Hidden non-Markovianity in open quantum systems

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(Received 1 October 2020; revised 24 November 2020; accepted 11 December 2020; published 7 January 2021)

We show that non-Markovian open quantum systems can exhibit exact Markovian dynamics up to an arbitrarily long time; the non-Markovianity of such systems is thus perfectly "hidden," i.e., not experimentally detectable by looking at the reduced dynamics alone. This shows that non-Markovianity is physically undecidable and extremely counterintuitive, since its features can change at any time, without precursors. Some interesting examples are discussed.

DOI: 10.1103/PhysRevA.103.012203

I. INTRODUCTION

The recent advance in quantum technology brought with it a renewed interest in the study of quantum noise. Never before have we built such complex high-dimensional quantum systems, which naturally come with spatially and temporally correlated noise, and never before have we demanded such purity in quantum dynamics required for scalable quantum computation. Simplistic error models no longer suffice to achieve optimal performance [1].

A particular noise feature, the analysis of the Markovianity (or lack thereof) of the system, is of primary interest. A continuous process is said to be Markovian if its evolution at any future time is determined unambiguously from the current state, rather than by the full history of the system that led it to the present state. The lack of Markovianity is inherently linked with the two-way exchange of information between the system and the bath; a Markovian description is legitimate, even if only as an approximation, whenever the observed time scale of the evolution is much larger than the correlation time that characterizes the interaction between system and bath. Non-Markovianity is a complex phenomenon which affects the system both in its dynamical and informational features; several nonequivalent definitions of non-Markovianity, each focusing on particular aspects of memory, have been given. For a recent review we refer to Ref. [2].

Non-Markovianity was discussed in a variety of physical systems and experimental platforms, such as cold atoms [3,4], superconducting qubits [5,6], photonic crystals [7,8], waveguide quantum electrodynamics [9–12], optical fibers [13], all-optical setups [14–17], and photonic waveguides [18,19], the list being far from exhaustive. Most of these systems are well described by a paradigmatic theoretical model: the spin-boson model, consisting of a two-level quantum system (qubit) interacting with a boson bath, the resulting rich phenomenology being ascribable to the structure of the bath and its interaction with the qubit.

Here we define a quantum evolution Λ_t to be Markovian if it is described by a quantum dynamical semigroup,

$$\Lambda_t = e^{-t\mathcal{L}}, \tag{1}$$

with a time-independent generator $\mathcal{L}$ [20,21]. This narrow definition of Markovianity is a common core of many of the inequivalent definitions in the literature, although it is worth pointing out that such a definition does not capture the effect of time-dependent driving and other interventions [22].

The question if a fixed-time quantum operation Λ_{t_0} ("snapshot") can be embedded into a Markovian evolution $e^{-t_0\mathcal{L}}$ was initiated in Ref. [23] and, remarkably, shown to be an NP-hard problem [24]. On the other hand, if more information, say the whole time evolution Λ_t for a time window $0 \leqslant t_1 < t < t_2$, is provided, it appears to be easy to decide Markovianity, simply by checking if the generator $-\Lambda_t^{-1}\frac{d}{dt}\Lambda_t$ exists and has time-independent Lindblad structure.

The main point of our contribution is to show that this is incorrect: deciding Markovianity remains hard for arbitrarily large windows. Without further knowledge on the environment or interventions on the dynamics it is, in fact, physically undecidable. See Fig. 1.

Recently, Tufarelli and co-authors [5] showed that there are systems which behave approximately Markovian up to a critical time T, and non-Markovian thereafter. Although for time windows which do not exceed T it is harder to assess non-Markovianity in such systems, they will still exhibit precursors (in the spirit of Ref. [25]) of non-Markovianity due to the coarse graining of the Markovian approximation. That is, there will be slight deviations from the exact semigroup structure which reveal and anticipate the non-Markovianity at later time.

In this article we will, however, show that in fact the spin-boson model can give rise to qubit evolutions which are

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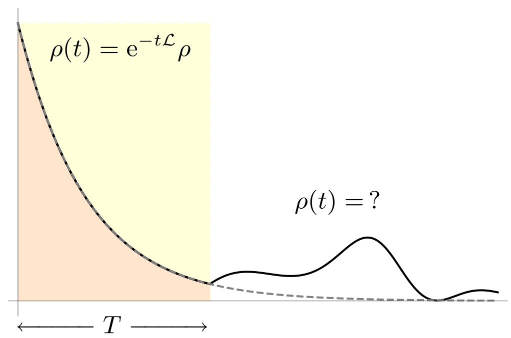

FIG. 1. If a pure Markovian evolution is observed up to a time $t = T$, will the dynamics be Markovian for $t > T$ (dashed line) or might the dynamics deviate from Markovianity (solid line)?

exactly Markovian up to some critical time T, without any precursor deviation of its dynamics. Since T can be arbitrarily large, we conclude that Markovianity of a quantum evolution cannot be assessed, even in the simplest case of a two-dimensional quantum system (qubit), by simply looking at the dynamics in a finite, however large, time window, as shown in Fig. 1.

In order to do so, we construct explicitly a full family of non-Markovian quantum channels, for a qubit interacting with a given boson bath, whose dynamics is indistinguishable from the one induced by an exactly Markovian evolution up to a finite time. The reduced evolution of the qubit is characterized by the following master equation:

$$\dot{\rho}(t) = -i\varepsilon(t)[H_q, \rho(t)] - \gamma(t)\mathcal{L}_{\mathrm{D}}(\rho(t)), \tag{2}$$

with $\rho(t) = \Lambda_t(\rho)$ being the density matrix of the qubit at time t, $\mathcal{L}_{\mathrm{D}}$ being the Lindblad superoperator [20,21] associated with an amplitude-damping channel, H_{q} the Hamiltonian of the qubit, and $\gamma(t)$, $\varepsilon(t)$ being two real functions that only depend on the characteristics of the coupling between system and bath.

