Greenberger-Horne-Zeilinger States and Few-Body Hamiltonians

Paolo Facchi,[1,2] Giuseppe Florio,[3,2] Saverio Pascazio,[3,2] and Francesco V. Pepe[3,2]
[1]Dipartimento di Matematica and MECENAS, Università di Bari, I-70125 Bari, Italy
[2]INFN, Sezione di Bari, I-70126 Bari, Italy
[3]Dipartimento di Fisica and MECENAS, Università di Bari, I-70126 Bari, Italy
(Received 3 May 2011; published 19 December 2011)

The generation of Greenberger-Horne-Zeilinger (GHZ) states is a crucial problem in quantum information. We derive general conditions for obtaining GHZ states as eigenstates of a Hamiltonian. We find that a necessary condition for an n-qubit GHZ state to be a nondegenerate eigenstate of a Hamiltonian is the presence of m-qubit couplings with $m \geq [(n+1)/2]$. Moreover, we introduce a Hamiltonian with a GHZ eigenstate and derive sufficient conditions for the removal of the degeneracy.

DOI: 10.1103/PhysRevLett.107.260502 PACS numbers: 03.67.Mn, 03.65.Ud, 75.10.Dg

The use of quantum mechanics for improving tasks such as communication, computation, and cryptography [1] is based on the availability of highly entangled states [2–5]. It is therefore of primary importance to obtain reliable strategies for their generation. Among others, Greenberger-Horne-Zeilinger (GHZ) states [6] represent a paradigmatic example of multipartite entangled states. In particular, in the case of three qubits, these states contain purely tripartite entanglement [7] and do not retain any bipartite entanglement when one of the qubits is traced out, thus maximizing the residual tangle [8].

The experimental realization of GHZ states [9–12], most recently with 14 qubits [13] has paved the way towards realistic implementation of quantum protocols. In these experiments a bottom-up approach is employed, whereby individual quantum systems (trapped particles, photons, cavities) are combined and manipulated. As the number of controllable qubits increases, the generation of GHZ states requires the use of quantum operations, whose feasibility strongly depends on the physical system used (optical, semiconductor, or superconductor based [14,15]). In the case of the recent trapped-ion implementation [13], the problem is additionally complicated by the presence of correlated Gaussian phase noise, that provokes "superdecoherence," by which decay scales quadratically with the number of qubits. It becomes therefore necessary to manipulate and control state fidelity and dynamics over sufficiently long time scales.

In principle, an alternative scheme for the implementation of GHZ states would consist in its encoding into one of the eigenstates (possibly the fundamental one) of a suitable Hamiltonian. For instance, in [16] it was shown that for the quantum Ising model in a transverse field the ground state is approximately a GHZ state if the strength of the field goes to infinity. Moreover, a proper choice of local fields for an Heisenberg-like spin model can yield a ground state which is, again, approximately GHZ [17,18].

On the other hand, it would be interesting to understand what are the requirements to obtain an exact GHZ state as an eigenstate of a quantum Hamiltonian. In this Letter we will address this problem and find rigorous conditions for the encoding of GHZ states into one of the eigenstates of a Hamiltonian that contains few-body coupling terms.

Let

$$|G^n_\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle^{\otimes n} \pm |1\rangle^{\otimes n}) \tag{1}$$

be GHZ states, where $\sigma^z|i\rangle = (-1)^i|i\rangle$ defines the computational basis, with $i = 0, 1$, and σ^z the third Pauli matrix. As a preliminary remark, we notice that it is trivial to find Hamiltonians involving n-body interaction terms, whose nondegenerate ground state is $|G^n_+\rangle$: the simplest example is $E_0|G^n_+\rangle\langle G^n_+|$, with $E_0 < 0$. On the other hand, we can ask whether it is possible for $|G^n_+\rangle$ to be the nondegenerate ground state, even if the Hamiltonian involves at most m-body interaction terms (with $m < n$). One can easily see that this is not possible. The reason lies in the fact that $|G^n_+\rangle$ and $|G^n_-\rangle$ share the same m-body reduced density matrices, and thus the same expectation values on m-body interaction terms. If $|G^n_+\rangle$ is a ground state, also $|G^n_-\rangle$ must be a ground state. This is a special case of a result proved in [19] and of a theorem concerning graph states in [20].

