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Inflation and Initial Conditions in the Pre-Big Bang Scenario

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The pre-big bang scenario describes the evolution of the Universe from an initial state approaching the flat, cold, empty, string perturbative vacuum. The choice of such an initial state is suggested by the present state of our Universe if we accept that the cosmological evolution is (at least partially) duality-symmetric. Recently, the initial conditions of the pre-big bang scenario have been criticized as they introduce large dimensionless parameters allowing the Universe to be "exponentially large from the very beginning". We agree that a set of initial parameters (such as the initial homogeneity scale, the initial entropy) larger than those determined by the initial horizon scale, H^{-1}, would be somewhat unnatural to start with. However, in the pre-big bang scenario, the initial parameters are all bounded by the size of the initial horizon. The basic question thus becomes: is a maximal homogeneity scale of order H^{-1} necessarily unnatural if the initial curvature is small and, consequently, H^{-1} is very large in Planck (or string) units? In the impossibility of experimental information one could exclude "a priori", for large horizons, the maximal homogeneity scale H^{-1} as a natural initial condition. In the pre-big bang scenario, however, pre-Planckian initial conditions are not necessarily washed out by inflation and are accessible (in principle) to observational tests, so that their naturalness could be also analyzed with a Bayesan approach, in terms of "a posteriori" probabilities.

Recently, the validity of the pre-big bang scenario as a viable inflationary model has been questioned on the grounds of its initial conditions [1].

The main criticism raised against models in which the Universe evolves from the flat, zero-interactions, string perturbative vacuum is mainly based on two points [1]. The first concerns the homogeneity problem, in particular the largeness of the initial homogeneous region in string (or Planckian) units; the second concerns the flatness problem, and in particular the two large dimensionless parameters (the inverse of the string coupling and of the curvature, in string units) characterizing the Universe at the beginning of inflation. The fact that, as a consequence of these large numbers, "the pre-big bang Universe must be very huge and homogeneous from the very beginning" is quoted as a serious problem, supporting the conclusion that "the current version of the pre-big bang scenario cannot replace usual inflation" [1].

I agree with the remarks concerning the initial size of the Universe (indeed, the need for an initial state with a Universe very large in Planck units was already noted in the first paper on the pre-big bang scenario [2] and, even before, in the context of string-driven superinflation [3]; in particular, the condition on the duration of inflation, reported in [1] as eq. (8), was already derived in [4]). The large initial size of the Universe is only part of the conditions to be imposed at the onset of pre-big bang inflation, and I also agree with the fact that a successful pre-big bang scenario requires an initial state characterized by very small (or very large) dimensionless ratios measuring the initial curvature and coupling constant, and possibly leading to a fine-tuning problem, as first pointed out in [5]. I disagree, however, with the conclusion presented in [1], and I would like to point out some arguments, hoping to clarify a different point of view on a large initial Universe.

I will concentrate, in particular, on the largeness of the initial horizon scale, which can be thought to be at the ground of the various objections discussed in [1]. The large dimensionless ratios of the initial state, when referred to the Einstein frame in which the Planck length is fixed, correspond indeed to a small initial curvature in Planck units, and then to a large horizon (in Planck length units), allowing a large homogeneous domain as initial condition. I do not pretend, of course, to provide a final answer to all problems. The modest aim of this paper is to stress that the problems raised in [1] reduce, in the end, to the question of whether the horizon scale, *irrespective of its size*, may be a natural scale for determining the inflationary initial conditions (in particular, the size of the initial homogeneous region), and to suggest the possibility that the answer is not negative "a priori", at least when the initial conditions are imposed well inside the classical regime, like in the case of the pre-big bang scenario.

Let me start recalling that the kinematical problems of the standard scenario can be solved by two classes of accelerated backgrounds [6]. Consider, for instance, the flatness problem, requiring a phase in which the ratio $r = k/a^2H^2 \sim \dot{a}^{-2}$ decreases, so as to compensate its growth up to the present value $r \lesssim 1$ during the subsequent phase of standard evolution. By parametrizing the scale factor as $a \sim |t|^\beta$, the decrease of $r \sim |t|^{2(1-\beta)}$ can be arranged either by 1) $\beta > 1$, $t \to +\infty$, or 2) $\beta < 1$, $t \to 0_-$. Both classes of backgrounds are accelerated, as sign $\dot{a}$ = sign $\ddot{a}$. The first class corresponds to power-inflation, and includes de Sitter inflation in the limit $\beta \to \infty$. The second class includes superinflation for $\beta < 0$, and accelerated contraction for $0 < \beta < 1$.

