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Low-Energy Quantum String Cosmology

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Abstract

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We introduce a Wheeler-De Witt approach to quantum cosmology based on the low-energy string effective action, with an effective dilaton potential included to account for non-perturbative effects and, possibly, higher-order corrections. We classify, in particular, four different classes of scattering processes in minisuperspace, and discuss their relevance for the solution of the graceful exit problem.

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1. Introduction

The effective action of string theory has recently suggested a "pre-big bang" cosmological scenario[1], in which the present state of our Universe is the result of a transition from the string perturbative vacuum. Such a transition necessarily involves the high-curvature and strong coupling regime, thus requiring, for a consistent description, the inclusion of higher derivatives and loops in the effective action[2]. These corrections cannot be simulated, classically, by a dilaton potential: it has been shown[3] that there are no smooth solutions of the lowest order string effective action interpolating between the pre- and post-big bang regime, for any choice of a local (and realistic) dilaton potential.

At the quantum level, however, the situation is different. With an appropriate potential added to the low-energy action, the transition probability from a pre-big bang to a post big-bang configuration, computed according to the Wheeler-De Witt (WDW) equation[4], has been shown to be non-vanishing even when the two configurations are classically disconnected by a curvature singularity[5].

This paper is devoted to report and quickly discuss some interesting aspect of such a low-energy approach to quantum string cosmology, in which no higher-order correction is taken into account, except those possibly encoded into an effective, non-perturbative dilaton potential. The decay of the string perturbative vacuum can be effectively described, in this context, as a scattering process of the WDW wave function in minisuperspace: we can identify, in particular, four different types of scattering along time-like or space-like directions, corresponding to expanding or contracting final geometric configurations. The wave function can be either damped or parametrically amplified, according to a "tunnelling" or "anti-tunnelling" transition of the string perturbative vacuum. In both cases the possible applications to the string cosmology scenario seem to be promising.

2. Duality and operator ordering

The low-energy approach to quantum string cosmology is based on the tree-level, lowest order in α', string effective action[6]. Working in the assumption that only the metric and the dilaton field ϕ contribute non-trivially to the background, the action becomes, in d spatial dimensions and in the string frame:

$$S = -\frac{1}{2\,\lambda_s^{d-1}} \int d^{d+1}x \sqrt{|g|}\, e^{-\phi} \left[R + \partial_\mu\phi\partial^\mu\phi + V(\phi)\right]. \qquad (2.1)$$

Here $\lambda_s = (\alpha')^{1/2}$ is the fundamental string length parameter, and V is a (possibly non-perturbative) dilaton potential. Considering an isotropic, spatially flat cosmological background, parametrized by

$$g_{\mu\nu} = \mathrm{diag}\left(N^2(t), -a^2(t)\delta_{ij}\right), \qquad a = \exp\left[\beta(t)/\sqrt{d}\right], \qquad \phi = \phi(t), \qquad (2.2)$$

and assuming spatial sections of finite volume, the action can be conveniently written as[5]:

$$S = \frac{\lambda_s}{2} \int dt\, \frac{e^{-\overline{\phi}}}{N} \left(\dot{\beta}^2 - \dot{\overline{\phi}}^2 - N^2\, V\right), \quad \overline{\phi} = \phi - \sqrt{d}\,\beta - \log\int \frac{d^d x}{\lambda_s^d} \qquad (2.3)$$

where $\overline{\phi}$ is the so-called shifted dilaton. The variation of the lapse function N leads then to the Hamiltonian constraint

$$\Pi_\beta^2 - \Pi_{\overline{\phi}}^2 + \lambda_s^2\, V(\beta, \overline{\phi})\, e^{-2\,\overline{\phi}} = 0\ , \qquad (2.4)$$

written in terms of the canonical momenta (in the cosmic time gauge $N = 1$):

$$\Pi_\beta = \frac{\delta S}{\delta\dot{\beta}} = \lambda_s\, \dot{\beta}\, e^{-\overline{\phi}}, \qquad \Pi_{\overline{\phi}} = \frac{\delta S}{\delta\dot{\overline{\phi}}} = -\lambda_s\, \dot{\overline{\phi}}\, e^{-\overline{\phi}}. \qquad (2.5)$$

It is important to stress that the corresponding WDW equation, implementing in superspace the Hamiltonian constraint through the differential representation $\Pi^2 = -\partial^2$, is manifestly free from problems of operator ordering, since the Hamiltonian (2.4) has a flat metric in momentum space. The ordering problem is trivially solved in this context because, thanks to the duality symmetry of the action (2.1), the corresponding minisuperspace is globally flat, and we can always choose a convenient parametrization leading to a flat minisuperspace metric. This is confirmed by the fact that, if we adopt a curvilinear parametrization of minisuperspace, the ordering fixed by the duality symmetry is exactly the same as the ordering imposed by the requirement of reparametrization invariance.