Thus, we relax our initial requirement and try to understand whether $|G^n_+\rangle$ can be a nondegenerate excited eigenstate for some m-body Hamiltonian. More specifically, we search for a limiting value m^*_n, depending on the number n of qubits in the system, such that, if the Hamiltonian involves m-body interaction terms (with $m < m^*_n$), $|G^n_+\rangle$ cannot be a nondegenerate eigenstate, otherwise the task becomes possible. The most generic m-body Hamiltonian acting on the Hilbert space of n qubits can be written as

$$H^{(m)} = \sum_{j_1<j_2<\cdots<j_m} \sum_{\alpha_1,\ldots,\alpha_m} J^{\alpha_1\ldots\alpha_m}_{j_1\ldots j_m} \sigma^{\alpha_1}_{j_1} \ldots \sigma^{\alpha_m}_{j_m}, \tag{2}$$

with $\alpha_i = 0, x, y, z$, $\sigma^0_i \equiv \mathbb{1}_i$ being the identity operator, σ^α_i the Pauli matrices acting on the Hilbert space of qubit i and J's real numbers. Terms involving only identities and an even number of σ^z's map $|G^n_+\rangle$ on the subspace spanned by itself. On the other hand, terms involving other (products of) Pauli matrices map $|G_+^n\rangle$ onto an orthogonal subspace. The action of $H^{(m)}$ on $|G_+^n\rangle$ is

$$H^{(m)}|G_+^n\rangle = \epsilon|G_+^n\rangle + |\Psi^{(m)}\rangle, \tag{3}$$

where ϵ is a multiplicative constant and $|\Psi^{(m)}\rangle$ is an unnormalized state vector satisfying

$$\langle\Psi^{(m)}|G_+^n\rangle = 0. \tag{4}$$

The vector $|\Psi^{(m)}\rangle$ can be expressed in a convenient way in an appropriate basis. For example, for a system of 5 qubits we can take basis states such as $(|00101\rangle \pm |11010\rangle)/\sqrt{2}$. We will introduce a new notation for these states. Let

$$\mathcal{N} = (1, 2, \ldots, n) \tag{5}$$

be the ordered set of naturals from 1 to n, and

$$\mathcal{I} = (i_1, i_2, \ldots, i_l) \tag{6}$$

denote a multi-index, whose elements range from 1 to n and satisfy $i_1 < i_2 < \ldots < i_l$. The cardinality $|\mathcal{I}| = l$ satisfies

$$1 \leq |\mathcal{I}| \leq m < n. \tag{7}$$

We now define a set of normalized state vectors, depending on the choice of the multi-index $\mathcal{I}$ and on the sign $s = \pm 1$:

$$|\tilde{G}_{s,\mathcal{I}}^n\rangle = \frac{1}{\sqrt{2}}(\mathbb{1} + s\sigma_1^x \ldots \sigma_n^x)\bigg(\bigotimes_{i\in\mathcal{I}}|1\rangle_i \bigotimes_{j\in\mathcal{N}/\mathcal{I}}|0\rangle_j\bigg). \tag{8}$$

The state $|\tilde{G}_{s,\mathcal{I}}^n\rangle$ differs from $|G_+^n\rangle$ in that spins corresponding to the indices in $\mathcal{I}$ are reversed in both computational basis vectors in the superposition $|G_+^n\rangle$. This means that $|\tilde{G}_{\pm,\mathcal{I}}^n\rangle = |G_\pm^n\rangle$ if $\mathcal{I}$ is the empty set. Moreover, the relative phase of the two vectors can be positive or negative, according to the sign s.

Terms in the Hamiltonian $H^{(m)}$ which are not the identity or the product of an even number of σ^z can act on $|G_+^n\rangle$ by (i) flipping some spins (at most m), (ii) changing the relative sign of the linear superposition, (iii) multiplying the state by an overall constant.

Thus, the vector $|\Psi^{(m)}\rangle$ in Eq. (3) can be expressed as

$$|\Psi^{(m)}\rangle = b_0|G_-^n\rangle + \sum_{|\mathcal{I}|\leq m}(a_{\mathcal{I}}|\tilde{G}_{+,\mathcal{I}}^n\rangle + b_{\mathcal{I}}|\tilde{G}_{-,\mathcal{I}}^n\rangle), \tag{9}$$

where the multi-indices $\mathcal{I}$ are referred to the spins being flipped by the terms in $H^{(m)}$.

The coefficients $a_{\mathcal{I}}$, $b_{\mathcal{I}}$, and b_0 are functions of the parameters of the Hamiltonian (2). It is obvious that, if they can all be set to zero by a proper choice of $H^{(m)}$, $|G_+^{(m)}\rangle$ will be an eigenstate of the Hamiltonian. A problem arises, however, if we take into account the antisymmetric state $|G_-^{(m)}\rangle$. The action of $H^{(m)}$ on this vector reads

$$H^{(m)}|G_-^n\rangle = \epsilon|G_-^n\rangle + |\Phi^{(m)}\rangle, \tag{10}$$

where $|\Phi^{(m)}\rangle$ is orthogonal to $|G_-^n\rangle$ and can be decomposed as

$$|\Phi^{(m)}\rangle = b_0|G_+^n\rangle + \sum_{|\mathcal{I}|\leq m}(a_{\mathcal{I}}|\tilde{G}_{-,\mathcal{I}}^n\rangle + b_{\mathcal{I}}|\tilde{G}_{+,\mathcal{I}}^n\rangle). \tag{11}$$

If all the coefficients in Eq. (9) are set to zero, this will result in the cancellation of $|\Phi^{(m)}\rangle$. As a consequence, $|G_+^n\rangle$ and $|G_-^n\rangle$ will be degenerate eigenstates (with eigenvalue ϵ). Thus, if the sufficient conditions

$$b_0 = 0, \tag{12}$$

$$a_{\mathcal{I}} = 0, \qquad b_{\mathcal{I}} = 0 \tag{13}$$

are also *necessary* for $|G_+^n\rangle$ to be an eigenstate of $H^{(m)}$, degeneracy is unavoidable. We notice that, since the following equality holds

$$\langle G_-^n|\tilde{G}_{s,\mathcal{I}}^n\rangle = 0 \quad \forall\, \mathcal{I} \quad \text{and} \quad \forall\, s, \tag{14}$$

Eq. (12) is always a necessary condition.

Let us start considering the case in which the Hamiltonian (2) contains interaction terms up to m body such that

$$m < m_n^* \equiv [(n+1)/2], \tag{15}$$

with $[\cdot]$ denoting the integer part. Following Eq. (7), the sum in the decomposition of $|\Psi^{(m)}\rangle$ and $|\Phi^{(m)}\rangle$ runs over all the multi-indices whose length satisfies

$$1 \leq |\mathcal{I}| \leq m < m_n^*. \tag{16}$$

If this inequality holds, the following orthogonality relations are verified:

$$\langle\tilde{G}_{s_1,\mathcal{I}_1}^n|\tilde{G}_{s_2,\mathcal{I}_2}^n\rangle = 0 \qquad \text{if } \mathcal{I}_1 \neq \mathcal{I}_2 \text{ or } s_1 \neq s_2. \tag{17}$$

Thus, Eq. (13) is a necessary condition to cancel $|\Psi^{(m)}\rangle$ and make $|G_+^n\rangle$ an eigenstate of $H^{(m)}$. In this case, however, $|G_+^n\rangle$ and $|G_-^n\rangle$ are eigenstates corresponding to the same eigenvalue. We can conclude that, if the Hamiltonian contains terms that couple less than $m_n^* = [(n+1)/2]$ spins, the GHZ state $|G_+^n\rangle$, and any equivalent state by local unitaries, cannot be a nondegenerate eigenstate. If $|G_+^n\rangle$ is an eigenstate for some Hamiltonian $H^{(m)}$, it must be at least twofold degenerate.

