$  couplings to fermions are flavor conserving, hence we can write

<sup>4</sup>In the same framework, relations among the coefficients of  $d = 6$  and  $d = 8$  operators have been worked out in [25].

$$\mathcal{L}_{\text{int,fermions}}^{Z'} = \sum_{\psi} \mathcal{L}_{\text{int}}^{Z'}, \quad (5)$$

where the sum extends over all the fermions generically denoted by  $\psi$  and

$$\mathcal{L}_{\text{int}}^{Z'} = g_Z z_{\psi} \bar{\psi} \gamma^{\mu} \psi Z'_{\mu}. \quad (6)$$

$g_Z$  is the  $U(1)'$  gauge coupling, and  $z_{\psi}$  the  $z$  hypercharge of the fermion  $\psi$ , i.e., the fermion quantum number related to the new symmetry group. In SM the fermions are chiral, hence it is useful to write (6) in terms of the left- and right-handed fermion fields  $\psi_{L(R)}$ :

$$\mathcal{L}_{\text{int}}^{Z'} = [(\Delta_L^{\psi})^{ij} \bar{\psi}_L^i \gamma^{\mu} \psi_L^j + (\Delta_R^{\psi})^{ij} \bar{\psi}_R^i \gamma^{\mu} \psi_R^j] Z'_{\mu}, \quad (7)$$

with

$$(\Delta_{L,R}^{\psi})^{ij} = g_Z z_{\psi_{L,R}} \delta^{ij}. \quad (8)$$

The  $Z'$  coupling to the SM Higgs field  $\varphi$  is described by  $\mathcal{L}_{\varphi}^{Z'}$ . We write the covariant derivative acting on the SM Higgs field as  $\bar{D}_{\mu} = D_{\mu} + i g_Z z_H Z'_{\mu}$ .  $D_{\mu}$  contains only the SM gauge fields and  $z_H$  is the Higgs  $z$  hypercharge. Therefore, we have

$$(\bar{D}_{\mu} \varphi)^{\dagger} (\bar{D}^{\mu} \varphi) = (D_{\mu} \varphi)^{\dagger} (D^{\mu} \varphi) + g_H (\varphi^{\dagger} i \overleftrightarrow{D}_{\mu} \varphi) Z'^{\mu} + g_H^2 Z'_{\mu} Z'^{\mu} (\varphi^{\dagger} \varphi), \quad (9)$$

where  $\varphi^{\dagger} i \overleftrightarrow{D}_{\mu} \varphi = \varphi^{\dagger} (i D_{\mu} \varphi) - (i D_{\mu} \varphi^{\dagger}) \varphi$  and we have defined

$$g_H = g_Z z_H. \quad (10)$$

The last term in (9) can be neglected in the present study since we are interested in  $d = 6$  operators arising at the EW scale when  $Z'$  is integrated out. The last term in (9) produces a  $d = 8$  operator, its coefficient would be inversely proportional to  $M_{Z'}^4$ . Therefore, in (3) we only include

$$\mathcal{L}_{\varphi}^{Z'} = g_H (\varphi^{\dagger} i \overleftrightarrow{D}_{\mu} \varphi) Z'^{\mu}. \quad (11)$$

### III. $\nu$ SMEFT OPERATORS GENERATED IN THE $U(1)'$ EXTENSION OF SM

In the Warsaw basis the operators are collected in classes according to their field content. The scalar field is denoted by  $\varphi$ , with  $\tilde{\varphi}$  defined as  $\tilde{\varphi}^j = \epsilon_{jk} (\varphi^k)^*$  [ $j, k$  are  $SU(2)_L$  indices]. The gauge field strengths are indicated by  $X, \tilde{X}$  being their duals. Fermions are denoted by  $\psi$ . Among the various terms in  $\mathcal{L}^{(6)}$  in Eq. (1), we focus on  $\mathcal{L}_{Z'}^{(6)}$ , the set of operators generated at the EW scale when the SM group is extended including  $U(1)'$  and the gauge boson  $Z'$  is integrated out.  $\mathcal{L}_{Z'}^{(6)}$  consists of the terms<sup>5</sup>

$$\begin{aligned} \mathcal{L}_{Z'}^{(6)} = & C_{\ell\ell} \mathcal{O}_{\ell\ell} + C_{qq}^{(1)} \mathcal{O}_{qq}^{(1)} + C_{ee} \mathcal{O}_{ee} + C_{uu} \mathcal{O}_{uu} + C_{dd} \mathcal{O}_{dd} + C_{\nu\nu}^{(6)} \mathcal{O}_{\nu\nu}^{(6)} \\ & + C_{\ell q}^{(1)} \mathcal{O}_{\ell q}^{(1)} + C_{ud}^{(1)} \mathcal{O}_{ud}^{(1)} + C_{eu} \mathcal{O}_{eu} + C_{ed} \mathcal{O}_{ed} + C_{\ell e} \mathcal{O}_{\ell e} + C_{\ell u} \mathcal{O}_{\ell u} + C_{\ell d} \mathcal{O}_{\ell d} + C_{qe} \mathcal{O}_{qe} \\ & + C_{qu}^{(1)} \mathcal{O}_{qu}^{(1)} + C_{qd}^{(1)} \mathcal{O}_{qd}^{(1)} + C_{ve} \mathcal{O}_{ve} + C_{vu} \mathcal{O}_{vu} + C_{vd} \mathcal{O}_{vd} + C_{\ell v} \mathcal{O}_{\ell v} + C_{qv} \mathcal{O}_{qv} \\ & + C_{\varphi\Box} \mathcal{O}_{\varphi\Box} + C_{\varphi D} \mathcal{O}_{\varphi D} + C_{e\varphi} \mathcal{O}_{e\varphi} + C_{u\varphi} \mathcal{O}_{u\varphi} + C_{d\varphi} \mathcal{O}_{d\varphi} + C_{\nu\varphi} \mathcal{O}_{\nu\varphi} \\ & + C_{\varphi\ell}^{(1)} \mathcal{O}_{\varphi\ell}^{(1)} + C_{\varphi e} \mathcal{O}_{\varphi e} + C_{\varphi q}^{(1)} \mathcal{O}_{\varphi q}^{(1)} + C_{\varphi u} \mathcal{O}_{\varphi u} + C_{\varphi d} \mathcal{O}_{\varphi d} + C_{\varphi\nu} \mathcal{O}_{\varphi\nu} + \text{H.c.} \end{aligned} \quad (12)$$

The various operators can be classified in the following classes defined in [3,14]:

(1) Four-fermion operators  $\mathcal{O}_{\psi_1\psi_2}$  (denoted as  $\mathcal{O}_{\psi\psi}$  if  $\psi_1 = \psi_2$ ) with structure  $(\bar{L}L)(\bar{L}L)$ :

$$\begin{aligned} [\mathcal{O}_{\ell\ell}]_{ijkp} &= (\bar{\ell}_L^i \gamma_{\mu} \ell_L^j) (\bar{\ell}_L^k \gamma^{\mu} \ell_L^p), \\ [\mathcal{O}_{qq}^{(1)}]_{ijkp} &= (\bar{q}_L^i \gamma_{\mu} q_L^j) (\bar{q}_L^k \gamma^{\mu} q_L^p), \\ [\mathcal{O}_{\ell q}^{(1)}]_{ijkp} &= (\bar{\ell}_L^i \gamma_{\mu} \ell_L^j) (\bar{q}_L^k \gamma^{\mu} q_L^p). \end{aligned} \quad (13)$$

<sup>5</sup>While in Eq. (2) the Wilson coefficients are dimensionless, in (12) it is convenient to include the mass dimension in the definition of the coefficients. The operator  $\mathcal{O}_{\nu\nu}^{(6)}$  is denoted by a superscript to distinguish it from the  $d = 5$  Weinberg operator  $\mathcal{O}_{\nu\nu}$ .