The main kinematic difference between the two classes is the behaviour of the event horizon, whose proper size is defined by

$$d_e(t) = a(t) \int_t^{t_M} dt' a^{-1}(t'). \qquad (1)$$

Here t_M is the maximal future extension of the cosmic time coordinate for the inflationary manifold. Therefore, $t_M = +\infty$ for the first class, and $t_M = 0$ for the second class of backgrounds. In both cases we find that the integral converges, and that $d_e(t) \sim |H|^{-1}(t)$, so that the horizon size is constant or growing for class 1), shrinking for class 2), following the inverse behaviour of the curvature scale. The phase of pre-big bang evolution, in particular, is dual to a phase of standard, decelerated evolution: its accelerated kinematics is characterized by a growing curvature scale (i.e. growing $|H|$), and may be represented as superinflation, in the string frame, or accelerated contraction, in the Einstein frame [6].

In order to recall the criticism of [1] we will now compare the kinematics of standard de Sitter inflation and pre-big bang superinflation, for an oversimplified cosmological model in which the standard radiation era begins at the Planck scale, and it is immediately preceeded by a phase of accelerated (inflationary) evolution. Also, for the sake of simplicity, we will identify at the end of inflation the present value of the string length L_s with the Planck length L_p (at tree-level, they are related by $L_p = \langle g \rangle L_s = \langle \exp \phi/2 \rangle L_s$, with a present dilaton expectation value $\langle g \rangle \sim 0.1 - 0.01$).

At the beginning of the radiation era the horizon size is thus controlled by the Planck length $L_p \sim L_s$, while the proper size of the homogeneous and causally connected region inside our present Hubble radius, rescaled down at the Planck epoch according to the standard decelerated evolution of $a(t)$, is unnaturally larger than the horizon by the factor $\sim 10^{30} L_p$. During the inflationary epoch, the ratio

$$\frac{\text{proper size horizon scale}}{\text{proper size homogeneous region}} \sim \frac{H^{-1}(t)}{a(t)} \sim \eta \qquad (2)$$

must thus decrease at least by the factor 10^{-30}, so as to push the homogeneous region outside the horizon, of the amount required by the subsequent decelerated evolution. Since the above ratio evolves linearly in conformal time $\eta \sim \int a^{-1} dt$, the condition of sufficient inflation can be written as

$$|\eta_f|/|\eta_i| \lesssim 10^{-30}, \qquad (3)$$

where η_i and η_f mark, respectively, the beginning and the end of the inflationary epoch.

Let us now compare de Sitter inflation, $a \sim (-\eta)^{-1}$, with a typical dilaton-dominated superinflation, $a \sim (-t)^{-1/\sqrt{3}} \sim (-\eta)^{-1/(\sqrt{3}+1)}$ (the same discussed in [1]).

In the standard de Sitter case the horizon and the Planck length are constant, $H^{-1} \sim L_s \sim L_p$; as we go back in time, according to eq. (3), $a(t)$ reduces by the factor 10^{-30} so that, at the beginning of inflation, we find a homogeneous region just of size L_p, like the horizon. In the superinflation case, on the contrary, during the conformal time interval (3), $a(t)$ is only reduced by the factor $a_i/a_f = 10^{-30/(1+\sqrt{3})} \sim 10^{-11}$, so that the size of the homogeneous region, at the beginning of inflation, is still large in string units, $\sim 10^{30\sqrt{3}/(1+\sqrt{3})} L_s \sim 10^{19} L_s$. The situation is even worse in Planck units since, at the beginning of inflation, the string coupling $\exp \phi/2$, and thus the Planck length L_p, are reduced with respect to their final values by the factor [4] $L_p/L_s = |\eta_f/\eta_i|^{\sqrt{3}/2} \sim 10^{-15\sqrt{3}}$, so that $10^{19} L_s \sim 10^{45} L_p$. This, by the way, is exactly the initial size of the homogeneous region evaluated in the Einstein frame in which L_p is constant, and the above dilaton-driven evolution is represented as a contraction, with $a \sim (-\eta)^{1/2}$ (see Fig. 1 for a qualitative illustration of the differences between de Sitter inflation and pre-big bang inflation).

According to [1], case (a) of Fig. 1 provides an acceptable example of inflationary scenario, as the initial homogeneity scale is contained within a single domain of Planckian size. Case (b), on the contrary, is not satisfactory because of the initial homogeneity on scales much greater than Planckian, $10^{19} L_s \sim 10^{45} L_p$. Quoting Ref. [1], this situation "is not much better than the situation in the non-inflationary big bang cosmology, where it was necessary to assume that the initial size of the homogeneous part of our Universe was greater than $10^{30} L_p$".

I would like to stress, however, that in case (b) the initial homogeneous region is large in Planck units, *but not larger than the horizon itself*. Indeed, during superinflation, the horizon scale shrinks linearly in cosmic time. As we go backwards in time, for the particular example that we are considering, the horizon increases by the factor $H_i^{-1}/H_f^{-1} = |t_i|/|t_f| = (\eta_i/\eta_f)^{\sqrt{3}/(1+\sqrt{3})}$, so that, at the beginning of inflation, $H^{-1} \sim 10^{30\sqrt{3}/(1+\sqrt{3})} L_s \sim 10^{19} L_s \sim 10^{45} L_p$, i.e. the horizon size is just the same as that of the homogeneous region (as illustrated in Fig.

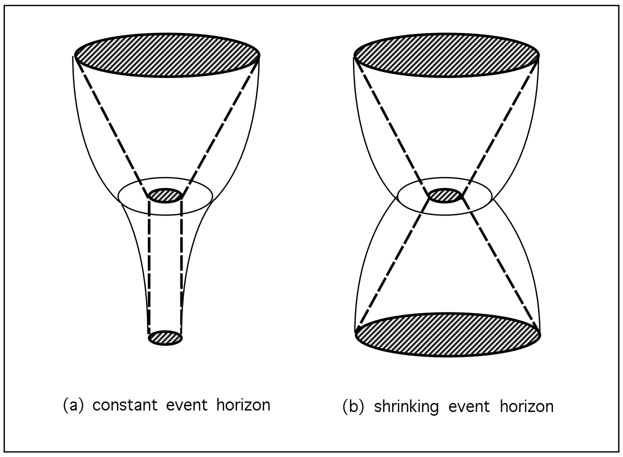

FIG. 1. *Qualitative evolution of the horizon scale and of the proper size of a homogeneous region for (a) standard de Sitter inflation, and (b) pre-big bang superinflation, represented in the Einstein frame as a contraction. The time direction coincides with the vertical axis. The three horizontal spatial sections corresponds, from top to bottom, to the present time, to the end and to the beginning of inflation. The shaded area represents the horizon, and the dashed lines its time evolution. The full curves represent the time evolution of the border of the homogeneous region, controlled by the scale factor.*

1). In this sense, both initial conditions, in cases (a) and (b), seem to be equally natural. The difference is that in case (b) the initial horizon is large in Planck units, while in case (a) it is of order one. This is an obvious consequence of the different curvature scales at the beginning of inflation.

The question about the naturalness of the initial conditions seems thus to concern the unit of length used, in particular, to measure the size of the initial homogeneous domain, and, more generally, to characterize the initial geometric configuration at the onset of inflation: which basic length scale has to be used, the Planck (or string) length, or the radius of the causal horizon?

This, I believe, is the question to be answered. Providing a definite answer may deserve a careful analysis, which is outside the scope of this brief paper. Let me note that, according to [1], it is the Planck (or string) scale that should provide the natural units for the size of the initial homogenous patches and for the initial curvature and coupling scale. This is certainly reasonable when initial conditions are imposed on a cosmological state approaching the high-curvature, quantum gravity regime. In the pre-big bang scenario, however, initial conditions are to be imposed when the Universe is deeply inside the low-curvature, weak coupling, classical regime. In that regime the Universe does not know about the Planck length, and the causal horizon H^{-1} could represent a natural candidate for controlling the set of initial conditions. For what concerns homogeneity, however, I am not suggesting that the horizon (which is the maximal homogeneity scale) should be always *assumed* as the natural scale of homogeneity. I am suggesting that this possibility should be discussed on the ground of some quantitative and objective criterium, as attempted for instance in [7], and not discarded a priori, as in [1] (see also [8] for a discussion of "generic" initial conditions in a string cosmology context).

One might think that, accepting the horizon size as a natural homogeneity scale, there is no need of inflation to explain our present homogeneous Universe [1]. This is not the case, however, because if we go back in time without inflation our Universe should start in the past from a homogeneous region unnaturally larger than the horizon (see Fig. 1). Only with inflation the homogeneous region, going back in time, re-enters inside the horizon. So, only if there is inflation, an initial homogeneity scale of the order of the horizon scale is enough to reproduce our present Universe.

Also, one might think, as noted in [1], that the classical homogeneity of the horizon might be destroyed by quantum fluctuations amplified during the contraction preceeding the onset of the inflationary era, in such a way as to prevent the formation of a large homogeneous domain. This problem has been recently discussed in [9] for the case of a homogeneous string cosmology background with negative spatial curvature: it has been shown that quantum fluctuations die off much faster than classical inhomogeneities as they approach the initial perturbative vacuum, and remain negligible throughout the perturbative pre-big bang phase. For classical perturbations, however, the situation is different, and no general result is presently available. The initial amplitude of the classical inhomogeneities is not normalized to a vacuum fluctuation spectrum, the results of [9] cannot be applied, and inflation can occurr successfully or not depending on the initial distribution of the classical amplitudes.

Finally, one might argue that a large initial horizon, assuming a saturation of the bound imposed by the holographic principle in a cosmological context [10], implies a large initial entropy, $S =$ (horizon area in Planck units), and thus a small probability for the initial configuration. Indeed, if S is large, the probability that such a configuration be obtained through a process of quantum tunnelling (proportional to $\exp[-S]$) is exponentially suppressed, as emphasized in [1]. However, in the pre-big bang scenario, quantum effects such as tunnelling or reflection of the Wheeler-De Witt wave function are expected to be important towards *the end* of inflation [11], and *not the beginning*, as they may be effective *to exit* [12], eventually, from the inflationary regime, *not to enter* it and to explain the origin of the initial state. A large entropy of the initial state, in the weakly coupled, highly classical regime, can only correspond to a large probability of such configuration, (proportional to $\exp[S]$), as expected for classical and macroscopic configurations.

In conclusion, let me come back on the large dimensionless parameters characterizing the initial state of pre-big bang inflation [1]. The physical meaning of those parameters, i.e. the fact that the initial string coupling and curvature are very small in string (or Planck) units, is to be understood as a consequence of the perturbative initial conditions, suggested by the underlying duality symmetries. On the other hand, whenever inflation starts at curvature scales smaller than Planckian, the initial state is necessarily characterized by a large dimensionless ratio – the inverse of the curvature in Planck units. If one believes that such large numbers should be avoided, then should be prepared to accept the fact that natural initial conditions are only possible in the context of models in which inflation starts at the Planck scale: for instance chaotic inflation, as pointed out in [1].

This is a rather strong conclusion, that rules out, as a satisfactory explanation of our present cosmological state, not only the pre-big bang scenario, but any model in which inflation starts at scales smaller than Planckian (unless we have a scenario with different stages of inflation responsible for solving different problems). Even for a single stage of inflation very close to the Planck scale, however, we are not free of problems, as we are led, eventually, to the following question: can we trust the naturalness of inflation models like chaotic inflation, in which classical general relativity is applied to set up initial conditions at Planckian curvature scales, i.e. deeply inside the non-perturbative, quantum gravity regime?

The Planckian regime is certainly problematic to deal with, both in the string and in the standard inflationary scenario: in string cosmology, in particular, it prevents a simple solution of the "graceful exit" problem [13]. The pre-big bang scenario, however, tries to look back in time beyond the Planck scale by using the powerful tools of superstring theory, in particular its duality symmetries. According to duality, the pre-Planckian Universe approaches initially the state of a low-energy system, and initial conditions are to be set up in a regime well described by the lowest order effective action, in which all quantum and higher-order corrections are small, and under control. It is true, however, that the presence of the Planckian regime can indirectly affect the initial conditions also in a string cosmology context, as it imposes a finite duration of the low-energy dilaton-driven phase: the initial homogeneity scale, as a consequence, has to be large enough to emerge with the required size at the Planck epoch, and to avoid the need for a further period of high-curvature, Planckian inflation [14].

It should be stressed, finally, that the main difference from the standard scenario, in which any tracks of the pre-Planckian cosmological state is washed out by inflation, is probably the fact that the pre-Planckian history may become visible, in the sense that its phenomenological consequences can be tested (at least in principle) even today [15]. So, while in the context of standard inflation the naturalness criterium can be safely applied to select an initial state at the Planck scale, it seems difficult (in my opinion) to apply the same criterium in a string cosmology context, and to discard a model of pre-Planckian evolution only on the grounds of the large parameters characterizing the initial conditions. Such initial conditions have consequences accessible to observational tests, and the analysis of the "a posteriori" probabilities with the Bayesan approach of [7] suggests that a state with a large initial horizon may become "a posteriori" natural, because of the duality symmetries intrinsic to the pre-big bang scenario.

However, much further work is certainly needed before a final conclusion is reached. Irrespective of the final results, such work will certainly improve our present understanding of string theory and of the physics of the early Universe.

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[1] N. Kaloper, A. Linde and R. Bousso, Phys. Rev. D **59**, 043508 (1999).
[2] G. Veneziano, Phys. Lett. B **265**, 287 (1991).
[3] M. Gasperini, N. Sanchez and G. Veneziano, Nucl. Phys. B **364** 365 (1991).
[4] M. Gasperini and G. Veneziano, Phys. Rev. D **50**, 2519 (1994).
[5] M. S. Turner and E. J. Weinberg, Phys. Rev. D **56**, 4604 (1997).
[6] M. Gasperini and G. Veneziano, Mod. Phys. Lett. A **8**, 3701 (1993).
[7] A. Buonanno, T. Damour and G. Veneziano, Nucl. Phys. B **543**, 275 (1999).
[8] D. Clancy, J. E. Lidsey and R. Tavakol, Phys. Rev. D **59**, 063511 (1999); Phys. