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UPPER LIMIT FOR A GRAVITATIONAL–WAVE STOCHASTIC BACKGROUND WITH THE EXPLORER AND NAUTILUS RESONANT DETECTORS

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Abstract

We discuss the sensitivity of resonant–mass gravitational–wave detectors to a cosmic stochastic background of gravitational waves. We report the experimental upper limits given by the gravitational wave detectors EXPLORER and NAUTILUS .

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Among the possible gravitational waves (gw) signals, a cosmic stochastic background is one of the most interesting, as it might give information on the very early stages of the Universe and its formation. Several sources of stochastic background have been considered in the past years [1]. We recall the effect of the superposition of many continuous waves generated by the pulsars, the overlapping bursts due to gravitational collapses and to coalescence of binary systems.

Nucleosynthesis considerations put an upper limit on the ratio Ω of the gw energy density to the critical density needed for a closed universe [1]. The upper limit is $\Omega \leq 10^{-5}$. As well known the critical density is given by

$$\rho_c = \frac{3\,H^2}{8\,\pi\,G} = 1.7\;10^{-8}\;\left(\frac{H}{100\ \mathrm{km\ s^{-1}\ Mpc^{-1}}}\right)^2 \frac{\mathrm{erg}}{\mathrm{cm}^3} \qquad (1)$$

where H is the Hubble constant.

Recently a source based on string theory has been more deeply investigated [2,3]. The interesting feature of this theory, from the actual observer point of view, is that it might predict relic gw's whose energy density increases, in a certain range, with the frequency f to the third power (remaining below the nucleosynthesis limit). In fact the previous models tend to predict gw's in the frequency range below 1 Hz, lower than the operating frequency of the present detectors already in operation (resonant bars) or entering in operation in the next four to five years (long–arm interferometers [4]). Only the newly proposed space experiment LISA could explore, with good sensitivity, a frequency range below 1 Hz, but such an experiment, if approved, will fly after the year 2016.

The predictions of the new string cosmology models depend on a number of parameters, such as the maximum frequency and the precise dependence of Ω on the frequency when it gets near to the maximum value. A measurement, even an upper limit, would help very much in delineating the exact model. At this stage is therefore very important to turn to the experiment.

The Rome group has operated the cryogenic resonant antenna EXPLORER [5] since 1990 and the ultracryogenic resonant antenna NAUTILUS [6] since 1995. Therefore it is worthwhile to study the recorded data to look for useful experimental information on relic gw.

A resonant gw detector consists of a carefully suspended resonant mass, usually a cylindrical bar, whose vibrational normal modes having the appropriate symmetry are excited by gw[1]. The mechanical oscillation of the resonant mass is transformed into an electrical signal by a motion transducer and then amplified by an electrical amplifier. Unavoidably, Brownian motion noise associated with dissipation in the resonant mass and the transducer, and electronic noise from the amplifier, limit the sensitivity of the detector.

The study of the problem of a stochastic background detection has shown that the presently available resonant detectors are suited for this type of measurement. As a matter of fact it turns out, as shown below, that the sensitivity of the resonant antennas to a stochastic background (for the present detectors operating with dcSQUID electronic amplifiers) depends essentially on the quantity

$$\frac{T}{M\,Q} \tag{2}$$

where T, M and Q are the detector thermodynamic temperature, mass and quality factor. The bandwidth of the apparatus enters to a minor extent, as it will be shown later. Thus what is the drawback of a resonant detector, namely the small bandwidth, does not jeopardize the measurement of the stochastic background.

The equation of motion for the bar end displacement $\xi(t)$ is

$$\ddot{\xi} + \frac{\omega_o}{Q}\dot{\xi} + \omega_o^2 = \frac{F}{m} \tag{3}$$

where F is the applied force, m the oscillator reduced mass (for a cylinder m = M/2), $\omega_o=2\pi f_o$ is the angular resonance frequency and Q is the merit factor.