- (2) Four-fermion operators  $\mathcal{O}_{\psi_1\psi_2}$  with structure  $(\bar{R}R)(\bar{R}R)$ :

$$\begin{aligned}
[\mathcal{O}_{ee}]_{ijkp} &= (\bar{e}_R^i \gamma_\mu e_R^j) (\bar{e}_R^k \gamma^\mu e_R^p), \\
[\mathcal{O}_{uu}]_{ijkp} &= (\bar{u}_R^i \gamma_\mu u_R^j) (\bar{u}_R^k \gamma^\mu u_R^p), \\
[\mathcal{O}_{dd}]_{ijkp} &= (\bar{d}_R^i \gamma_\mu d_R^j) (\bar{d}_R^k \gamma^\mu d_R^p), \\
[\mathcal{O}_{ud}^{(1)}]_{ijkp} &= (\bar{u}_R^i \gamma_\mu u_R^j) (\bar{d}_R^k \gamma^\mu d_R^p), \\
[\mathcal{O}_{eu}]_{ijkp} &= (\bar{e}_R^i \gamma_\mu e_R^j) (\bar{u}_R^k \gamma^\mu u_R^p), \\
[\mathcal{O}_{ed}]_{ijkp} &= (\bar{e}_R^i \gamma_\mu e_R^j) (\bar{d}_R^k \gamma^\mu d_R^p), \\
[\mathcal{O}_{\nu\nu}^{(6)}]_{ijkp} &= (\bar{\nu}_R^i \gamma_\mu \nu_R^j) (\bar{\nu}_R^k \gamma^\mu \nu_R^p), \\
[\mathcal{O}_{\nu e}]_{ijkp} &= (\bar{\nu}_R^i \gamma_\mu \nu_R^j) (\bar{e}_R^k \gamma^\mu e_R^p), \\
[\mathcal{O}_{\nu u}]_{ijkp} &= (\bar{\nu}_R^i \gamma_\mu \nu_R^j) (\bar{u}_R^k \gamma^\mu u_R^p), \\
[\mathcal{O}_{\nu d}]_{ijkp} &= (\bar{\nu}_R^i \gamma_\mu \nu_R^j) (\bar{d}_R^k \gamma^\mu d_R^p). \quad (14)
\end{aligned}$$

- (3) Four-fermion operators  $\mathcal{O}_{\psi_1\psi_2}$  with structure  $(\bar{L}L)(\bar{R}R)$ :

$$\begin{aligned}
[\mathcal{O}_{\ell e}]_{ijkp} &= (\bar{\ell}_L^i \gamma_\mu \ell_L^j) (\bar{e}_R^k \gamma^\mu e_R^p), \\
[\mathcal{O}_{qe}]_{ijkp} &= (\bar{q}_L^i \gamma_\mu q_L^j) (\bar{e}_R^k \gamma^\mu e_R^p), \\
[\mathcal{O}_{\ell u}]_{ijkp} &= (\bar{\ell}_L^i \gamma_\mu \ell_L^j) (\bar{u}_R^k \gamma^\mu u_R^p), \\
[\mathcal{O}_{\ell d}]_{ijkp} &= (\bar{\ell}_L^i \gamma_\mu \ell_L^j) (\bar{d}_R^k \gamma^\mu d_R^p), \\
[\mathcal{O}_{qu}^{(1)}]_{ijkp} &= (\bar{q}_L^i \gamma_\mu q_L^j) (\bar{u}_R^k \gamma^\mu u_R^p), \\
[\mathcal{O}_{qd}^{(1)}]_{ijkp} &= (\bar{q}_L^i \gamma_\mu q_L^j) (\bar{d}_R^k \gamma^\mu d_R^p), \\
[\mathcal{O}_{\ell\nu}]_{ijkp} &= (\bar{\ell}_L^i \gamma_\mu \ell_L^j) (\bar{\nu}_R^k \gamma^\mu \nu_R^p), \\
[\mathcal{O}_{q\nu}]_{ijkp} &= (\bar{q}_L^i \gamma_\mu q_L^j) (\bar{\nu}_R^k \gamma^\mu \nu_R^p). \quad (15)
\end{aligned}$$

- (4) Operators  $\mathcal{O}_{\phi\phi}$  involving the Higgs field  $\phi$ , classified as  $\phi^4 D^2$  in the Warsaw basis:

$$\begin{aligned}
\mathcal{O}_{\phi\Box} &= (\phi^\dagger \phi) \Box (\phi^\dagger \phi), \\
\mathcal{O}_{\phi D} &= (\phi^\dagger D^\mu \phi) ((D_\mu \phi)^\dagger \phi). \quad (16)
\end{aligned}$$

- (5) Operators  $\mathcal{O}_{\psi\phi}$  involving the Higgs field  $\phi$  and the fermion fields, classified as  $\psi^2 \phi^3$ :

$$\begin{aligned}
[\mathcal{O}_{e\phi}]_{ij} &= (\phi^\dagger \phi) (\bar{\ell}_L^i \phi e_R^j), \\
[\mathcal{O}_{u\phi}]_{ij} &= (\phi^\dagger \phi) (\bar{q}_L^i \phi u_R^j), \\
[\mathcal{O}_{d\phi}]_{ij} &= (\phi^\dagger \phi) (\bar{q}_L^i \phi d_R^j), \\
[\mathcal{O}_{\nu\phi}]_{ij} &= (\phi^\dagger \phi) (\bar{\ell}_L^i \phi \nu_R^j). \quad (17)
\end{aligned}$$

- (6) Operators  $\mathcal{O}_{\phi\psi}$  comprising the Higgs field  $\phi$  and the fermion fields, classified as  $\psi^2 \phi^2 D$ :

$$\begin{aligned}
[\mathcal{O}_{\phi\ell}^{(1)}]_{ij} &= (\phi^\dagger i\overleftrightarrow{D}_\mu \phi) (\bar{\ell}_L^i \gamma^\mu \ell_L^j), \\
[\mathcal{O}_{\phi e}]_{ij} &= (\phi^\dagger i\overleftrightarrow{D}_\mu \phi) (\bar{e}_R^i \gamma^\mu e_R^j), \\
[\mathcal{O}_{\phi q}^{(1)}]_{ij} &= (\phi^\dagger i\overleftrightarrow{D}_\mu \phi) (\bar{q}_L^i \gamma^\mu q_L^j), \\
[\mathcal{O}_{\phi u}]_{ij} &= (\phi^\dagger i\overleftrightarrow{D}_\mu \phi) (\bar{u}_R^i \gamma^\mu u_R^j), \\
[\mathcal{O}_{\phi d}]_{ij} &= (\phi^\dagger i\overleftrightarrow{D}_\mu \phi) (\bar{d}_R^i \gamma^\mu d_R^j), \\
[\mathcal{O}_{\phi\nu}]_{ij} &= (\phi^\dagger i\overleftrightarrow{D}_\mu \phi) (\bar{\nu}_R^i \gamma^\mu \nu_R^j). \quad (18)
\end{aligned}$$

$i, j, k, p$  are generation indices.