On the other hand, if $m = m_n^*$ degeneracy can be avoided. Actually, in this case some conditions in Eq. (13) are no longer necessary and, therefore, the orthogonality relations in Eq. (17) hold if inequality (16) is satisfied. However, a new relation emerges connecting $|\tilde{G}_{s,\mathcal{I}}^n\rangle$ states corresponding to multi-indices of length m_n^* and $(n - m_n^*)$ (which is equal to m_n^* for even n and to $m_n^* - 1$ for odd n). Indeed, reversing m_n^* spins in $|G_+^n\rangle$ is completely equivalent to reversing the other $n - m_n^*$ ones. Instead, if the same operations are applied on the antisymmetric state $|G_-^n\rangle$, they will differ only by an overall sign. Thus, we have the following relations:

$$|\tilde{G}^n_{\pm,\mathcal{I}}\rangle = \pm|\tilde{G}^n_{\pm,\mathcal{N}/\mathcal{I}}\rangle \qquad \text{if } |\mathcal{I}| = m^*_n,\ n - m^*_n. \quad (18)$$

While conditions (13) still hold for $|\mathcal{I}| < \min(m^*_n, n - m^*_n)$, for larger values of $|\mathcal{I}|$ one should use

$$a_{\mathcal{I}} = -a_{\mathcal{N}/\mathcal{I}},\ b_{\mathcal{I}} = b_{\mathcal{N}/\mathcal{I}} \qquad \text{if } |\mathcal{I}| = m^*_n,\ n - m^*_n. \quad (19)$$

Thus, in order to cancel $|\Psi^{(m)}\rangle$, it is no longer necessary to set all the coefficient $a_{\mathcal{I}} = 0$ and $b_{\mathcal{I}} = 0$ in Eq. (9) because this would give a degeneracy (remember that, by the same conditions, one would have $|\Phi^{(m)}\rangle = 0$). Instead, by using Eq. (19), the vector $|\Phi^{(m)}\rangle$ in Eq. (11) becomes

$$|\bar{\Phi}^{(m)}\rangle \equiv \sum_{|\mathcal{I}|=m^*_n, n-m^*_n} (a_{\mathcal{I}}|\tilde{G}^n_{-,\mathcal{I}}\rangle + b_{\mathcal{I}}|\tilde{G}^n_{+,\mathcal{I}}\rangle), \quad (20)$$

which is generally different from the null vector. If, for some values of the parameters in the Hamiltonian (2), the conditions (19) are satisfied without canceling $|\bar{\Phi}^{(m)}\rangle$, the GHZ state $|G^n_+\rangle$ can, at least in principle, be a nondegenerate eigenstate of an Hamiltonian with interaction terms coupling no less than $m^*_n = [(n+1)/2]$ qubits.

Relying on the above considerations, that yield necessary conditions for the GHZ state $|G^n_+\rangle$ to be a nondegenerate eigenstate, one can also obtain a *sufficient* condition. We will introduce a technique, independent of the number of qubits, to build a one-parameter family of m^*_n-qubit perturbations of the Ising Hamiltonian that admits GHZ as a nondegenerate eigenstate. Moreover, we will demonstrate that it is possible to exactly determine the parameter range such that GHZ is the first excited state. Let us consider an Ising ferromagnetic Hamiltonian, acting on a system of $n > 2$ qubits on a circle

$$H_0 = -\sum_{j=1}^{n} \sigma^z_j \sigma^z_{j+1} \quad (\text{with } \boldsymbol{\sigma}_{n+1} \equiv \boldsymbol{\sigma}_1). \quad (21)$$

The states $|G^n_\pm\rangle$, being superpositions of computational basis states with all spins along the z axis, are degenerate ground states of (21) with eigenvalue $E^{(0)} = -n$. The states $|\tilde{G}^n_{s,\mathcal{I}}\rangle$ defined in (8) are excited eigenstates, grouped in degenerate multiplets corresponding to the energy levels

$$E^{(k)} = -n + 4k, \qquad \text{with } k = 1, 2, \ldots, 2[n/2] \quad (22)$$

where k is half the (even) number of "domain walls" generated by the inversion of some spins with respect to $|00\ldots0\rangle$ or $|11\ldots1\rangle$. It is clear that both computational basis states forming the superposition $|\tilde{G}^n_{s,\mathcal{I}}\rangle$ have the same number of domain walls. The energy levels of the Ising Hamiltonian (21) are thus spaced by $\Delta E = -4$.