In general, the quantum channel solving the master equation (2) will satisfy the semigroup property $\Lambda_{t+s} = \Lambda_t\Lambda_s$ at *all* times $t, s \geqslant 0$ only if the coupling is engineered in such a way that $\gamma(t)$ and $\varepsilon(t)$ are constant functions, which, as will be explained later, can only be obtained with an (essentially) unique choice of the coupling. However, there are infinitely many ways to engineer the coupling in such a way that the semigroup property is satisfied only up to a finite time T:

$$\Lambda_{t+s} = \Lambda_t\Lambda_s \quad \text{for all } t, s \geqslant 0, \quad t + s \leqslant T, \tag{3}$$

with T itself only depending on the choice of coupling. This can be obtained by choosing it in such a way that the reduced dynamics of the system satisfies Eq. (2) with $\gamma(t)$ and $\varepsilon(t)$ being constant only up to $t = T$. Such a system is by definition non-Markovian, but its non-Markovianity is hidden: no observation at times $t \leqslant T$ will detect any deviation from Markovianity.

In fact, we show that this phenomenon can occur even in the extreme case of non-Markovian systems, that is, spatially confined systems. Even though the Hamiltonian will have a discrete spectrum and therefore the system will display recurring dynamics [26] and (partial) quantum revivals [27,28], by an appropriate choice of the coupling, its dynamics can be made perfectly Markovian for all times less than T, as if the system were spatially unbounded. The system will start exhibiting an oscillatory behavior, typical of a discrete spectrum, only after the time T.

II. THE MODEL

We consider a qubit with ground state $|1\rangle$ and excited state $|0\rangle$ at energy ω_0, interacting with a boson quantum bath with creation and annihilation operators $b_\omega^\dagger$ and b_ω satisfying the commutation relations $[b_\omega, b_{\omega'}^\dagger] = \delta(\omega - \omega')$. The Hamiltonian has the form $H = H_0 + H_{\mathrm{int}}$, where

$$H_0 = \omega_0 H_{\mathrm{q}} \otimes \mathbb{1} + \mathbb{1} \otimes H_{\mathrm{B}} \tag{4}$$

is the free Hamiltonian with

$$H_{\mathrm{q}} = |0\rangle\langle 0|, \quad H_{\mathrm{B}} = \int d\omega\, \omega\, b_\omega^\dagger b_\omega \tag{5}$$

being the Hamiltonians of the qubit and the field, respectively. The qubit-field interaction has the form

$$H_{\mathrm{int}} = \sigma_+ \otimes B(g) + \sigma_- \otimes B(g)^\dagger, \quad B(g) = \int d\omega\, g(\omega)^* b_\omega, \tag{6}$$

where $\sigma_+ = |0\rangle\langle 1|$, $\sigma_- = |1\rangle\langle 0|$, and the function $g(\omega)$, the form factor, weights the strength of the interaction of the qubit with a boson of energy ω. Interaction (6) has a rotating-wave form: a boson with wavefunction $g(\omega)$ is created if the qubit undergoes the transition $|0\rangle \to |1\rangle$, and is annihilated if the qubit is excited $|1\rangle \to |0\rangle$. As a consequence, the excitation number

$$N = \sigma_+\sigma_- + \int d\omega\, b_\omega^\dagger b_\omega \tag{7}$$

is conserved, $[N, H] = 0$, so that the sectors with given excitation number are invariant under the evolution. In particular, the component of the Hamiltonian in the one-excitation sector, known as the Friedrichs-Lee model [29,30], has very rich mathematical properties that have been extensively studied in Refs. [31,32].

We focus on the reduced dynamics induced by this Hamiltonian on a state $\rho \otimes |\mathrm{vac}\rangle\langle\mathrm{vac}|$ by tracing out the bath, with the vector $|\mathrm{vac}\rangle$ being the vacuum of the boson field characterized by $b_\omega|\mathrm{vac}\rangle = 0$ for all ω. Define

$$\rho(t) = \Lambda_t(\rho) = \mathrm{tr}_{\mathrm{bath}}(e^{-itH}\rho \otimes |\mathrm{vac}\rangle\langle\mathrm{vac}|\, e^{itH}). \tag{8}$$

It can be shown that $\rho(t)$ is given by [33]

$$\rho(t) = \begin{pmatrix} |a(t)|^2\rho_{00} & a(t)\rho_{01} \\ a(t)^*\rho_{10} & \rho_{11} + (1 - |a(t)|^2)\rho_{00} \end{pmatrix}, \tag{9}$$

where $a(t)$ is a complex function with $a(0) = 1$, $|a(t)| \leqslant 1$ that is solely determined by the coupling function $|g(\omega)|^2$ and the energy of the state $|0\rangle$ (see the Appendix). Physically, $a(t)$ is the survival amplitude of the state $|0\rangle$.

The density matrix $\rho(t)$ satisfies the master equation (2) with

$$\mathcal{L}_{\mathrm{D}}(\rho) = -\sigma_-\rho\sigma_+ + \tfrac{1}{2}\{\sigma_+\sigma_-, \rho\}, \tag{10}$$

$H_q = |0\rangle\langle 0|$, and the functions $\gamma(t)$ and $\varepsilon(t)$ being defined via

$$a(t) = e^{-\int_0^t ds\,\left(\frac{\gamma(s)}{2}+i\varepsilon(s)\right)}. \tag{11}$$

In general, this system will not satisfy the semigroup property unless we select $g(\omega)$ in such a way that $\gamma(t)$ and $\varepsilon(t)$ are constant. This can be accomplished by taking a flat form factor, i.e.,

$$|g(\omega)|^2 = \frac{\gamma_0}{2\pi}, \tag{12}$$

for some $\gamma_0 > 0$: the qubit couples with the same strength to all frequencies of the boson field. Although the Hamiltonian is singular in such a case, and expressions (4)–(6) are only formal, in the Appendix we prove that they yield a bona fide unitary evolution of the total system (the corresponding Hamiltonian H is self-adjoint [31]). In fact, the qubit density matrix $\rho(t)$ satisfies Eq. (2) with $\gamma(t) = \gamma_0$ and $\varepsilon(t) = \varepsilon_0$ both being constant, where $\varepsilon_0 = \omega_0 + \delta\omega_0$ is the dressed energy of the excited state $|0\rangle$ in interaction with the boson field [31], thus

$$a(t) = e^{-(\frac{\gamma_0}{2}+i\varepsilon_0)t}, \tag{13}$$

and, in particular, the channel satisfies the semigroup property at *all* times $t, s \geqslant 0$; i.e., it is Markovian.