#### IV. RELATIONS AMONG THE WILSON COEFFICIENTS

The coefficients of the operators in Sec. III can be expressed in terms of the couplings in Eq. (7) [27]. For four-fermion operators they read:

$$[C_{\ell\ell}]_{ijkp} = -\frac{(\Delta_L^\ell)^{ij} (\Delta_L^\ell)^{kp}}{2M_{Z'}^2}, \quad (19)$$

$$[C_{qq}^{(1)}]_{ijkp} = -\frac{(\Delta_L^q)^{ij} (\Delta_L^q)^{kp}}{2M_{Z'}^2}, \quad (20)$$

$$[C_{ee}]_{ijkp} = -\frac{(\Delta_R^e)^{ij} (\Delta_R^e)^{kp}}{2M_{Z'}^2}, \quad (21)$$

$$[C_{uu}]_{ijkp} = -\frac{(\Delta_R^u)^{ij} (\Delta_R^u)^{kp}}{2M_{Z'}^2}, \quad (22)$$

$$[C_{dd}]_{ijkp} = -\frac{(\Delta_R^d)^{ij} (\Delta_R^d)^{kp}}{2M_{Z'}^2}, \quad (23)$$

$$[C_{\nu\nu}^{(6)}]_{ijkp} = -\frac{(\Delta_R^\nu)^{ij} (\Delta_R^\nu)^{kp}}{2M_{Z'}^2}, \quad (24)$$

$$[C_{\ell q}^{(1)}]_{ijkp} = -\frac{(\Delta_L^\ell)^{ij} (\Delta_L^q)^{kp}}{M_{Z'}^2}, \quad (25)$$

$$[C_{ud}^{(1)}]_{ijkp} = -\frac{(\Delta_R^u)^{ij} (\Delta_R^d)^{kp}}{M_{Z'}^2}, \quad (26)$$

$$[C_{eu}]_{ijkp} = -\frac{(\Delta_R^e)^{ij} (\Delta_R^u)^{kp}}{M_{Z'}^2}, \quad (27)$$

$$[C_{ed}]_{ijkp} = -\frac{(\Delta_R^e)^{ij} (\Delta_R^d)^{kp}}{M_{Z'}^2}, \quad (28)$$

$$[C_{\ell e}]_{ijkp} = -\frac{(\Delta_L^\ell)^{ij}(\Delta_R^e)^{kp}}{M_{Z'}^2}, \quad (29)$$

$$[C_{\ell u}]_{ijkp} = -\frac{(\Delta_L^\ell)^{ij}(\Delta_R^u)^{kp}}{M_{Z'}^2}, \quad (30)$$

$$[C_{\ell d}]_{ijkp} = -\frac{(\Delta_L^\ell)^{ij}(\Delta_R^d)^{kp}}{M_{Z'}^2}, \quad (31)$$

$$[C_{qe}]_{ijkp} = -\frac{(\Delta_L^q)^{ij}(\Delta_R^e)^{kp}}{M_{Z'}^2}, \quad (32)$$

$$[C_{qu}^{(1)}]_{ijkp} = -\frac{(\Delta_L^q)^{ij}(\Delta_R^u)^{kp}}{M_{Z'}^2}, \quad (33)$$

$$[C_{qd}^{(1)}]_{ijkp} = -\frac{(\Delta_L^q)^{ij}(\Delta_R^d)^{kp}}{M_{Z'}^2}, \quad (34)$$

$$[C_{\nu e}]_{ijkp} = -\frac{(\Delta_R^\nu)^{ij}(\Delta_R^e)^{kp}}{M_{Z'}^2}, \quad (35)$$

$$[C_{\nu u}]_{ijkp} = -\frac{(\Delta_R^\nu)^{ij}(\Delta_R^u)^{kp}}{M_{Z'}^2}, \quad (36)$$

$$[C_{\nu d}]_{ijkp} = -\frac{(\Delta_R^\nu)^{ij}(\Delta_R^d)^{kp}}{M_{Z'}^2}, \quad (37)$$

$$[C_{\ell \nu}]_{ijkp} = -\frac{(\Delta_L^\ell)^{ij}(\Delta_R^\nu)^{kp}}{M_{Z'}^2}, \quad (38)$$

$$[C_{q\nu}]_{ijkp} = -\frac{(\Delta_L^q)^{ij}(\Delta_R^\nu)^{kp}}{M_{Z'}^2}. \quad (39)$$

The coefficients of the operators  $\mathcal{O}_{\phi\Box}$  and  $\mathcal{O}_{\phi D}$  are given by

$$C_{\phi\Box} = -\frac{g_H^2}{2M_{Z'}^2}, \quad (40)$$

$$C_{\phi D} = -\frac{2g_H^2}{M_{Z'}^2}, \quad (41)$$

so that

$$C_{\phi D} = 4C_{\phi\Box} \quad (42)$$

and  $C_{\phi D} < 0$ .

The couplings to fermions enter in the coefficients of  $\mathcal{O}_{e\varphi}$ ,  $\mathcal{O}_{u\varphi}$ , and  $\mathcal{O}_{d\varphi}$ . However, when the UV completion consists only of the new  $U(1)'$  group, as considered in the present study, such coefficients vanish. The coefficients of  $\mathcal{O}_{\phi\ell}^{(1)}$ ,  $\mathcal{O}_{\phi e}$ ,  $\mathcal{O}_{\phi q}^{(1)}$ ,  $\mathcal{O}_{\phi u}$ , and  $\mathcal{O}_{\phi d}$  are given by

$$[C_{\phi\ell}^{(1)}]_{ij} = -\frac{(\Delta_L^\ell)^{ij}g_H}{M_{Z'}^2}, \quad (43)$$

$$[C_{\phi e}]_{ij} = -\frac{(\Delta_R^e)^{ij}g_H}{M_{Z'}^2}, \quad (44)$$

$$[C_{\phi q}^{(1)}]_{ij} = -\frac{(\Delta_L^q)^{ij}g_H}{M_{Z'}^2}, \quad (45)$$

$$[C_{\phi u}]_{ij} = -\frac{(\Delta_R^u)^{ij}g_H}{M_{Z'}^2}, \quad (46)$$

$$[C_{\phi d}]_{ij} = -\frac{(\Delta_R^d)^{ij}g_H}{M_{Z'}^2}, \quad (47)$$

$$[C_{\phi\nu}]_{ij} = -\frac{(\Delta_R^\nu)^{ij}g_H}{M_{Z'}^2}. \quad (48)$$

For  $N$  generations, the coefficients in Eqs. (19)–(39) are generally complex matrices in a  $N^4$ -dimensional space. However, the coefficients in (19)–(24) correspond to Hermitian operators; hence they are real and have  $N^4$  components. In principle, the coefficients in Eqs. (43)–(48) involve  $2N^2$  independent parameters. This parameter counting changes for the UV completion obtained extending the SM gauge group with the new  $U(1)'$ . We derive relations among the coefficients before SSB, with unrotated fermion fields and diagonal  $Z'$  couplings to fermions. Moreover, in this case all coefficients are real, since they are expressed in terms of the (real)  $z$  hypercharges and of  $g_H$  which is real as from (10).

Relations exist among the remaining coefficients. We denote by  $C_{\psi\psi}$  a generic coefficient among those in Eqs. (19)–(24), and by  $C_{\psi_1\psi_2}$  a coefficient among those in Eqs. (25)–(39). The coefficients in Eqs. (43)–(48) are generically denoted as  $C_{\phi\psi}$  (in all cases  $\psi = \ell, q, \nu, e, u, d$ ). We have

$$[C_{\psi_1\psi_2}]_{ijkp} = \pm 2\sqrt{[C_{\psi_1\psi_1}]_{ijij}[C_{\psi_2\psi_2}]_{kpkp}}, \quad (49)$$

$$[C_{\psi\psi}]_{ijkp} = \frac{[C_{\phi\psi}]_{ij}[C_{\phi\psi}]_{kp}}{C_{\phi D}}, \quad (50)$$

$$[C_{\psi_1\psi_2}]_{ijkp} = 2\frac{[C_{\phi\psi_1}]_{ij}[C_{\phi\psi_2}]_{kp}}{C_{\phi D}}. \quad (51)$$

Considering Eq. (8), only the components  $C_{iikk}$  are non-vanishing among the coefficients in (19)–(39). Moreover, the coefficients in (19)–(24) are symmetric under the exchange  $ii \leftrightarrow kk$ , so they comprise only six independent components. It is convenient to use the notation  $\underline{i} = ii$ ,  $\underline{k} = kk$ . As for the coefficients  $[C_{\phi\psi}]_{ij}$  in Eqs. (43)–(48),

they are nonvanishing only for  $i = j$ . We denote them as  $[C_{\varphi\psi}]_{\underline{i}}$ .<sup>6</sup>