In order to lift the degeneracy of the ground state, a suitable perturbation should be added to H_0. It is clear from the above considerations that any additional term lifting the degeneracy, with $|G^n_+\rangle$ still an eigenstate, must couple at least $[(n+1)/2]$ qubits. In order to fulfill these requirements, consider

$$H(\lambda) = H_0 + \lambda H_1, \quad (23)$$

with

$$H_1 = \sigma^x_1 \sigma^x_2 \ldots \sigma^x_{[n/2]} - \sigma^x_{[n/2]+1} \ldots \sigma^x_n, \quad (24)$$

made up of two strings of spin flipping matrices σ^x acting on one half of the system. The perturbation (24) is optimal, since it is neither possible to reduce the number of addenda in (24) to one, nor to reduce the range of the couplings. The Hamiltonian $H(\lambda)$ can be analyzed independently of the dimensionality of the system. Here we are not interested in the complete diagonalization, but only in the analysis of its spectrum, with the aim of determining the position of the eigenstate $|G^n_+\rangle$ in the spectrum and conditions on parameter λ that ensure its nondegeneracy. As stated, $H(\lambda)$ is constructed under the requirement that $|G^n_+\rangle$ be an eigenstate. Indeed, it is easy to check that

$$H(\lambda)|G^n_+\rangle = -n|G^n_+\rangle. \quad (25)$$

All positive linear combinations $|\tilde{G}^n_{+,\mathcal{I}}\rangle$, defined in (8), are eigenstates of $H(\lambda)$, since $H_1|\tilde{G}^n_{+,\mathcal{I}}\rangle = 0$. The two-dimensional spaces spanned by the two negative linear combinations $\{|\tilde{G}^n_{-,\mathcal{I}}\rangle, |\tilde{G}^n_{-,\mathcal{I}'}\rangle\}$, with the last $[(n+1)/2]$ spins inverted, are left invariant by $H(\lambda)$. One of these states is mapped by H_1 into the other with a multiplicative factor -2. For example, the action of H_1 on $|G^n_-\rangle$, which is a degenerate ground state of H_0, is

$$H_1|G^n_-\rangle = -2(|0\rangle_1 \ldots |0\rangle_{[n/2]}|1\rangle_{[n/2]+1} \ldots |1\rangle_n - |1\rangle_1 \ldots |1\rangle_{[n/2]}|0\rangle_{[n/2]+1} \ldots |0\rangle_n)/\sqrt{2}. \quad (26)$$

Thus, the energy spectrum of $H(\lambda)$ is completely determined by diagonalizing these two-dimensional sectors. Since the action of H_1 on a state $|\tilde{G}^n_{-,\mathcal{I}}\rangle$ can either leave the number of domain walls unchanged or change it by two, we have two possible cases. In the first case, the two states of the basis (8) coupled by H_1 belong to the same energy level $E^{(k)}$ of the unperturbed Hamiltonian H_0. Therefore, the diagonalization of the sector yields two eigenvalues with a linear dependence on λ:

$$\epsilon^{(k)}_\pm = E^{(k)} \pm 2\lambda. \quad (27)$$

In the second case, one of the states coupled by H_1 has unperturbed energy $E^{(k)}$, while the other one has energy $E^{(k+1)} = E^{(k)} + 4$. Therefore, the eigenvalues of the sector read

$$\eta^{(k)}_- = E^{(k)} - 2\left(\sqrt{1+\lambda^2} - 1\right), \quad (28)$$

$$\eta^{(k+1)}_+ = E^{(k+1)} + 2\left(\sqrt{1+\lambda^2} - 1\right). \quad (29)$$

It is clear that $\epsilon^{(k)}_- < \eta^{(k)}_- < \eta^{(k)}_+ < \epsilon^{(k)}_+$ for $\lambda \neq 0$. The twofold degenerate ground state of H_0 is split in two