However, we can choose the coupling $g(\omega)$ in such a way that $a(t)$ is exactly exponential only up to a finite time T, and is no longer exponential afterwards; see Fig. 1. In such a way, Eq. (3) holds and we obtain a non-Markovian system whose non-Markovianity is, however, *hidden*: no experiment performed within the time horizon T will be able to detect any deviation from the exponential law. This can be accomplished by choosing a periodic coupling $|g(\omega)|^2$, whose Fourier series reads

$$|g(\omega)|^2 = \frac{\gamma_0}{2\pi}\left(1 + 2\sum_{n=1}^{\infty} c_n \cos(nT\omega)\right). \tag{14}$$

The damping function $a(t)$ corresponding to this coupling is evaluated in the Appendix and reads

$$a(t) = e^{-(i\varepsilon_0+\frac{\gamma_0}{2})t} + \sum_{n=1}^{\infty} e^{-(i\varepsilon_0+\frac{\gamma_0}{2})(t-nT)} \times \phi_n(\gamma_0(t-nT))\,\theta(t-nT), \tag{15}$$

where θ is the Heaviside step function; this function is exactly exponential up to $t = T$, while nonexponential terms start adding up at times nT, for $n = 1, 2, \ldots$. In detail, here $\phi_n(x)$ is a polynomial of degree n whose coefficient can be analytically computed in terms of the coefficients c_n.

Physically, the above behavior is a consequence of the time-energy uncertainty relation

$$\Delta t\,\Delta\omega \geqslant \tfrac{1}{2} \tag{16}$$

(a general property of the Fourier transform). Any measurements performed within the time window $[0, T]$ cannot resolve energy differences $\Delta\omega$ below $1/(2T)$. Therefore, the observation of the decay in a time window of width T will depend on a coarse graining of the form factor. A coarse-grained periodic coupling will be indistinguishable from a flat one if the resolution is larger than its periodicity, hence the first line in Eq. (15). Only for times larger than T the system will start to resolve the finer details of a nonflat coupling and the underlying non-Markovianity will start to become manifest via the additional terms in the second line of Eq. (15).

In the following we furnish two explicit examples of form factors $g(\omega)$ for which all terms in Eq. (15) can be evaluated explicitly.

III. TWO EXAMPLES

The simplest nontrivial example can be obtained by setting, in Eq. (14),

$$c_1 = -\frac{\alpha}{2}, \quad c_n = 0 \ \text{ for all } n \geqslant 2, \tag{17}$$

for some $|\alpha| \leqslant 1$; in this case, $|g(\omega)|^2$ is a sinusoidal function whose amplitude is maximal for $\alpha = \pm 1$ and null for $\alpha = 0$. Physically, the choice $\alpha = 1$ can be associated with a quantum emitter coupled with a semi-infinite waveguide with a perfect mirror at one end. In such a case non-Markovianity can be explained by the presence of a delay due to a finite propagation speed: the information is transferred from the emitter to the photon, which moves away from the emitter, bounces back from the mirror at a finite distance, and returns to the emitter after a finite time T. Therefore, the evolution is described by a delay differential equation (DDE), which was first obtained through some approximations in Ref. [3], while the non-Markovianity of the system was thoroughly investigated in Refs. [5,10] via non-Markovianity measures. The case $\alpha = 0$ corresponds again to a flat coupling, and thus to a Markovian evolution at all times.

All polynomials $\phi_n(x)$ in Eq. (15) have the simple form

$$\phi_n(x) = \frac{1}{n!}\left(\frac{\alpha x}{2}\right)^n, \tag{18}$$

(see Appendix) and thus the function $a(t)$ can be evaluated at all times (see Fig. 2). The results can be summarized as follows. With respect to the pure exponential decay at $\alpha = 0$, the decay will be either enhanced or slowed down depending on the values of the parameters α, ε_0, and γ_0, and, in particular, for any fixed α the decay will be slowest when $g(\varepsilon_0)$ is smallest, i.e., when $\varepsilon_0 T = 2\nu\pi$ for some integer ν. In particular, if $\alpha = 1$ and $\varepsilon_0 = 2\nu\pi$, $a(t)$ does not decay at all: a bound state is obtained. In the physical implementation of the model in waveguide QED, the emitter is at a distance of an integer number of half wavelengths from the mirror and the photon is trapped between emitter and mirror. The departure from Markovianity is thus maximal.

Another instance of periodic coupling for which $a(t)$ can be computed exactly is obtained by setting, in Eq. (14),

$$c_n = e^{-\beta n} \tag{19}$$

for some $\beta \geqslant 0$. If $\beta = 0$, this is a comb of Dirac functions placed at integer values of the energy, while for $\beta > 0$ it is a "smoothed" comb. A physical implementation of the discrete case $\beta = 0$ can be obtained by considering a closed-loop waveguide or a one-dimensional optical cavity: indeed, when confining the boson field in a finite space, the emitter will only interact with a countable set of boson states. Interestingly enough, the DDE for $\beta = 0$ was already obtained in 1983 by Milonni and co-authors [34] in a different framework, and