Summarizing, the following structures of coefficients are realized:

$$C_{\varphi\psi} = ([C_{\varphi\psi}]_{\underline{1}} \quad [C_{\varphi\psi}]_{\underline{2}} \quad [C_{\varphi\psi}]_{\underline{3}}), \quad (52)$$

$$C_{\psi\psi} = \frac{1}{C_{\varphi D}} \begin{pmatrix} ([C_{\varphi\psi}]_{\underline{1}})^2 & [C_{\varphi\psi}]_{\underline{1}}[C_{\varphi\psi}]_{\underline{2}} & [C_{\varphi\psi}]_{\underline{1}}[C_{\varphi\psi}]_{\underline{3}} \\ [C_{\varphi\psi}]_{\underline{2}}[C_{\varphi\psi}]_{\underline{1}} & ([C_{\varphi\psi}]_{\underline{2}})^2 & [C_{\varphi\psi}]_{\underline{2}}[C_{\varphi\psi}]_{\underline{3}} \\ [C_{\varphi\psi}]_{\underline{3}}[C_{\varphi\psi}]_{\underline{1}} & [C_{\varphi\psi}]_{\underline{3}}[C_{\varphi\psi}]_{\underline{2}} & ([C_{\varphi\psi}]_{\underline{3}})^2 \end{pmatrix}, \quad (53)$$

$$C_{\psi_1\psi_2} = \frac{2}{C_{\varphi D}} \begin{pmatrix} [C_{\varphi\psi_1}]_{\underline{1}}[C_{\varphi\psi_2}]_{\underline{1}} & [C_{\varphi\psi_1}]_{\underline{1}}[C_{\varphi\psi_2}]_{\underline{2}} & [C_{\varphi\psi_1}]_{\underline{1}}[C_{\varphi\psi_2}]_{\underline{3}} \\ [C_{\varphi\psi_1}]_{\underline{2}}[C_{\varphi\psi_2}]_{\underline{1}} & [C_{\varphi\psi_1}]_{\underline{2}}[C_{\varphi\psi_2}]_{\underline{2}} & [C_{\varphi\psi_1}]_{\underline{2}}[C_{\varphi\psi_2}]_{\underline{3}} \\ [C_{\varphi\psi_1}]_{\underline{3}}[C_{\varphi\psi_2}]_{\underline{1}} & [C_{\varphi\psi_1}]_{\underline{3}}[C_{\varphi\psi_2}]_{\underline{2}} & [C_{\varphi\psi_1}]_{\underline{3}}[C_{\varphi\psi_2}]_{\underline{3}} \end{pmatrix}. \quad (54)$$

The number of independent coefficients in the dimension-six Lagrangian density (12) is reduced to 19. They can be the 18 coefficients  $[C_{\psi\psi}]_{\underline{i}\underline{j}}$  for  $\underline{i} = \underline{1}, \underline{2}, \underline{3}$  and the six  $\psi = \ell, q, \nu, e, u, d$ , and  $C_{\varphi D}$ ; alternatively, they can be the 18 coefficients  $[C_{\varphi\psi}]_{\underline{i}}$  and  $C_{\varphi D}$ . In the next section we describe the constraints for such coefficients obtained requiring that the extended gauge group is free of gauge anomalies.

## V. CONSTRAINTS FROM GAUGE ANOMALY CANCELLATION

The issue of gauge anomaly cancellation in presence of a new  $U(1)'$  symmetry has been considered in many studies [26,28–34]. In case of a new  $Z'$  gauge boson, six gauge anomalies are generated. They can be expressed introducing the quantities  $z_{\psi}^{(n)}$  defined in terms of the sums

$$z_{\psi}^{(n)} = \sum_{i=1}^3 z_{\psi_i}^n, \quad (55)$$

with  $\psi_i$  a fermion in the  $i$  generation [28]. The  $[SU(3)_C]^2 U(1)'$ ,  $[SU(2)_L]^2 U(1)'$ , and  $[U(1)_Y]^2 U(1)'$  anomaly cancellation conditions involve the linear combinations of hypercharges in (55) and read as follows:

$$A_{33z} = 2z_q^{(1)} - z_u^{(1)} - z_d^{(1)} = 0, \quad (56)$$

$$A_{22z} = 3z_q^{(1)} + z_{\ell}^{(1)} = 0, \quad (57)$$

$$A_{11z} = \frac{1}{6}z_q^{(1)} - \frac{4}{3}z_u^{(1)} - \frac{1}{3}z_d^{(1)} + \frac{1}{2}z_{\ell}^{(1)} - z_e^{(1)} = 0. \quad (58)$$

The triangular graph involving two gravitons and  $Z'$  also produces a relation linear in the  $z$  hypercharges:

<sup>6</sup>To avoid confusion, when pedices refer to pairs of indices or to a single index we write  $\underline{i} = \underline{1}, \underline{2}, \underline{3}$  and  $i = 1, 2, 3$ , respectively.

$$A_{GGz} = 3[2z_q^{(1)} - z_u^{(1)} - z_d^{(1)}] + 2z_{\ell}^{(1)} - z_e^{(1)} - z_{\nu}^{(1)} = 0, \quad (59)$$

which can be simplified using Eq. (56):

$$A_{GGz} = 2z_{\ell}^{(1)} - z_e^{(1)} - z_{\nu}^{(1)} = 0. \quad (60)$$

The  $U(1)_Y[U(1)']^2$  anomaly cancellation condition involves the quadratic sums in (55):

$$A_{1zz} = [z_q^{(2)} - 2z_u^{(2)} + z_d^{(2)}] - [z_{\ell}^{(2)} - z_e^{(2)}] = 0. \quad (61)$$

The  $[U(1)']^3$  anomaly cancellation condition involves the cubic sums in (55):

$$A_{zzz} = 3[2z_q^{(3)} - z_u^{(3)} - z_d^{(3)}] + [2z_{\ell}^{(3)} - z_{\nu}^{(3)} - z_e^{(3)}] = 0. \quad (62)$$

The previous equations provide constraints to the coefficients in (12). We define

$$\tilde{C}_{\varphi\psi}^{(n)} = \sum_{i=1}^3 ([C_{\varphi\psi}]_{\underline{i}})^n, \quad (63)$$

denoting for simplicity  $\tilde{C}_{\varphi\psi}^{(1)} = \tilde{C}_{\varphi\psi}$ . Using Eqs. (43)–(48) we have

$$z_{\psi_i} = -\frac{M_{Z'}^2}{g_Z g_H} [C_{\varphi\psi}]_{\underline{i}}, \quad (64)$$

and Eq. (55) becomes

$$z_{\psi}^{(n)} = \left(-\frac{M_{Z'}^2}{g_Z g_H}\right)^n \tilde{C}_{\varphi\psi}^{(n)}. \quad (65)$$

With such definitions, the equations of the gauge anomaly cancellation conditions read as follows:

$$A_{33z} \rightarrow 2\tilde{C}_{\varphi q} - \tilde{C}_{\varphi u} - \tilde{C}_{\varphi d} = 0, \quad (66)$$

$$A_{22z} \rightarrow 3\tilde{C}_{\varphi q} + \tilde{C}_{\varphi\ell} = 0, \quad (67)$$

$$A_{11z} \rightarrow \tilde{C}_{\varphi q} - 8\tilde{C}_{\varphi u} - 2\tilde{C}_{\varphi d} + 3\tilde{C}_{\varphi\ell} - 6\tilde{C}_{\varphi e} = 0, \quad (68)$$

$$A_{GGz} \rightarrow 2\tilde{C}_{\varphi\ell} - \tilde{C}_{\varphi e} - \tilde{C}_{\varphi\nu} = 0. \quad (69)$$

They produce the relations:

$$\tilde{C}_{\varphi q} = \frac{\tilde{C}_{\varphi u} + \tilde{C}_{\varphi d}}{2}, \quad (70)$$

$$\tilde{C}_{\varphi\ell} = -3\tilde{C}_{\varphi q} = -3\frac{\tilde{C}_{\varphi u} + \tilde{C}_{\varphi d}}{2}, \quad (71)$$

$$\tilde{C}_{\varphi e} = -2\tilde{C}_{\varphi u} - \tilde{C}_{\varphi d}, \quad (72)$$

$$\tilde{C}_{\varphi\nu} = -\tilde{C}_{\varphi u} - 2\tilde{C}_{\varphi d}. \quad (73)$$