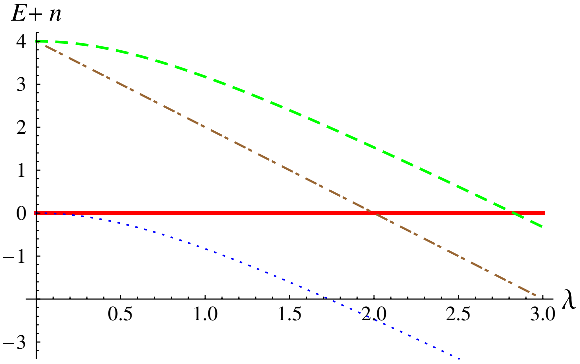

FIG. 1 (color online). Energy levels of $H(\lambda)$ as a function of λ. The solid (red) line represents the constant energy of the eigenstate $|G_-^n\rangle$. The dotted (blue) line is the ground state energy. The dot-dashed (brown) line and the dashed (green) line correspond to the energy levels $\epsilon_-^{(1)}$ and $\eta_-^{(1)}$, that cross the GHZ energy, respectively, at $\lambda = 2$ and $\lambda = 2\sqrt{2}$.

energy levels. The upper one corresponds to the GHZ eigenstate $|G_+^n\rangle$, whose energy is independent of λ and satisfies Eq. (25). The lower one corresponds to the energy $\eta_-^{(0)} = -n - 2(\sqrt{1+\lambda^2} - 1)$, and represents the nondegenerate ground state of $H(\lambda)$ for all $\lambda \neq 0$. Therefore, the GHZ state $|G_+^n\rangle$ is the nondegenerate first-excited state of $H(\lambda)$ for $0 < \lambda < 2$ (and $-2 < \lambda < 0$), since for small λ the degeneracy of the ground state of H_0 is lifted, and $|G_+^n\rangle$ continues to be nondegenerate for increasing values of λ up to $\lambda = 2$, where a crossing with the energy level $\epsilon_-^{(1)} = -n + 4 - 2\lambda$ occurs. Figure 1 displays a plot of the lowest energy levels as a function of λ.

We have obtained a sufficient condition ensuring that a GHZ state is the nondegenerate first-excited state of an $[(n+1)/2]$-body Hamiltonian. More generally, from (27)–(29), it is also possible to determine all values of λ for which the GHZ state $|G_+^n\rangle$ is a nondegenerate excited eigenstate of $H(\lambda)$. The eigenstate $|G_+^n\rangle$ is surely nondegenerate if

$$\lambda \neq \pm 2k \quad \text{and} \quad \lambda \neq \pm 2\sqrt{k(k+1)} \qquad (30)$$

with $k = 0, 1, \ldots, 2[n/2]$. We stress that these results are independent of the number of qubits of the system. Moreover, interestingly, the energy gap between $|G_+^n\rangle$, the ground state and the second-excited states for $0 < \lambda < 2$ does not depend on the size of the system.

We observe that many-body interaction terms have been recently engineered in a spin lattice by implementing an effective dynamics that makes use of an ancilla and stroboscopiclike interactions [21]. This shows that the Hamiltonian (24) can be experimentally realized for reasonable values of n.

In conclusion, we investigated general conditions such that GHZ states (1) are nondegenerate in the spectrum of a Hamiltonian. We showed that if the Hamiltonian acting on the Hilbert space of n qubits involves terms that couple at most m qubits, it is impossible to have a nondegenerate GHZ eigenstate if $m < m_n^*$ with $m_n^* = [(n+1)/2]$. We then derived sufficient conditions for the GHZ to be the nondegenerate first-excited eigenstate of the m_n^*-qubit perturbation (23) of the Ising Hamiltonian.

The difficulty in obtaining GHZ states as ground states (or even eigenstates) of Hamiltonians that involve only few-body interactions is in accord with previous results [22] and seems to be a characteristic trait of multipartite entanglement. It would be interesting, also in view of applications, to investigate the existence of general conditions for obtaining approximate GHZ states for an arbitrary number of qubits by making use of few-body Hamiltonians.