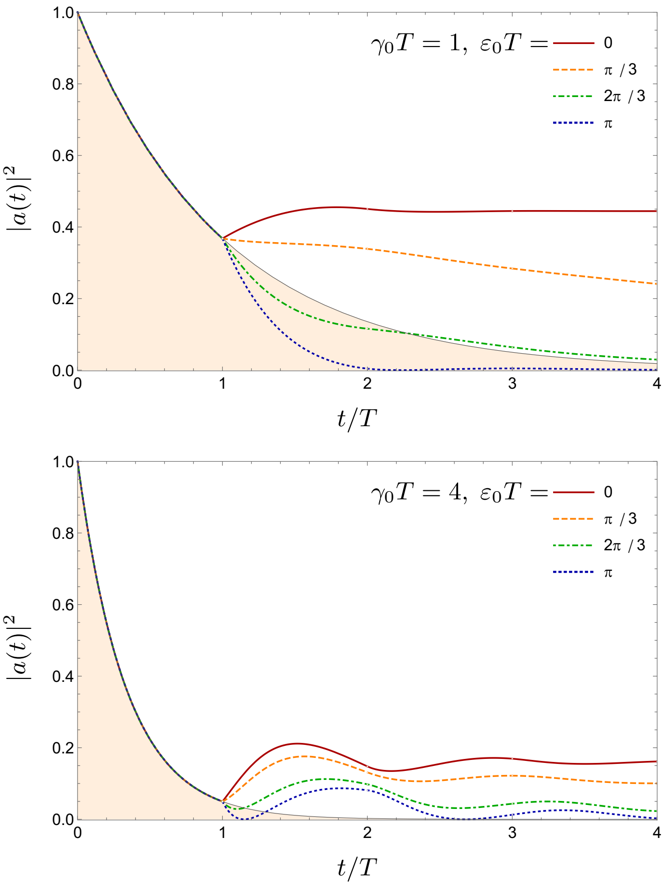

FIG. 2. Survival amplitude $a(t)$ corresponding to a periodic coupling with Fourier coefficients as given in Eq. (17), with $\alpha = 1$ and $\varepsilon_0 T = 0, \pi/3, 2\pi/3, \pi \pmod{2\pi}$.

has been rediscovered afterwards a couple of times. The flat coupling is recovered in the opposite limit $\beta \to +\infty$.

Again, with this class of couplings the dynamics is exactly computable at all times (see Appendix): Eq. (15) holds with

$$\phi_n(x) = e^{-\beta n} \sum_{m=1}^{n} \binom{n-1}{m-1} \frac{(-x)^m}{m!}, \tag{20}$$

implying that the non-Markovian contributions to the survival amplitude have the same functional expression for all β, up to a total weight $e^{-\beta n}$ which suppresses such contributions as n grows, provided that $\beta > 0$. As a result, the larger β, the quicker such contributions "switch off," whereas for small β those contributions are non-negligible for a longer time. In particular, in the limit $\beta \to \infty$ all non-Markovian contributions vanish and we recover the exponential decay at all times. In the opposite limit $\beta \to 0$, where the coupling is discrete, no exponential suppression of such contribution happens and we have recurring dynamics with revivals at all times as shown in Fig. 3.

IV. CONCLUSIONS

In this article we show that no finite-time measurement can establish Markovianity of an open quantum system: the non-Markovianity may indeed be hidden, in the sense that non-Markovian effects may only switch on after some time

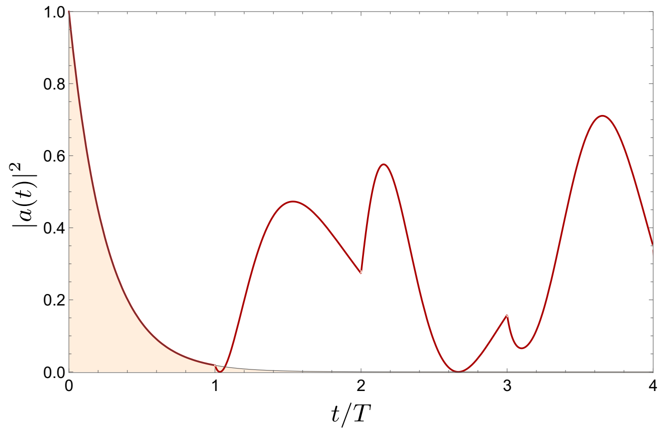

FIG. 3. Survival amplitude $a(t)$ corresponding to a periodic coupling with Fourier coefficients as given in Eq. (19), with $\beta = 0$, $\varepsilon_0 T = 0$, and $\gamma_0 T = 4$.

threshold. To show this, we have considered a model of interaction between a qubit and a boson bath which reduces to an amplitude-damping channel for the former, with a survival amplitude which can be tuned by properly choosing the form factor of the coupling; whenever the latter is a periodic function, non-Markovian effects will only arise after a finite time. Remarkably, such corrections can be computed exactly: two particular examples have been discussed.

ACKNOWLEDGMENTS

D.B. acknowledges support by the Australian Research Council (Project No. FT190100106). P.F., M.L., and D.L. were partially supported by the Italian National Group of Mathematical Physics (GNFM-INdAM). P.F. and D.L. were partially supported by Istituto Nazionale di Fisica Nucleare (INFN) through the project "QUANTUM," and by Regione Puglia and QuantERA ERA-NET Cofund in Quantum Technologies (Grant Agreement No. 731473), project PACE-IN.

APPENDIX

1. The model

We consider a qubit, with ground and excited states $|1\rangle$ and $|0\rangle$, respectively, interacting with a bosonic quantum bath at zero temperature. The microscopic Hamiltonian is $H = H_0 + H_{\rm int}$, where

$$H_0 = \omega_0 H_{\rm q} \otimes \mathbb{1} + \mathbb{1} \otimes H_{\rm B} \tag{A1}$$

and

$$H_{\rm q} = \sigma_+\sigma_- = |0\rangle\langle 0|\,, \quad H_{\rm B} = \int d\omega\, \omega\, b_\omega^\dagger b_\omega, \tag{A2}$$

are the qubit Hamiltonian and the bath Hamiltonian, respectively, while

$$H_{\rm int} = \sigma_+ \otimes B(g) + \sigma_- \otimes B^\dagger(g) \tag{A3}$$

is the interaction Hamiltonian with

$$\sigma_+ = \sigma_-^\dagger = |0\rangle\langle 1|\,, \quad B(g) = \int d\omega\, g(\omega)^* b_\omega, \tag{A4}$$

and b_ω and $b_\omega^\dagger$ are the bosonic annihilation and creation operators, satisfying the canonical commutation relations $[b_\omega, b_{\omega'}^\dagger] = \delta(\omega - \omega')$, $[b_\omega, b_{\omega'}] = 0$. That is,

$$H = \omega_0 |0\rangle\langle 0| \otimes \mathbb{1} + \mathbb{1} \otimes \int d\omega\, \omega\, b_\omega^\dagger b_\omega + |0\rangle\langle 1| \otimes \int d\omega\, g(\omega)^* b_\omega + |1\rangle\langle 0| \otimes \int d\omega\, g(\omega) b_\omega^\dagger. \tag{A5}$$