We also have

$$A_{1zz} \rightarrow \tilde{C}_{\varphi q}^{(2)} - 2\tilde{C}_{\varphi u}^{(2)} + \tilde{C}_{\varphi d}^{(2)} - \tilde{C}_{\varphi\ell}^{(2)} + \tilde{C}_{\varphi e}^{(2)} = 0, \quad (74)$$

$$A_{zzz} \rightarrow 3[2\tilde{C}_{\varphi q}^{(3)} - \tilde{C}_{\varphi u}^{(3)} - \tilde{C}_{\varphi d}^{(3)}] + [2\tilde{C}_{\varphi\ell}^{(3)} - \tilde{C}_{\varphi\nu}^{(3)} - \tilde{C}_{\varphi e}^{(3)}] = 0. \quad (75)$$

Examples on how the equations representing the anomaly cancellation conditions (ACE) can be exploited are discussed below, considering models with specific  $z$ -hypercharge assignments.

## VI. APPLICATIONS TO MODELS WITH SPECIFIC $z$ -HYPERCHARGE ASSIGNMENTS

The anomaly cancellation equations involve 18 parameters:  $[C_{\varphi\psi}]_{\underline{i}}$  for  $\psi = q_i, \ell_i, u_i, d_i, \nu_i, e_i$ , and  $\underline{i} = \underline{1}, \underline{2}, \underline{3}$ . Taking into account the constraints from the six ACE, there are 12 independent parameters. With other assumptions, further constraints can be imposed, as discussed below for selected cases.

### A. $Z'$ only coupled to the third generation, and $Z'$ universally coupled to the three generations

If  $Z'$  only couples to one of the generations, e.g., the third one, then we have  $z_{\psi_3} = z_{\psi}$  and  $z_{\psi_1} = z_{\psi_2} = 0$ . The number of parameters involved in the ACE is 6, denoted  $\tilde{C}_{\varphi\psi}$ :

$$[C_{\varphi\psi}]_{\underline{3}} = \tilde{C}_{\varphi\psi}, \quad [C_{\varphi\psi}]_{\underline{1}} = [C_{\varphi\psi}]_{\underline{2}} = 0. \quad (76)$$

It follows that

$$\tilde{C}_{\varphi\psi}^{(n)} = (\tilde{C}_{\varphi\psi})^n. \quad (77)$$

Before discussing the ACE, let us consider the scenario in which  $Z'$  universally couples to the three generations:

$z_{\psi_1} = z_{\psi_2} = z_{\psi_3} = z_{\psi}$ , as in models where the  $z$  hypercharge is a linear combination of the SM hypercharge  $Y$  and of  $B - nL$ , with  $B$  and  $L$  the baryon and total lepton number and  $n$  an integer number [28,35,36].<sup>7</sup> The six parameters involved in the ACE are denoted again by  $\tilde{C}_{\varphi\psi}$ :

$$[C_{\varphi\psi}]_{\underline{1}} = [C_{\varphi\psi}]_{\underline{2}} = [C_{\varphi\psi}]_{\underline{3}} = \tilde{C}_{\varphi\psi}, \quad (78)$$

and the relation holds:

$$\tilde{C}_{\varphi\psi}^{(n)} = 3(\tilde{C}_{\varphi\psi})^n. \quad (79)$$

The factor 3 factorizes in the ACE, hence the two cases are identical from the viewpoint of solving the equations and can be discussed together.

Since Eqs. (74)–(75) are automatically satisfied, they do not represent additional constraints, hence there are two independent coefficients. One can express all coefficients in terms of  $\tilde{C}_{\varphi d}$  and  $\tilde{C}_{\varphi e}$ :

$$\begin{aligned} \tilde{C}_{\varphi q} &= \frac{\tilde{C}_{\varphi d} - \tilde{C}_{\varphi e}}{4}, \\ \tilde{C}_{\varphi u} &= -\frac{\tilde{C}_{\varphi d} + \tilde{C}_{\varphi e}}{2}, \\ \tilde{C}_{\varphi\ell} &= -\frac{3(\tilde{C}_{\varphi d} - \tilde{C}_{\varphi e})}{4}, \quad \tilde{C}_{\varphi\nu} = \frac{-3\tilde{C}_{\varphi d} + \tilde{C}_{\varphi e}}{2}. \end{aligned} \quad (80)$$

Correlations among the four coefficients depending on the two independent ones are obtained, as shown in Fig. 1 varying  $\tilde{C}_{\varphi d}$  and  $\tilde{C}_{\varphi e}$ .

In this specific scenario, information can also be obtained on  $[C_{\psi\psi}]_{\underline{3}\underline{3}}$ . Indeed, Eqs. (19)–(23) imply

$$z_{\psi_i} = \pm \left( 2 \frac{M_{Z'}^2}{g_Z^2} \right)^{1/2} (-[C_{\psi\psi}]_{\underline{i}\underline{i}})^{1/2}. \quad (81)$$

As done for  $C_{\varphi\psi}$  we define  $[C_{\psi\psi}]_{\underline{3}\underline{3}} = \tilde{C}_{\psi\psi}$ . The ACE can be used to relate the nonvanishing  $z$  hypercharges:

$$\begin{aligned} z_{q_3} &= \frac{1}{4}(z_{d_3} - z_{e_3}), \\ z_{u_3} &= -\frac{1}{2}(z_{d_3} + z_{e_3}), \\ z_{\ell_3} &= -\frac{3}{4}(z_{d_3} - z_{e_3}), \\ z_{\nu_3} &= \frac{1}{2}(z_{e_3} - 3z_{d_3}). \end{aligned} \quad (82)$$

<sup>7</sup>Replacing  $L$  with a family lepton number or a combination of  $L_e, L_\mu, L_\tau$  different from  $L$  does not belong to the generation independent category [35,37].

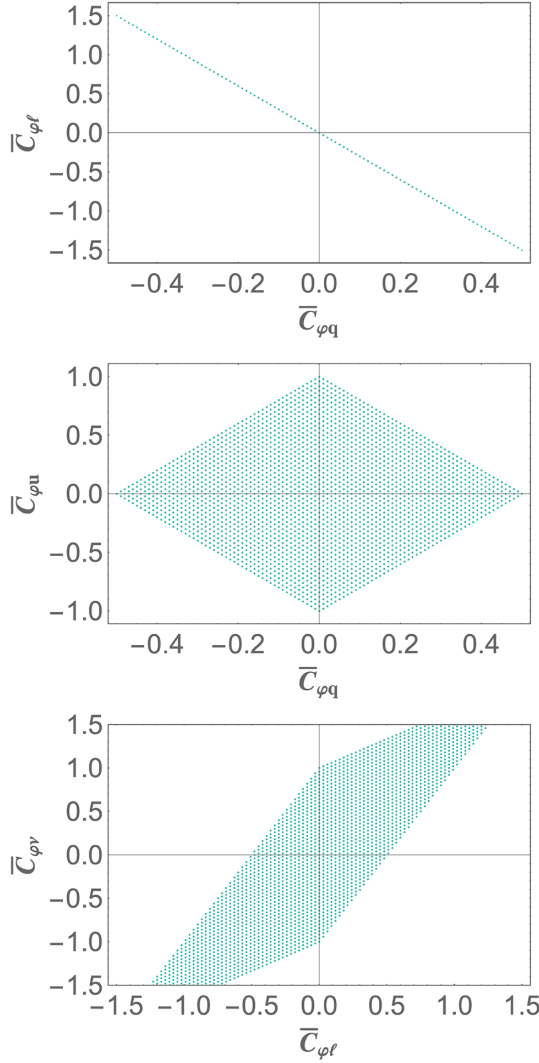


FIG. 1.  $Z'$  only coupled to the third fermion generation: Correlations among nonvanishing coefficients, varying  $\tilde{C}_{\phi d}$  and  $\tilde{C}_{\phi e}$  in the range  $[-1, 1]$ .