P. F. and G. F. acknowledge support through the project IDEA of University of Bari.

[1] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, 2000).
[2] W. K. Wootters, Quantum Inf. Comput. **1**, 27 (2001).
[3] L. Amico, R. Fazio, A. Osterloh, and V. Vedral, Rev. Mod. Phys. **80**, 517 (2008).
[4] R. Horodecki, P. Horodecki, M. Horodecki, and K. Horodecki, Rev. Mod. Phys. **81**, 865 (2009).
[5] G. Adesso and F. Illuminati, J. Phys. A **40**, 7821 (2007).
[6] D. M. Greenberger, M. A. Horne, and A. Zeilinger, *Bell's Theorem, Quantum Theory, and Conceptions of the Universe*, edited by M. Kafatos (Kluwer Academic, Dordrecht, 1989), pp. 69–72.
[7] W. Dur, G. Vidal, and J. I. Cirac, Phys. Rev. A **62**, 062314 (2000).
[8] V. Coffman, J. Kundu, and W. K. Wootters, Phys. Rev. A **61**, 052306 (2000).
[9] D. Bouwmeester, J. W. Pan, M. Daniell, H. Weinfurter, and A. Zeilinger, Phys. Rev. Lett. **82**, 1345 (1999).
[10] D. Leibfried, E. Knill, S. Seidelin, J. Britton, R. B. Blakestad, J. Chiaverini, D. B. Hume, W. M. Itano, J. D. Jost, C. Langer, R. Ozeri, R. Reichle, and D. J. Wineland, Nature (London) **438**, 639 (2005).
[11] W.-B. Gao, C.-Y. Lu, X.-C. Yao, P. Xu, O. Guhne, A. Goebel, Y.-A. Chen, C.-Z. Peng, Z.-B. Chen, and J.-W. Pan, Nature Phys. **6**, 331 (2010).
[12] R. Prevedel, G. Cronenberg, M. S. Tame, M. Paternostro, P. Walther, M. S. Kim, and A. Zeilinger, Phys. Rev. Lett. **103**, 020503 (2009).
[13] T. Monz, P. Schindler, J. T. Barreiro, M. Chwalla, D. Nigg, W. A. Coish, M. Harlander, W. Hänsel, M. Hennrich, and R. Blatt, Phys. Rev. Lett. **106**, 130506 (2011).
[14] K. Mølmer and A. Sørensen, Phys. Rev. Lett. **82**, 1835 (1999).
[15] L. F. Wei, Yu-xi Liu, and F. Nori, Phys. Rev. Lett. **96**, 246803 (2006); Ying-Dan Wang, S. Chesi, D. Loss, and C. Bruder, Phys. Rev. B **81**, 104524 (2010).
[16] P. Stelmachovic and V. Buzek, Phys. Rev. A **70**, 032313 (2004).

[17] B. Rothlisberger, J. Lehmann, D. S. Saraga, P. Traber, and D. Loss, Phys. Rev. Lett. **100**, 100502 (2008).
[18] B. Rothlisberger, J. Lehmann, and D. Loss, Phys. Rev. A **80**, 042301 (2009).
[19] H. L. Haselgrove, M. A. Nielsen, and T. J. Osborne, Phys. Rev. A **69**, 032303 (2004).
[20] M. Van den Nest, K. Luttmer, W. Dür, and H. J. Briegel, Phys. Rev. A **77**, 012301 (2008).
[21] H. P. Büchler, A. Micheli, and P. Zoller, Nature Phys. **3**, 726 (2007); H. Weimer, M. Müller, I. Lesanovsky, P. Zoller, and H. P. Büchler, Nature Phys. **6**, 382 (2010); J. T. Barreiro, M. Müller, P. Schindler, D. Nigg, T. Monz, M. Chwalla, M. Hennrich, C. F. Roos, P. Zoller, and R. Blatt, Nature (London) **470**, 486 (2011).
[22] P. Facchi, G. Florio, S. Pascazio, and F. Pepe, Phys. Rev. A **82**, 042313 (2010).