Expressing the effect of the Brownian and electronic noise in term of displacement of the bar ends, we obtain [6] the noise power spectrum :

$$S_\xi^n(f) = \frac{S_F}{(2\pi)^4 m^2}\;\frac{1 + \Gamma\left[Q^2\left(1-\left(\frac{f}{f_o}\right)^2\right)^2 + \left(\frac{f}{f_o}\right)^2\right]}{\left(f^2 - f_o^2\right)^2 + \frac{f^2 f_o^2}{Q^2}} \qquad \left[\frac{m^2}{Hz}\right] \tag{4}$$

with

$$S_F = \frac{2\,\omega_o}{Q}\,m\,k\,T_e \tag{5}$$

where T_e is the equivalent temperature that includes the effect of the back–action from the electronic amplifier, and Γ is the spectral ratio between electronic and brownian noise [7] (usually $\Gamma \ll 1$):

$$\Gamma \approx \frac{T_n}{\beta\,Q\,T_e} \tag{6}$$

T_n is the amplifier noise temperature and β the coupling parameter of the transducer to the bar ($\beta \approx 10^{-2}$-10^{-3}). The power spectrums are expressed in two–sided form.

When a gravitational wave with amplitude h and optimum polarization impinges perpendicularly to the bar axis, the bar displacement corresponds [7] to the action of a force

$$F = \frac{2}{\pi^2}\,m\,L\,\ddot{h} \tag{7}$$

For a gw excitation with power spectrum $S_h(f)$, the spectrum of the corresponding bar end displacement is

$$S_\xi(f) = \frac{4\,L^2\,f^4\,S_h}{\pi^4}\;\frac{1}{(f^2 - f_o^2)^2 + \dfrac{f^2 f_o^2}{Q^2}} \qquad \left[\frac{m^2}{Hz}\right] \tag{8}$$

We notice that the power spectrum of the bar displacement for a constant spectrum of gw is similar to that due to the action of the Brownian force. Therefore, if only the Brownian noise were present, we would have an infinite bandwidth, in terms of signal to noise ratio (SNR).

By taking the ratio of the noise spectrum (4) and the signal spectrum (8) we obtain the signal to noise ratio (SNR)

$$SNR(f) = \frac{S_\xi(f)}{S_\xi^0(f)} = \frac{64\,L^2\,f^4\,m^2\,S_h(f)}{S_F}\;\frac{1}{1 + \Gamma\left[Q^2\left(1-\left(\frac{f}{f_o}\right)^2\right)^2 + \left(\frac{f}{f_o}\right)^2\right]} \tag{9}$$

By equating to unity the above ratio we obtain the gw spectrum detectable with SNR=1,that is the detector noise spectrum referred to the input:

$$S_h(f) = \frac{\pi}{8}\,\frac{k\,T_e}{M\,Q\,L^2}\,\frac{f_o}{f^4}\left\{1 + \Gamma\left[Q^2\left(1-\left(\frac{f}{f_o}\right)^2\right)^2 + \left(\frac{f}{f_o}\right)^2\right]\right\} \tag{10}$$

At the resonance f_o we have (being $\Gamma \ll 1$)

$$S_h(f_o) = \frac{\pi}{8}\,\frac{k\,T_e}{M\,Q\,L^2}\,\frac{1}{f_o^3} \tag{11}$$

We remark that the equivalent temperature T_e reduces to T if the backaction from the electromechanical transducer can be neglected, as in the case of a dcSQUID.

The above quantity $S_h(f)$ must be related to the quantity $\Omega(f)$ predicted by the theory, where

$$\Omega(f) = \frac{d\Omega}{d\ln(f)} \tag{12}$$

It turns out that [3]:

$$S_h(f) = \frac{3\,H^2}{4\,\pi^2}\,\frac{\Omega(f)}{f^3} = \frac{8.0\;10^{-37}}{Hz}\,\frac{\Omega(f)}{f^3}\,\left(\frac{H}{100\;km\;s^{-1}\;Mpc^{-1}}\right)^2 =$$

$$=(@\;920\;Hz)\;\left\{\frac{3.2\;10^{-23}}{\sqrt{Hz}}\,\frac{H}{100\;km\;s^{-1}\;Mpc^{-1}}\sqrt{\Omega(f)}\right\}^2 \tag{13}$$

The target sensitivity of ultracryogenic antennas like NAUTILUS [8] or AURIGA [9] with f=920 Hz, M=2300 kg, T=0.1 K and Q=5 10^6 is $S_h(f)=(8.6\;10^{-23}/\sqrt{Hz})^2$. This is not sufficient to reach the limit imposed by the nucleosynthesis bound of $\Omega\leq10^{-5}$, but it gives an upper limit.