Two different cases can be analyzed, depending whether  $z_{d_3}$  and  $z_{e_3}$  have same or opposite signs:

- (1)  $z_{d_3} > 0, z_{e_3} > 0$  and  $z_{d_3} < 0, z_{e_3} < 0$ : we have

$$\begin{aligned}
 \tilde{C}_{qq} &= -\left(\frac{\sqrt{-\tilde{C}_{dd}} - \sqrt{-\tilde{C}_{ee}}}{4}\right)^2, \\
 \tilde{C}_{uu} &= -\left(\frac{\sqrt{-\tilde{C}_{dd}} + \sqrt{-\tilde{C}_{ee}}}{2}\right)^2, \\
 \tilde{C}_{\ell\ell} &= -\frac{9}{16}\left(\sqrt{-\tilde{C}_{dd}} - \sqrt{-\tilde{C}_{ee}}\right)^2, \\
 \tilde{C}_{\nu\nu} &= -\left(\frac{\sqrt{-\tilde{C}_{ee}} - 3\sqrt{-\tilde{C}_{dd}}}{2}\right)^2.
 \end{aligned} \tag{83}$$

- (2)  $z_{d_3} < 0, z_{e_3} > 0$ , and  $z_{d_3} > 0, z_{e_3} < 0$ : we have

$$\begin{aligned}
 \tilde{C}_{qq} &= -\left(\frac{\sqrt{-\tilde{C}_{dd}} + \sqrt{-\tilde{C}_{ee}}}{4}\right)^2, \\
 \tilde{C}_{uu} &= -\left(\frac{\sqrt{-\tilde{C}_{dd}} - \sqrt{-\tilde{C}_{ee}}}{2}\right)^2, \\
 \tilde{C}_{\ell\ell} &= -\frac{9}{16}\left(\sqrt{-\tilde{C}_{dd}} + \sqrt{-\tilde{C}_{ee}}\right)^2, \\
 \tilde{C}_{\nu\nu} &= -\left(\frac{\sqrt{-\tilde{C}_{ee}} + 3\sqrt{-\tilde{C}_{dd}}}{2}\right)^2.
 \end{aligned} \tag{84}$$

Correlations among the four coefficients are obtained varying  $\tilde{C}_{dd}$  and  $\tilde{C}_{ee}$ , as shown in Fig. 2.

### B. $Z'$ only coupled to left-handed fermions

The possibility that  $Z'$  only couples to fermions of a given chirality has been considered, e.g., in [38]. If  $Z'$  only couples to left-handed fermions the nonvanishing coefficients are  $[C_{\phi\psi}]_{\underline{i}}$  for  $\psi = \{q_i, \ell_i\}$ , hence six parameters. The number of constraints is reduced to 4 since Eqs. (67) and (68) are redundant. The linear equations (66) and (69) provide the relations

$$\tilde{C}_{\phi q} = \tilde{C}_{\phi\ell} = 0. \tag{85}$$

The quadratic and cubic ACE provide further relations, hence the number of independent coefficients is 2. Varying  $[C_{\phi\ell}]_{\underline{3}}$  and  $[C_{\phi q}]_{\underline{3}}$ , correlations are obtained among the remaining coefficients. They are shown in Fig. 3.

Also in this case the ACE can be exploited to derive correlations among  $[C_{\psi\psi}]$ , choosing  $[C_{\ell\ell}]_{\underline{33}}, [C_{qq}]_{\underline{33}}$  as independent coefficients, for same-sign or opposite-sign  $z_{\ell_3}$  and  $z_{q_3}$ . The correlations between the remaining coefficients  $[C_{\ell\ell}]_{\underline{33}}$  and  $[C_{qq}]_{\underline{33}}$  are shown in Fig. 4.

### C. $Z'$ only coupled to right-handed fermions

If  $Z'$  only couples to right-handed fermions, then the nonvanishing coefficients are  $[C_{\phi\psi}]_{\underline{i}}$  for  $\psi = \{u_i, d_i, e_i, \nu_i\}$ , hence 12 parameters. The number of constraints is reduced to 5 since Eq. (67) is automatically satisfied. Equations (66), (68), and (69) provide the relations

$$\tilde{C}_{\phi u} = -\tilde{C}_{\phi d} = -\tilde{C}_{\phi e} = \tilde{C}_{\phi\nu}. \tag{86}$$

The quadratic and cubic ACE give further relations, so that the number of independent coefficients is 7.

### D. Leptophobic $Z'$

If  $Z'$  only couples to quarks, then the nonvanishing coefficients are  $[C_{\phi\psi}]_{\underline{i}}$  for  $\psi = \{q_i, u_i, d_i\}$ , therefore nine parameters. The number of constraints is 5, since Eq. (69) is



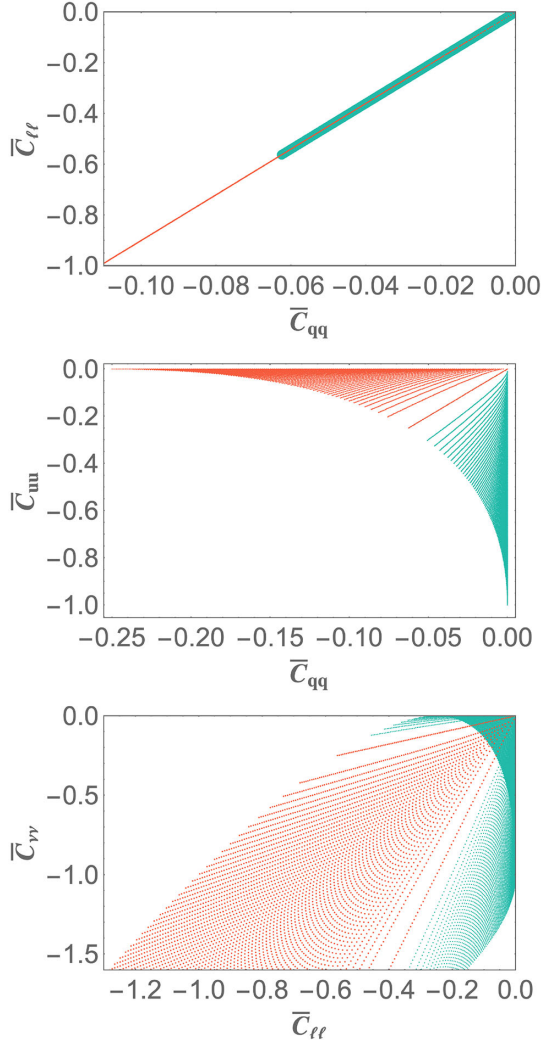


FIG. 2.  $Z'$  only coupled to the third generation: Correlation among nonvanishing coefficients, varying  $\tilde{C}_{dd}$  and  $\tilde{C}_{ee}$  in the range  $[-1, 0]$ . The green points refer to same-sign  $\tilde{C}_{dd}$  and  $\tilde{C}_{ee}$ , the orange points to the case of opposite signs.

automatically verified. The other linear equations provide the relations

$$\tilde{C}_{\phi q} = \tilde{C}_{\phi u} = \tilde{C}_{\phi d} = 0, \quad (87)$$

while the quadratic and cubic ACE read

$$\tilde{C}_{\phi q}^{(2)} - 2\tilde{C}_{\phi u}^{(2)} + \tilde{C}_{\phi d}^{(2)} = 0, \quad (88)$$

$$2\tilde{C}_{\phi q}^{(3)} - \tilde{C}_{\phi u}^{(3)} - \tilde{C}_{\phi d}^{(3)} = 0. \quad (89)$$

Consequently, there are four independent coefficients.