Here $g(\omega)$ is a complex function that weights the strength of the interaction; the interaction term is constructed in such a way that a boson is created if the qubit undergoes the transition $|0\rangle \to |1\rangle$, and is annihilated if the qubit undergoes the transition $|1\rangle \to |0\rangle$. The excitation number

$$N = \sigma_+\sigma_- \otimes \mathbb{1} + \mathbb{1} \otimes \int d\omega\, b_\omega^\dagger b_\omega \tag{A6}$$

is conserved, $[N, H] = 0$; thus, the eigenspaces of N with eigenvalues $0, 1, 2, \dots$ are reducing subspaces for the Hamiltonian H which splits into a direct sum of operators. The eigenspace corresponding to the eigenvalue $N = 0$ is one dimensional and is spanned by the vector $|1, \mathrm{vac}\rangle := |1\rangle \otimes |\mathrm{vac}\rangle$, while the eigenspace corresponding to the eigenvalue $N = 1$ (one-excitation sector) is the linear span of the vectors $|0, \mathrm{vac}\rangle := |0\rangle \otimes |\mathrm{vac}\rangle$ and $|1, \omega\rangle := |1\rangle \otimes b_\omega^\dagger |\mathrm{vac}\rangle$. The latter subspace is isomorphic to $\mathbb{C} \oplus L^2(\omega)$ and the restriction of H to it is known as the Friedrichs-Lee Hamiltonian [32]; its properties are extensively studied in the references pointed out in the main text. The eigenspaces with higher excitation numbers are spanned by states with at least one photon. For example, the two-excitation sector is spanned by $|0\rangle \otimes b_\omega^\dagger |\mathrm{vac}\rangle$ and $|1\rangle \otimes b_\omega^\dagger b_{\omega'}^\dagger |\mathrm{vac}\rangle$.

We assume that the initial state of the bath is the vacuum, $|\mathrm{vac}\rangle$. Thus, we focus on the reduced dynamics induced by Hamiltonian (A5) on a state $\rho \otimes |\mathrm{vac}\rangle\langle \mathrm{vac}|$, by tracing out the bath. We evaluate the following quantity:

$$\rho(t) = \Lambda_t(\rho) = \mathrm{tr}_{\mathrm{bath}}(e^{-itH} \rho \otimes |\mathrm{vac}\rangle\langle \mathrm{vac}|\, e^{itH}), \tag{A7}$$

with ρ being an arbitrary density matrix of the qubit,

$$\rho = \sum_{j=0}^{1} \rho_{j\ell} |j\rangle\langle \ell| = \begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix} \tag{A8}$$

with

$$\rho = \rho^*, \quad \rho \geqslant 0, \quad \mathrm{tr}(\rho) = \rho_{00} + \rho_{11} = 1. \tag{A9}$$

As such, the evolved density matrix $\rho(t)$ reads

$$\rho(t) = \sum_{j,\ell=0}^{1} \rho_{j\ell}\, \mathrm{tr}_{\mathrm{bath}}(e^{-itH} |j, \mathrm{vac}\rangle\langle \ell, \mathrm{vac}|\, e^{itH}), \tag{A10}$$

where $|j, \mathrm{vac}\rangle := |j\rangle \otimes |\mathrm{vac}\rangle$, for all $j = 0, 1$. Consequently, we need to compute

$$e^{-itH} |0, \mathrm{vac}\rangle, \quad e^{-itH} |1, \mathrm{vac}\rangle \tag{A11}$$

for all t, i.e., the evolution of $|0, \mathrm{vac}\rangle$ and $|1, \mathrm{vac}\rangle$ under the action of the Hamiltonian H. First of all, notice that

$$\begin{aligned} H |1, \mathrm{vac}\rangle &= 0, \\ H |0, \mathrm{vac}\rangle &= \omega_0 |0, \mathrm{vac}\rangle + \int d\omega\, g(\omega) |1, \omega\rangle, \\ H |1, \omega\rangle &= \omega |1, \omega\rangle + g(\omega)^* |0, \mathrm{vac}\rangle. \end{aligned} \tag{A12}$$

Therefore, the evolution of state $|1, \mathrm{vac}\rangle$ is trivial,

$$e^{-itH} |1, \mathrm{vac}\rangle = |1, \mathrm{vac}\rangle, \tag{A13}$$

while the components $|1, \omega\rangle$ and $|0, \mathrm{vac}\rangle$ evolve nontrivially, without mixing with the previous component.

2. Evolution in the one-excitation sector

The Schrödinger equation for a global time-dependent state of the form

$$|\Psi(t)\rangle = a(t) |0, \mathrm{vac}\rangle + \int d\omega\, c(t, \omega) |1, \omega\rangle \tag{A14}$$

reads

$$\begin{aligned} i\,\dot{a}(t) |0, \mathrm{vac}\rangle + i \int d\omega\, \dot{c}(t, \omega) |1, \omega\rangle &= \int d\omega\, (a(t)\, g(\omega) + \omega\, c(t, \omega)) |1, \omega\rangle \\ &\quad + \left(\omega_0\, a(t) + \int d\omega\, g(\omega)^* c(t, \omega)\right) |0, \mathrm{vac}\rangle, \end{aligned} \tag{A15}$$

finally yielding a system of coupled differential equations in $a(t)$ and $c(t, \omega)$:

$$\begin{aligned} i\,\dot{a}(t) &= \omega_0\, a(t) + \textstyle\int d\omega'\, g(\omega')^* c(t, \omega') \\ i\,\dot{c}(t, \omega) &= g(\omega)\, a(t) + \omega\, c(t, \omega). \end{aligned} \tag{A16}$$