In order to illustrate this point, consider the pair of minisuperspace coordinates $(a, \overline{\phi})$, different from the previous pair $(\beta, \overline{\phi})$ used in eq. (2.3). The kinetic part of the action (2.1) leads then to the classical (kinetic part of the) Hamiltonian

$$H = \frac{a^2}{d}\Pi_a^2 - \Pi_{\overline{\phi}}^2 \equiv \gamma^{AB}\Pi_A\Pi_B, \qquad (2.6)$$

corresponding to the non-trivial 2×2 metric:

$$\gamma_{AB} = \mathrm{diag}\left(\frac{d}{a^2}, -1\right). \tag{2.7}$$

The quantum operator corresponding to the Hamiltonian (2.6) has to be ordered, and its differential representation can be written in general as

$$H = \frac{\partial^2}{\partial\overline{\phi}^2} - \frac{1}{d}\left(a^2\frac{\partial^2}{\partial a^2} + \epsilon a\frac{\partial}{\partial a}\right), \tag{2.8}$$

where ϵ is a c-number parameter depending on the ordering. Note that there are no contributions to the ordered Hamiltonian from the minisuperspace scalar curvature[7], which is vanishing for the metric (2.7).

Reparametrization invariance now imposes on the Hamiltonian the covariant Dalembertian form $H = -\Box = -\nabla_A\nabla^A$, and consequently fixes $\epsilon = 1$. The action (2.1), on the other hand, is invariant under the T-duality transformation[8]

$$a \to \tilde{a} = a^{-1}, \qquad \overline{\phi} \to \overline{\phi}, \tag{2.9}$$

which implies, for the Hamiltonian (2.8),

$$H(a) = H(\tilde{a}) + \frac{2}{d}(\epsilon - 1)\tilde{a}\frac{\partial}{\partial\tilde{a}}. \tag{2.10}$$

The invariance of the Hamiltonian requires $\epsilon = 1$, and thus fixes the same quantum ordering as the general covariance condition.

A similar relation between quantum ordering and duality symmetry can be easily established for more general effective actions including an antisymmetric tensor background[5,9], and a larger class of non-minimal gravi-dilaton couplings[10].

3. Wave scattering in minisuperspace

In the convenient parametrization corresponding to β and $\overline{\phi}$, the Hamiltonian constraint (2.4) leads to the second-order WDW equation

$$\left[\partial_{\overline{\phi}}^2 - \partial_\beta^2 + \lambda_s^2\, V(\beta,\overline{\phi})\, e^{-2\overline{\phi}}\right]\, \psi(\beta,\overline{\phi}) = 0\,. \tag{3.1}$$

In the absence of the dilaton potential we thus obtain a free Klein-Gordon equation. The four independent solutions

$$\psi \sim e^{\pm ik\beta \pm ik\overline{\phi}} \tag{3.2}$$

span a plane wave representation of the four branches of the classical solutions, characterized by $\Pi_\beta = \pm\Pi_{\overline{\phi}}$, and corresponding respectively to expansion, $\Pi_\beta > 0$, contraction, $\Pi_\beta < 0$, growing dilaton, $\Pi_{\overline{\phi}} < 0$, decreasing dilaton, $\Pi_{\overline{\phi}} > 0$ (see the definitions (2.5)). It may be useful, in particular, to recall the physical correspondence[1,2,5]

- expanding pre-big bang $\Longrightarrow$ $\Pi_\beta > 0,\ \Pi_{\overline{\phi}} < 0$;
- expanding post-big bang $\Longrightarrow$ $\Pi_\beta > 0,\ \Pi_{\overline{\phi}} > 0$;
- contracting pre-big bang $\Longrightarrow$ $\Pi_\beta < 0,\ \Pi_{\overline{\phi}} < 0$;
- contracting post-big bang $\Longrightarrow$ $\Pi_\beta < 0,\ \Pi_{\overline{\phi}} > 0$.