We give now the results of measurements made with the antennas EXPLORER in the years 1991 (Fig.1) and 1994 (Fig.2) and NAUTILUS operating at T=1.3 K in the year 1995 (Fig.3). NAUTILUS is capable to operate below 0.1 K, but in 1995 we operated it at 1.3 K because we had some excess noise. We recall that both EXPLORER and NAUTILUS employ a resonant electromechanical transducer, thus showing two resonances which may have different sensitivity according to the noise on each one and to the tuning of the transducer to the bar.

With one detector only, the sensitivity does not depend on the length of the measuring time. Increasing the time of measurement would just reduce the error in the spectral determination, leaving practically unchanged the level of the spectrum. We notice that in 1994 we obtained at both the resonances (907 Hz and 923.32 Hz) a measurement 6 $10^{-22}/\sqrt{Hz}$. The upper limit from these measurements turns out to be still very high, about Ω=300.

At the frequency of 923.8 Hz we obtain from NAUTILUS 7 $10^{-22}/\sqrt{Hz}$.

Better sensitivity can be obtained by cross correlating the output of two antennas, because the local noises are uncorrelated and the sensitivity improves with a longer measuring time. It can be shown [10] that, in such a case, if the two identical antennas with respective spectral outputs S_{1h} and S_{2h} are close to each other, within a distance much smaller than the gw wavelength [11], the sensitivity is

$$\delta S_{gw}(\omega) = \frac{\sqrt{S_{1h}\,S_{2h}}}{\sqrt{t_m \Delta f}} \tag{14}$$

where t_m is the measuring time and Δf is the antenna bandwidth.

We see in the above formula the effect of the bandwidth, which enters as the 1/4 power for the usual sensitivity expressed in units $1/\sqrt{Hz}$. With the present resonant detectors at T=0.1 K having Δf=1 Hz and for a measuring time of one year one can reach $\delta S_h(f)=(1.1\;10^{-24}/\sqrt{Hz})^2$ corresponding to $\Omega=1.3\;10^{-3}$. at 920 Hz.

In order to reach the limit of $\Omega=10^{-5}$, two resonant detectors cooled to 10 mK and with a ten times larger mass would be required. If such two resonant detectors operate at their quantum limit then the bandwidth may become as large as Δf=50 Hz thus allowing to reach $\Omega=5\;10^{-6}$ at f=920 Hz.

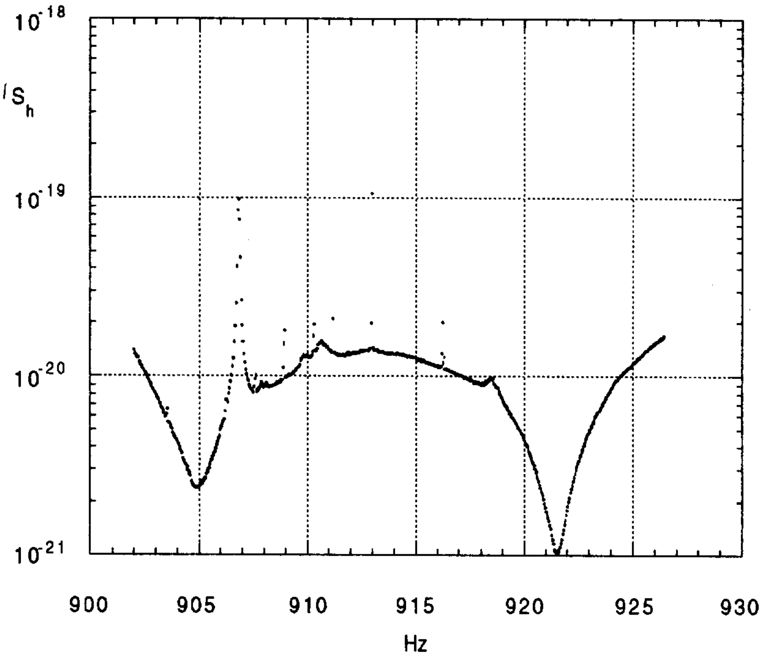

Fig. 1 – Sensitivity to stochastic gw background with SNR=1 for EXPLORER. T=2.9 K, M=2300 kg, Q=10^6, average spectrum over 31.4 days (1991).