This is exactly the same differential equation that is obtained in Ref. [32], albeit in a much more general case, for the generic state of a Friedrichs-Lee Hamiltonian; in this sense, as stated in the main text, our system is a "variation" of the Friedrichs-Lee model [29,30]. The solution of this system was found explicitly in Ref. [33]. In particular, by choosing as an initial condition the state $|\Psi(0)\rangle = |0, \mathrm{vac}\rangle$, i.e., $a(0) = 1$ and $c(0, \omega) = 0$, and by taking the Fourier-Laplace transform, for $z \in \mathbb{C}$ with $\mathrm{Im}\, z > 0$,

$$\begin{aligned} \hat{a}(z) &= i \int_0^{+\infty} dt\, e^{itz} a(t), \\ \hat{c}(z, \omega) &= i \int_0^{+\infty} dt\, e^{itz} c(t, \omega), \end{aligned} \tag{A17}$$

we get

$$\begin{aligned} z\hat{a}(z) + 1 &= \omega_0\, \hat{a}(z) + \textstyle\int d\omega'\, g(\omega')^* \hat{c}(z, \omega') \\ z\hat{c}(z, \omega) &= g(\omega)\, \hat{a}(z) + \omega\, \hat{c}(z, \omega). \end{aligned} \tag{A18}$$

By plugging the second equation into the first we have

$$\hat{a}(z) = \frac{1}{\omega_0 - z - \Sigma_0(z)}, \tag{A19}$$

where

$$\Sigma_0(z) = \int d\omega\, \frac{|g(\omega)|^2}{\omega - z} \tag{A20}$$

is the bare self-energy function. The latter is well defined for $\mathrm{Im}\, z > 0$ as far as $\int d\omega\, |g(\omega)|^2/(|\omega| + 1) < +\infty$, which is the case if the form factor $g(\omega)$ is a square-integrable function.

3. Singular couplings and renormalization

If the form factor $g(\omega)$ is not a square-integrable function [e.g., flat form factor $g(\omega) = \text{const}$] the bare self-energy $\Sigma_0(z)$ in Eq. (A20) diverges and a renormalization procedure is required. Such form factors and the corresponding Hamiltonians will be called *singular*.

The renormalization procedure consists in expressing the propagator $\hat{a}(z)$ in terms of dressed quantities, $\tilde{\omega}_0$ and $\Sigma(z)$, instead of bare ones, ω_0 and $\Sigma_0(z)$, namely,

$$\hat{a}(z) = \frac{1}{\tilde{\omega}_0 - z - \Sigma(z)}, \tag{A21}$$

where

$$\tilde{\omega}_0 = \omega_0 + \delta\omega_0, \quad \Sigma(z) = \Sigma_0(z) + \delta\omega_0, \tag{A22}$$

with $\delta\omega_0$ a suitable (infinite) renormalization constant (see, e.g., the clear discussion in Ref. [35], Sec. 10.3). By choosing for convenience the subtraction point at $z = i$, that is,

$$\delta\omega_0 = \text{Re}\, \Sigma_0(i), \tag{A23}$$

the dressed self-energy function is

$$\Sigma(z) = \int d\omega\, |g(\omega)|^2 \left(\frac{1}{\omega - z} - \frac{\omega}{\omega^2 + 1} \right) = \int d\omega\, |g(\omega)|^2 \frac{1 + \omega z}{(\omega - z)(\omega^2 + 1)}. \tag{A24}$$

Notice that the dressed self-energy $\Sigma(z)$ in Eq. (A24) is well defined even for a flat form factor $g(\omega) = \text{const}$. In this case the bare quantities ω_0 and $\Sigma_0(z)$, as well as the energy shift $\delta\omega_0$, diverge, and the microscopic Hamiltonian (A5) is just a formal expression with an infinite bare qubit energy ω_0. However, the sums in Eqs. (A22) are finite and give a well-defined dynamics characterized by $\Sigma(z)$ [and hence $g(\omega)$] and by the dressed qubit energy $\tilde{\omega}_0$ (whose value is independent of ω_0).

Indeed, by transforming back to the time domain one finally gets

$$a(t) = \frac{1}{2\pi i} \int_{\mathbb{R}+iy} \frac{e^{-izt}}{\tilde{\omega}_0 - z - \Sigma(z)}\, dz, \tag{A25}$$

with an arbitrary $y > 0$.

The above heuristic derivation can be made fully rigorous [32] and one can show that for every dressed energy $\tilde{\omega}_0 \in \mathbb{R}$ and for every form factor $g(\omega)$ satisfying the growth condition

$$\int d\omega\, \frac{|g(\omega)|^2}{\omega^2 + 1} < +\infty, \tag{A26}$$

which includes non-square-integrable singular form factors (such as the flat one), the Friedrichs-Lee Hamiltonian is self-adjoint, and thus yields a unitary evolution with a survival amplitude given by Eq. (A25).

4. Reduced dynamics

We have proved that

$$e^{-itH} |0, \text{vac}\rangle = a(t)\, |0, \text{vac}\rangle + \int d\omega\, c(t, \omega)\, |1, \omega\rangle\,, \tag{A27}$$

with $a(t)$ given by Eq. (A25). Notice that, since the global evolution is unitary,

$$\int |c(t, \omega)|^2\, d\omega = 1 - |a(t)|^2. \tag{A28}$$

Having evaluated both $e^{-itH} |0, \text{vac}\rangle$ and $e^{-itH} |1, \text{vac}\rangle$ [see Eqs. (A13) and (A27)], we have

$$e^{-itH} |0, \text{vac}\rangle\langle 0, \text{vac}|\, e^{itH} = |a(t)|^2\, |0, \text{vac}\rangle\langle 0, \text{vac}| + \iint d\omega\, d\omega'\, c(t, \omega) c(t, \omega')^*\, |1, \omega\rangle\langle 1, \omega'| + a(t) \int d\omega\, c(t, \omega)^*\, |0, \text{vac}\rangle\langle 1, \omega| + a(t)^* \int d\omega\, c(t, \omega)\, |1, \omega\rangle\langle 0, \text{vac}|\,, \tag{A29}$$

$$e^{-itH} |1, \text{vac}\rangle\langle 1, \text{vac}|\, e^{itH} = |1, \text{vac}\rangle\langle 1, \text{vac}|\,, \tag{A30}$$

$$e^{-itH} |0, \text{vac}\rangle\langle 1, \text{vac}|\, e^{itH} = a(t)\, |0, \text{vac}\rangle\langle 1, \text{vac}| + \int d\omega\, c(t, \omega)\, |1, \omega\rangle\langle 0, \text{vac}|\,. \tag{A31}$$