In this context, a transition from pre– to post-big bang is represented as a transition from an asymptotic state $\psi_{\overline{\phi}}^{(-)}$, characterized by a negative eigenvalue of $\Pi_{\overline{\phi}}$, to an asymptotic state $\psi_{\overline{\phi}}^{(+)}$ characterized by a positive eigenvalue of $\Pi_{\overline{\phi}}$,

$$\Pi_{\overline{\phi}}\, \psi_{\overline{\phi}}^{(\pm)} = \pm k\, \psi_{\overline{\phi}}^{(\pm)}. \tag{3.3}$$

Similarly, a transition from expansion to contraction is a transition from $\psi_\beta^{(+)}$ to $\psi_\beta^{(-)}$, where

$$\Pi_\beta\, \psi_\beta^{(\pm)} = \pm k\, \psi_\beta^{(\pm)}. \tag{3.4}$$

This suggests to look at the quantum transition responsible for the evolution of our Universe from the string perturbative vacuum, namely from $\beta = -\infty, \overline{\phi} = -\infty$, as a process of scattering of the WDW wave function, induced by an appropriate dilaton potential, in the two-dimensional minisuperspace spanned by β and $\overline{\phi}$. With the boundary conditions chosen so as to fix the perturbative vacuum as the initial state of the Universe, we have four possible types of processes, depending on the effective dilaton potential $V(\beta, \overline{\phi})$ and on the choice of the time-like coordinate in the $(\beta, \overline{\phi})$ plane. They will be discussed in the following Section.

4. Four scattering processes

The four possible types of scattering for the wave function of the string perturbative vacuum are illustrated in Fig. 1.

The process (a) describes the transition from expanding pre-big bang to expanding post-big bang configurations[5], represented as a reflection along the spatial direction $\overline{\phi}$, induced by an effective dilaton potential. The simplest case is the reflection induced by a cosmological constant, $V = \Lambda$; see Ref. [5] for more complicated potentials, and Ref. [11] for a rigorous definition of scalar products in the appropriate Hilbert space.

The process (b) describes the transition from expanding pre-big bang to contracting pre-big bang configurations[12], represented in a third quantization formalism as the production of a universe–anti-universe pair (one expanding, the other contracting) out of the vacuum. A step potential $V = \Lambda\theta(\overline{\phi})$, corresponding to a cosmological constant generated non-perturbatively in the strong coupling regime, is already enough to trigger the pair production[12].

The process (c) describes again the transition from expanding pre-big bang to contracting pre-big bang, represented however as a reflection[13] along the spatial

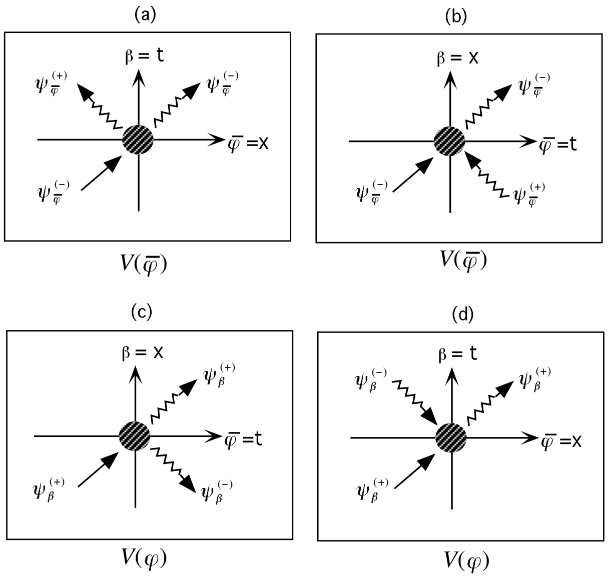

Figure 1: *Four possible classes of scattering processes in minisuperspace. The boundary conditions are fixed by the choice of the perturbative vacuum as the initial cosmological configuration.*

direction β, induced by a local potential which depends, more realistically, on ϕ (instead of $\overline{\phi}$).

The last process (d) describes the transition from expanding pre-big bang to expanding post-big bang, represented as the production from the vacuum of a pair of universes, one evolving towards the low-energy post-big bang regime, the other falling inside the pre-big bang singularity. This type of process has not yet been analyzed in detail but, potentially, is the more promising for a solution of the graceful exit problem[3], i.e. for driving a forced evolution of the initial perturbative vacuum into the standard cosmological configuration. In this class of processes, in fact, the WDW wave function is parametrically amplified, and the transition probability may easily approach unity, instead of being exponentially suppressed like in case (a). This process requires however a complicated potential, which has to break duality invariance in order to allow both positive and negative Π_β, and has to be volume-dependent in order to define asymptotically free states, in the limit $\beta \to \pm\infty$. Finding such a potential is certainly not impossible, but may be hard to be justified naturally in a string theory context.

It is important to note that, as illustrated in Fig.1, the transitions from pre- to post-big bang configurations are those in which β plays the role of the time-like coordinate, (a) and (d), while the transitions from expanding to contracting configurations require $\overline{\phi}$ as the time-like coordinate, (b) and (c). Also, for $V = V(\overline{\phi})$ all the final asymptotic states are characterized by a positive eigenvalue of Π_β (consistently with the initial conditions since, in that case, $[\Pi_\beta, H] = 0$); a reflection along β, as in processes (c) and (d), is only allowed if V depends on both β and $\overline{\phi}$, so as to break invariance under the duality transformation (2.9).

Finally, in processes of type (a) and (c), which describe a spatial reflection, there are only outgoing waves at the singular space-time boundary, $\overline{\phi} \to +\infty$. This is the analogous of tunnelling boundary conditions[14,15], imposed in the context of the standard inflationary scenario (indeed, the transition probability turns out to be very similar). We can thus look at these processes as at a "tunnelling from the string perturbative vacuum"[5], instead of a 'tunnelling from nothing"[14]. The processes (b) and (d), which describe pair production, can be seen instead as an "anti-tunnelling from the string perturbative vacuum"[12]. In fact the wave function, instead of being damped, is parametrically amplified in superspace, and the probability of the process is controlled by the inverse of the quantum-mechanical transmission coefficient.

5. Concluding remarks: self-reproducing Universe from the string perturbative vacuum?

Recently, in the context of the chaotic inflationary scenario, it has been proposed a model of "self-reproducing" Universe[16], based on the quantum production of a foam of infinitely many universes, distributed over a wide range of curvature scales. This scenario is interesting not only in itself, but also because it may provide a mechanism for explaining, consistently with inflation, a present value of the large-scale density different from one in critical units.

The self-reproduction process requires that the quantum nucleation of universes be exponentially suppressed at low curvature scales, and is thus implemented in the context of "tunnelling from nothing" boundary conditions. Such conditions are certainly appropriate for the standard cosmological scenario, in which the Universe evolves from the big-bang singularity. In a string cosmology context, however, the required initial distribution of "baby" universes could be nucleated not "from nothing", but from a well defined pre-big bang phase, starting from the perturbative vacuum.

The reflection corresponding to the scattering process (a), of Fig. 1, describes in fact the "birth" of a class of expanding post-big bang configurations, with a probability distribution P fixed by the reflection coefficient as[5]:

$$P = \exp\left[-\frac{\Omega_s}{g_s^2\lambda_s^3 f(\lambda_s^2\Lambda_s)}\right]. \tag{5.1}$$

Here Ω is the proper spatial volume of the nucleated Universe, $g = e^{\phi/2}$ is the string coupling constant, and $f(\Lambda)$ is a complicated function of the constant dilaton potential $V = \Lambda$ triggering the transition (all quantities are referred to the string curvature scale $\dot{\beta} = \lambda_s^{-1}$, where the transition is expected to occur).

This probability is exponentially suppressed, as required by the self-reproduction scenario, unless the volume Ω_s is very small and the cosmological constant Λ_s very large in string units (which seems unnatural in string theory context). The reflection probability (5.1) is thus in qualitative agreement with the tunnelling probability[14,15]. The only basic difference is that the coupling depends on the dilaton, and it is thus running in Planckian units. As a consequence, the universes tend to emerge from the nucleation process in the strong coupling regime, with a typical instanton-like distribution $P \sim \exp(-g_s^{-2})$.

In conclusion, the low-energy string effective action provides an adequate classical description of the initial, very early cosmological evolution from the string perturbative vacuum. Such an action cannot directly describe the strong coupling, high-curvature regime, without the inclusion of higher-order corrections. However, when at least some of these corrections and/or possible non-perturbative effects are accounted for by an appropriate dilaton potential, the WDW equation obtained from the low-energy action action permits a quantum analysis of the background evolution, and points out new possible interesting ways for the Universe to reach the present cosmological configuration.

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