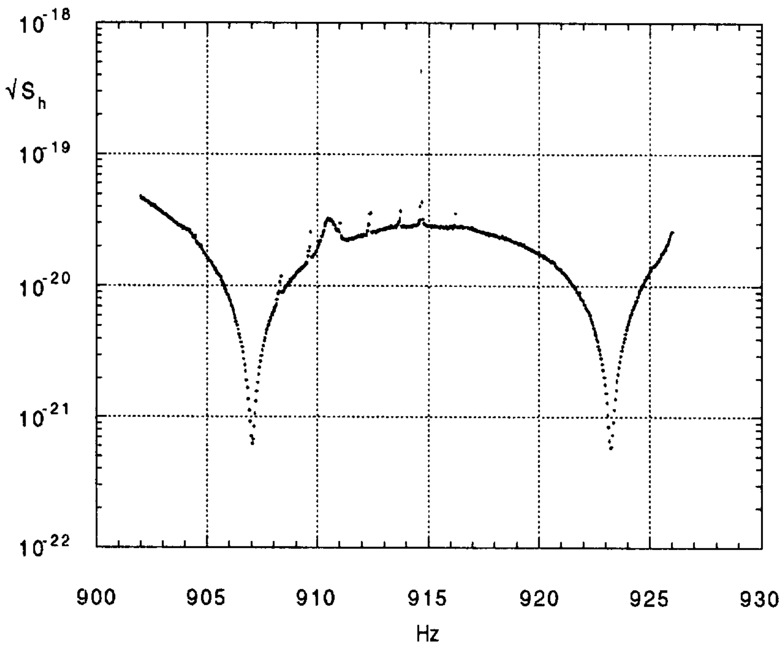

Fig. 2 – Sensitivity to stochastic gw background with SNR=1 for EXPLORER.T=2.4 K, M=2300 kg, Q=5 10^6, average spectrum over 36 hours (1994).

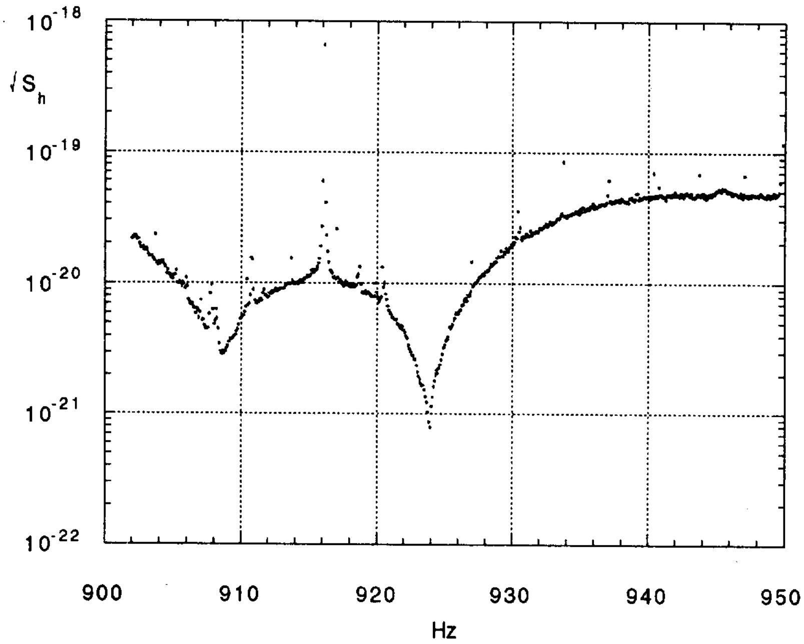

Fig. 3 – Sensitivity to stochastic gw background with SNR=1 for NAUTILUS. T=1.3 K, M=2300 kg, Q=2.6 10^6, average spectrum over 2.3 hours (1995).

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