By tracing out the bath we get

$$\text{tr}_{\text{bath}}(e^{-itH} |0, \text{vac}\rangle\langle 0, \text{vac}|\, e^{itH}) = |a(t)|^2\, |0\rangle\langle 0| + \int d\omega\, |c(t, \omega)|^2\, |1\rangle\langle 1|\,, \tag{A32}$$

$$\text{tr}_{\text{bath}}(e^{-itH} |1, \text{vac}\rangle\langle 1, \text{vac}|\, e^{itH}) = |1\rangle\langle 1|\,, \tag{A33}$$

$$\text{tr}_{\text{bath}}(e^{-itH} |0, \text{vac}\rangle\langle 1, \text{vac}|\, e^{itH}) = a(t)\, |0\rangle\langle 1|\,, \tag{A34}$$

and recalling Eqs. (A10) and (A28) we finally get

$$\rho(t) = \Lambda_t(\rho) = \begin{pmatrix} |a(t)|^2 \rho_{00} & a(t)\rho_{01} \\ a(t)^* \rho_{10} & \rho_{11} + (1 - |a(t)|^2)\rho_{00} \end{pmatrix}. \tag{A35}$$

Now we define two real functions $\gamma(t)$ and $\varepsilon(t)$ such that $a(t)$ can be rewritten as

$$a(t) = \exp\left(-\int_0^t ds \left(\frac{\gamma(s)}{2} + i\varepsilon(s) \right) \right), \tag{A36}$$

from which

$$\frac{\dot{a}(t)}{a(t)} = -\frac{\gamma(t)}{2} - i\varepsilon(t). \tag{A37}$$

By a simple computation one gets

$$\gamma(t) = -\frac{2}{|a(t)|} \frac{d}{dt} |a(t)|, \quad \varepsilon(t) = \frac{i}{\text{sgn}(a(t))} \frac{d}{dt} \text{sgn}(a(t)) \tag{A38}$$

with $\mathrm{sgn}(z) = z/|z|$. Using these functions the derivative of $\rho(t)$ reads

$$\dot{\rho}(t) = \begin{pmatrix} -\gamma(t)\rho_{00}(t) & \left(-\frac{\gamma(t)}{2} - i\varepsilon(t)\right)\rho_{01}(t) \\ \left(-\frac{\gamma(t)}{2} + i\varepsilon(t)\right)\rho_{10}(t) & \gamma(t)\rho_{00}(t) \end{pmatrix}, \tag{A39}$$

where $\rho_{00}(t) := |a(t)|^2\rho_{00}$, $\rho_{01}(t) := a(t)\rho_{01}$, $\rho_{10}(t) := a(t)^*\rho_{10}$, and $\rho_{11}(t) := \rho_{11} + (1 - |a(t)|^2)\rho_{00}$. Therefore, $\dot{\rho}(t)$ can be written as

$$\dot{\rho}(t) = -i\varepsilon(t)[H_{\mathrm{q}}, \rho(t)] - \gamma(t)\mathcal{L}_{\mathrm{D}}(\rho(t)), \tag{A40}$$

with

$$H_{\mathrm{q}} = |0\rangle\langle 0|\,, \tag{A41}$$

and

$$\mathcal{L}_{\mathrm{D}}(\rho) = -\sigma_-\rho\sigma_+ + \tfrac{1}{2}\{\sigma_+\sigma_-, \rho\}, \tag{A42}$$

where, as usual, the square brackets denote the commutator while the curly brackets denote the anticommutator. Therefore, the quantum channel Λ_t in Eq. (A7) describing the evolution of the qubit has a generator in the Gorini–Kossakowski–Lindblad-Sudarshan form,

$$-i\varepsilon(t)\,\mathrm{ad}_{H_{\mathrm{q}}} - \gamma(t)\mathcal{L}_{\mathrm{D}}, \tag{A43}$$

with time-dependent coefficients $\varepsilon(t)$ and $\gamma(t)$, where $\mathrm{ad}_{H_{\mathrm{q}}}(\rho) = [H_{\mathrm{q}}, \rho]$. Since $\mathrm{ad}_{H_{\mathrm{q}}}$ and $\mathcal{L}$ commute, we have

$$\begin{aligned}\Lambda_t &= \exp\left(-\int_0^t ds\,(\gamma(s)\mathcal{L}_{\mathrm{D}} + i\varepsilon(s)\,\mathrm{ad}_{H_{\mathrm{q}}})\right) \\ &= \exp(\ln(|a(t)|^2)\mathcal{L}_{\mathrm{D}} + i\arg(a(t))\,\mathrm{ad}_{H_{\mathrm{q}}}).\end{aligned} \tag{A44}$$

In the case $\gamma(t) = \gamma_0 = \mathrm{const}$ and $\varepsilon(t) = \varepsilon_0 = \mathrm{const}$, i.e., $a(t) = e^{-(\gamma_0/2 + i\varepsilon_0)t}$, we have

$$\Lambda_t = e^{-t(\gamma_0\mathcal{L}_{\mathrm{D}} + i\varepsilon_0\,\mathrm{ad}_{H_{\mathrm{q}}})} \tag{A45}$$

and the semigroup property, i.e., $\Lambda_t\Lambda_s = \Lambda_{t+s}$ for all $t, s \geqslant 0$, is satisfied and hence the channel is Markovian; this is the amplitude-damping channel. More generally, the semigroup property would be satisfied if and only if $a(t+s) = a(t)a(s)$ for all $t, s \geqslant 0$, which does not hold in general, thus preventing the channel from being Markovian.