### E. Hadrophobic $Z'$

The situation is specular to the leptophobic  $Z'$ . The expressions of the ACE are

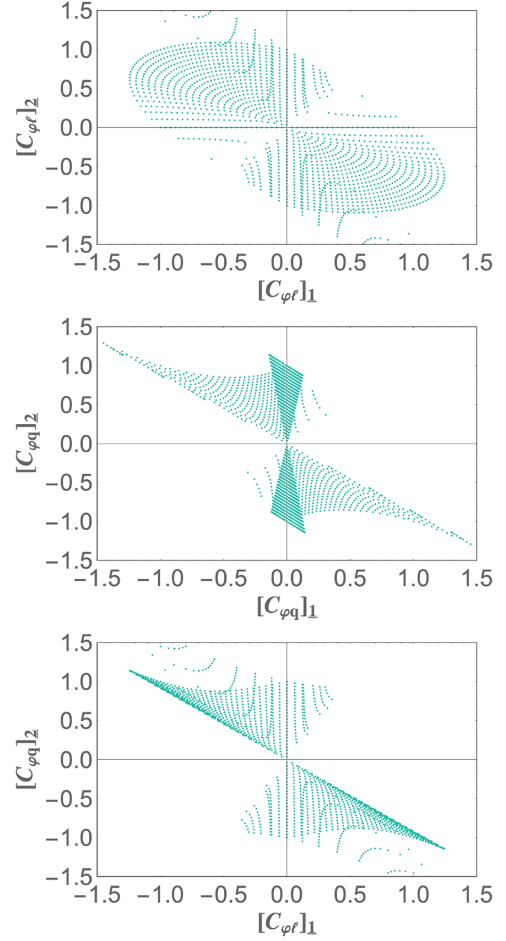


FIG. 3.  $Z'$  only coupled to left-handed fermions: Correlation among nonvanishing coefficients, varying  $[C_{\phi\ell}]_3$  and  $[C_{\phi q}]_3$  in the range  $[-1, 1]$ .

$$\begin{aligned} \tilde{C}_{\phi\ell} &= \tilde{C}_{\phi\nu} = \tilde{C}_{\phi e} = 0, \\ -\tilde{C}_{\phi\ell}^{(2)} + \tilde{C}_{\phi e}^{(2)} &= 0, \\ 2\tilde{C}_{\phi\ell}^{(3)} - \tilde{C}_{\phi\nu}^{(3)} - \tilde{C}_{\phi e}^{(3)} &= 0. \end{aligned} \quad (90)$$

The number of independent coefficients is 4.

For such models the experimental bounds are weaker than in previous cases, and allow a relatively light  $Z'$ . Moreover,  $Z'$  can contribute to lepton-flavor violating decays and to the lepton anomalous magnetic moments [39–46], an issue of great interest at present [47,48]. Models gauging  $L_a - L_b$  ( $a, b$  being the lepton flavors) belong to this class, namely models gauging  $L_\mu - L_\tau$  [49–54].

As an example of a hadrophobic model, we can also consider the  $Z'$  only coupled to right-handed neutrinos, a scenario belonging to the class of neutrinophilic NP models [55–57]. As for the ACE, setting all  $z$  hypercharges to 0 but for right-handed neutrinos, we have that Eq. (90) is satisfied only if at least one of the three right-handed neutrinos is sterile under  $U(1)'$ . Choosing  $z_{\nu_3} = 0$ , the ACE imply  $[C_{\phi\nu}]_3 = 0$  and  $[C_{\phi\nu}]_1 = -[C_{\phi\nu}]_2$ .

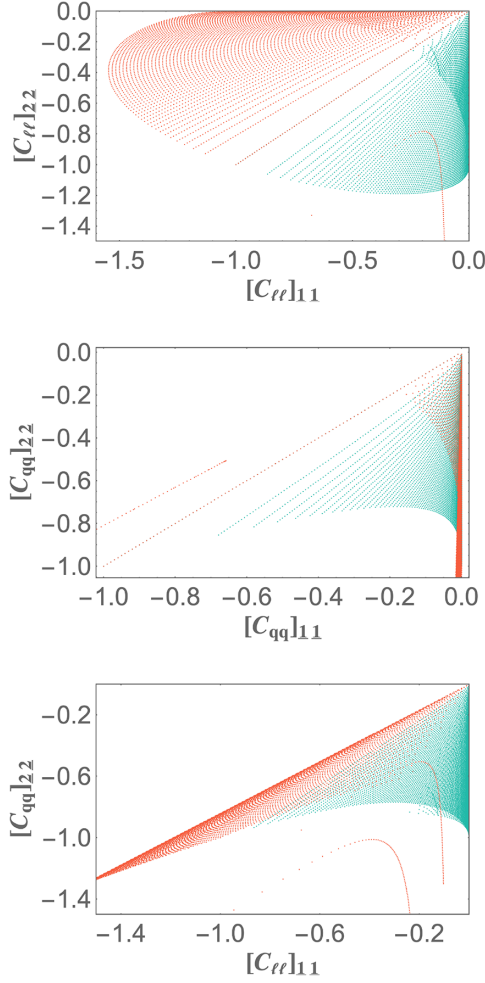


FIG. 4.  $Z'$  coupled only to left-handed fermions: Correlations among coefficients, varying  $[C_{\ell\ell}]_{33}$  and  $[C_{qq}]_{33}$  in the range  $[-1, 0]$ . The color code is the same as in Fig. 2.

### F. ABCD model [26]

A model with a heavy gauge boson  $Z'$  with flavor nonuniversal quark and lepton couplings has been considered in [26]. The assignment of the  $z$  hypercharge to a generic fermion  $\psi_i = \{q_i, u_i, d_i, \ell_i, \nu_i, e_i\}$  ( $i$  a generation index) is

$$z_{\psi_i} = y_{\psi} + \epsilon_i. \quad (91)$$

$y_{\psi}$  denote the generation universal SM hypercharges,  $\epsilon_i$  are parameters generation dependent, but universal within a given generation. This construction produces quark-lepton correlations. As shown in [26], all ACE are satisfied provided

$$\sum_{i=1}^3 \epsilon_i = 0. \quad (92)$$

The assignment implies the relation

$$\tilde{C}_{\phi\psi} = 3 \left( -\frac{g_Z^2 C_{\phi D}}{2M_{Z'}^2} \right)^{1/2} y_{\psi}. \quad (93)$$

For right-handed neutrinos one has  $\tilde{C}_{\phi\nu} = 0$  since  $y_{\nu} = 0$ . Nontrivial relations among the SMEFT coefficients are predicted:

$$-6 \frac{\tilde{C}_{\phi q}}{\tilde{C}_{\phi e}} = -\frac{3 \tilde{C}_{\phi u}}{2 \tilde{C}_{\phi e}} = 3 \frac{\tilde{C}_{\phi d}}{\tilde{C}_{\phi e}} = 2 \frac{\tilde{C}_{\phi \ell}}{\tilde{C}_{\phi e}}. \quad (94)$$

## VII. CONCLUSIONS

The possibility of gaining information on possible extensions of the SM, in a bottom-up approach, is largely based on the SM effective field theory framework. It is important to obtain the widest information from the phenomenological analysis of the coefficients of the operators in the effective field theory Lagrangian. We have discussed the set of constraints and relations among the coefficients of the  $d = 6$  operators if the SM extension includes a nonanomalous  $U(1)'$ . In particular, we have investigated how the anomaly cancellation equations, involving the  $z$  hypercharges, can be translated into constraints for the  $\nu$ SMEFT Wilson coefficients. Such constraints become more stringent if particular features are assumed for the  $Z'$  couplings to fermions.

We have discussed examples on how the constraints can be exploited, and which correlations among the coefficients emerge. Correlations among different coefficients imply relations among different physical processes, which can be searched and tested in experiment. Such processes could also involve neutrinos, which motivates our choice of considering the  $\nu$ SMEFT formulation. The correlations could also be included in global fit analyses using the data already available, or that will be collected in the near future. This provides us with a way for accessing the long-sighted extension of the Standard Model.

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- [1] F. De Fazio, [arXiv:2311.02987](https://arxiv.org/abs/2311.02987).
- [2] W. Buchmuller and D. Wyler, *Nucl. Phys.* **B268**, 621 (1986).
- [3] B. Grzadkowski, M. Iskrzynski, M. Misiak, and J. Rosiek, *J. High Energy Phys.* **10** (2010) 085.
- [4] I. Brivio and M. Trott, *Phys. Rep.* **793**, 1 (2019).
- [5] G. Isidori, F. Wilsch, and D. Wyler, *Rev. Mod. Phys.* **96**, 015006 (2024).
- [6] A. Helset and A. Kobach, *Phys. Lett. B* **800**, 135132 (2020).
- [7] W. Dekens and P. Stoffer, *J. High Energy Phys.* **10** (2019) 197; **11** (2022) 148(E).
- [8] E. E. Jenkins, A. V. Manohar, and M. Trott, *J. High Energy Phys.* **10** (2013) 087.
- [9] E. E. Jenkins, A. V. Manohar, and M. Trott, *J. High Energy Phys.* **01** (2014) 035.
- [10] R. Alonso, E. E. Jenkins, A. V. Manohar, and M. Trott, *J. High Energy Phys.* **04** (2014) 159.
- [11] F. del Aguila, S. Bar-Shalom, A. Soni, and J. Wudka, *Phys. Lett. B* **670**, 399 (2009).
- [12] A. Aparici, K. Kim, A. Santamaria, and J. Wudka, *Phys. Rev. D* **80**, 013010 (2009).
- [13] S. Bhattacharya and J. Wudka, *Phys. Rev. D* **94**, 055022 (2016); **95**, 039904(E) (2017).
- [14] Y. Liao and X.-D. Ma, *Phys. Rev. D* **96**, 015012 (2017).
- [15] J. Alcaide, S. Banerjee, M. Chala, and A. Titov, *J. High Energy Phys.* **08** (2019) 031.
- [16] M. Chala and A. Titov, *J. High Energy Phys.* **09** (2020) 188.
- [17] A. Datta, J. Kumar, H. Liu, and D. Marfatia, *J. High Energy Phys.* **02** (2021) 015.
- [18] A. Datta, J. Kumar, H. Liu, and D. Marfatia, *J. High Energy Phys.* **05** (2021) 037.
- [19] S. Weinberg, *Phys. Rev. Lett.* **43**, 1566 (1979).
- [20] A. Leike, *Phys. Rep.* **317**, 143 (1999).
- [21] P. Langacker and M. Plumacher, *Phys. Rev. D* **62**, 013006 (2000).
- [22] T. Appelquist, B. A. Dobrescu, and A. R. Hopper, *Phys. Rev. D* **68**, 035012 (2003).
- [23] T. G. Rizzo, in *Theoretical Advanced Study Institute in Elementary Particle Physics: Exploring New Frontiers Using Colliders and Neutrinos* (2006), pp. 537–575; [arXiv:hep-ph/0610104](https://arxiv.org/abs/hep-ph/0610104).
- [24] S. Navas *et al.* (Particle Data Group), *Phys. Rev. D* **110**, 030001 (2024).
- [25] S. Dawson, M. Forslund, and M. Schnubel, *Phys. Rev. D* **110**, 015002 (2024).
- [26] J. Aebischer, A. J. Buras, M. Cerdà-Sevilla, and F. De Fazio, *J. High Energy Phys.* **02** (2020) 183.
- [27] J. de Blas, J. C. Criado, M. Perez-Victoria, and J. Santiago, *J. High Energy Phys.* **03** (2018) 109.
- [28] M. Carena, A. Daleo, B. A. Dobrescu, and T. M. P. Tait, *Phys. Rev. D* **70**, 093009 (2004).
- [29] A. Celis, J. Fuentes-Martin, M. Jung, and H. Serodio, *Phys. Rev. D* **92**, 015007 (2015).
- [30] A. Ekstedt, R. Enberg, G. Ingelman, J. Löfgren, and T. Mandal, *J. High Energy Phys.* **11** (2016) 071.
- [31] A. Ismail, W.-Y. Keung, K.-H. Tsao, and J. Unwin, *Nucl. Phys.* **B918**, 220 (2017).
- [32] J. Ellis, M. Fairbairn, and P. Tunney, *J. High Energy Phys.* **08** (2017) 053.
- [33] B. C. Allanach, J. Davighi, and S. Melville, *J. High Energy Phys.* **02** (2019) 082; **08** (2019) 64.
- [34] B. C. Allanach, B. Gripaios, and J. Tooby-Smith, *Phys. Rev. D* **101**, 075015 (2020).
- [35] E. Salvioni, A. Strumia, G. Villadoro, and F. Zwirner, *J. High Energy Phys.* **03** (2010) 010.
- [36] E. Accomando, A. Belyaev, L. Fedeli, S. F. King, and C. Shepherd-Themistocleous, *Phys. Rev. D* **83**, 075012 (2011).
- [37] A. Greljo, P. Stangl, and A. E. Thomsen, *Phys. Lett. B* **820**, 136554 (2021).
- [38] A. J. Buras, F. De Fazio, and J. Girrbach, *J. High Energy Phys.* **02** (2013) 116.
- [39] J. Heeck and W. Rodejohann, *Phys. Rev. D* **84**, 075007 (2011).
- [40] K. Harigaya, T. Igari, M. M. Nojiri, M. Takeuchi, and K. Tobe, *J. High Energy Phys.* **03** (2014) 105.
- [41] B. Allanach, F. S. Queiroz, A. Strumia, and S. Sun, *Phys. Rev. D* **93**, 055045 (2016); **95**, 119902(E) (2017).
- [42] W. Altmannshofer, C.-Y. Chen, P. S. Bhupal Dev, and A. Soni, *Phys. Lett. B* **762**, 389 (2016).
- [43] S. Iguro, Y. Omura, and M. Takeuchi, *J. High Energy Phys.* **09** (2020) 144.
- [44] Y. Cheng, X.-G. He, and J. Sun, *Phys. Lett. B* **827**, 136989 (2022).
- [45] D. W. P. Amaral, D. G. Cerdeno, A. Cheek, and P. Foldenauer, *Eur. Phys. J. C* **81**, 861 (2021).
- [46] J. Kriewald, J. Orloff, E. Pinsard, and A. M. Teixeira, *Eur. Phys. J. C* **82**, 844 (2022).
- [47] B. Abi *et al.* (Muon g-2 Collaboration), *Phys. Rev. Lett.* **126**, 141801 (2021).
- [48] D. P. Aguillard *et al.* (Muon g-2 Collaboration), *Phys. Rev. Lett.* **131**, 161802 (2023).
- [49] W. Altmannshofer, S. Gori, M. Pospelov, and I. Yavin, *Phys. Rev. D* **89**, 095033 (2014).
- [50] A. Crivellin, G. D’Ambrosio, and J. Heeck, *Phys. Rev. Lett.* **114**, 151801 (2015).
- [51] K. Asai, K. Hamaguchi, N. Nagata, S.-Y. Tseng, and K. Tsumura, *Phys. Rev. D* **99**, 055029 (2019).
- [52] A. Biswas and A. Shaw, *J. High Energy Phys.* **05** (2019) 165.
- [53] J. Davighi, M. Kirk, and M. Nardecchia, *J. High Energy Phys.* **12** (2020) 111.
- [54] A. J. Buras, A. Crivellin, F. Kirk, C. A. Manzari, and M. Montull, *J. High Energy Phys.* **06** (2021) 068.
- [55] K. Huitu, S. Khalil, H. Okada, and S. K. Rai, *Phys. Rev. Lett.* **101**, 181802 (2008).
- [56] L. Basso, A. Belyaev, S. Moretti, and C. H. Shepherd-Themistocleous, *Phys. Rev. D* **80**, 055030 (2009).
- [57] W. Abdallah, A. K. Barik, S. K. Rai, and T. Samui, *Phys. Rev. D* **104**, 095031 (2021).