5. Coupling and evolution

Equation (A25) implies that the value of $a(t)$ is ultimately determined by the self-energy $\Sigma(z)$, which in turn depends on the square modulus $|g(\omega)|^2$ of the form factor via Eq. (A24). In fact, the correspondence between $\Sigma(z)$ and $|g(\omega)|^2$ is unique, as discussed in Ref. [32] and references therein: $|g(\omega)|^2$ can be reconstructed from the self-energy via

$$|g(\omega)|^2 = \frac{1}{\pi}\lim_{\delta\downarrow 0}\mathrm{Im}\ \Sigma(\omega + i\delta). \tag{A46}$$

As a first example, by setting $g(\omega) = \sqrt{\gamma_0/2\pi}$ for some $\gamma_0 > 0$, we have $\Sigma(z) = i\frac{\gamma_0}{2}$ whenever $\mathrm{Im}\,z > 0$ and thus, substituting in Eq. (A25), one immediately obtains

$$a(t) = e^{-(\frac{\gamma_0}{2} + i\varepsilon_0)t}, \tag{A47}$$

with $\varepsilon_0 = \tilde{\omega}_0$; i.e., a flat coupling yields an exponential decay of the damping rate $a(t)$ at all times.

Let us examine the case of a periodic coupling, written in a Fourier cosine series as

$$|g(\omega)|^2 = \frac{\gamma_0}{2\pi}\left(1 + 2\sum_{n=1}^{\infty} c_n\cos nT\omega\right) \tag{A48}$$

for some family of real coefficients $\{c_n\}_{n=1}^{\infty}$ chosen in such a way that the series is absolutely convergent and positive for all ω. The corresponding self-energy reads

$$\Sigma(z) = \frac{i\gamma_0}{2}\left(1 + 2\sum_{n=1}^{\infty} c_n e^{inTz}\right), \tag{A49}$$

which can be verified immediately by Eq. (A46). With this choice of self-energy, from

$$\hat{a}(z) = \frac{1}{\varepsilon_0 - z - \Sigma(z)}, \tag{A50}$$

by a simple calculation we get

$$\hat{a}(z) = \frac{1}{\varepsilon_0 - z - \frac{i\gamma_0}{2}} + i\gamma_0\sum_{n=1}^{\infty} c_n e^{inTz}\frac{\hat{a}(z)}{\varepsilon_0 - z - \frac{i\gamma_0}{2}}, \tag{A51}$$

which implies

$$\begin{aligned}a(t) &= e^{-(\frac{\gamma_0}{2} + i\varepsilon_0)t} - \gamma\sum_{n=1}^{\infty} c_n\,\theta(t - nT) \\ &\quad\times [a \star e^{-(\frac{\gamma_0}{2} + i\varepsilon_0)(\cdot)}](t - nT),\end{aligned} \tag{A52}$$

where $\theta(t)$ is the Heaviside step function, and $\star$ is the convolution product evaluated at $t - nT$. From this equation it is already clear that $a(t)$ will be exactly exponential up to $t = T$; thereafter, nonexponential corrections will add up.

The solution of this equation can be found by means of a proper ansatz:

$$\begin{aligned}a(t) &= e^{-(i\varepsilon_0 + \frac{\gamma_0}{2})t} + \sum_{n=1}^{\infty} e^{-(i\varepsilon_0 + \frac{\gamma_0}{2})(t - nT)}\theta(t - nT) \\ &\quad\times \phi_n(\gamma_0(t - nT)),\end{aligned} \tag{A53}$$

where $\phi_n(x)$ is some function to be evaluated. By imposing Eq. (A52) for the function in Eq. (A53), one obtains a solvable recursion equation in n for the functions $\phi_n(x)$ which finally yields

$$\phi_n(x) = \sum_{m=1}^{n} b_n^{(m)}\frac{(-x)^m}{m!}, \tag{A54}$$

where the coefficients $b_n^{(m)}$ for $m = 1, \ldots, n$ are

$$b_n^{(m)} = \sum_{(h_1,\ldots,h_m)\in I_n^m}\left(\prod_{i=1}^{m} c_{h_i}\right), \tag{A55}$$

with I_n^m being the set of all ordered m-tuples of strictly positive integers that sum to n, i.e.,

$$I_n^m = \{(h_1, \ldots, h_m) \in \mathbb{N}^m \setminus \{0\} : h_1 + \cdots + h_m = n\}, \tag{A56}$$

that is, the positive integer elements of the m-dimensional simplex with edge length n. Notice that the cardinality of this set is

$$\#(I_n^m) = \binom{n-1}{m-1}, \tag{A57}$$

as can be proven through the usual stars-and-bars argument. By these formulas, we are finally able to compute the polynomials $\phi_n(x)$ for the two examples in the main text.

a. Single nonzero coefficient (sinusoidal measure)

In the case

$$c_1 = -\frac{\alpha}{2}, \quad c_n = 0 \quad \forall n \geqslant 2, \tag{A58}$$

for some $|\alpha| \leqslant 1$, we have only one nonzero coefficient; therefore, the only elements which must be taken into account in the sum are m-tuples in the form $(1, 1, \ldots, 1)$, which do belong to the simplex I_n^m if and only if $n = m$. As a result, the only nonzero coefficients $b_n^{(m)}$ are those with $n = m$, with

$$b_n^{(n)} = \prod_{i=1}^{n} c_{1,\alpha} = \frac{(-\alpha)^n}{2^n}, \tag{A59}$$

hence

$$\phi_n(x) = \frac{1}{n!}\left(\frac{\alpha x}{2}\right)^n. \tag{A60}$$

b. Exponentially decaying coefficients (smoothed Dirac measure)

In the case

$$c_n = e^{-\beta n} \quad \forall n \in \mathbb{N}, \tag{A61}$$

for some $\beta \geqslant 0$, the coefficients satisfy the property

$$\prod_{i=1}^{m} c_{h_i} = c_{h_1+h_2+\cdots+h_m} \tag{A62}$$

and hence, by Eqs. (A55) and (A57),

$$b_n^{(m)} = \binom{n-1}{m-1} e^{-\beta n}, \tag{A63}$$

thus implying

$$\phi_n(x) = e^{-\beta n} \sum_{m=1}^{n} \binom{n-1}{m-1} \frac{(-x)^m}{m!}. \tag{A64}$$

This implies that the non-Markovian contributions to the survival amplitude have the same functional expression for all β, up to a total weight $e^{-\beta n}$ which suppresses such contributions as n grows, provided that $\beta > 0$; as a result, the larger β, the quicker such contributions "switch off," whereas for small β those contributions are non-negligible for a longer time. In particular, in the limit $\beta \to \infty$ all non-Markovian contributions vanish and we recover the exponential decay at all times.

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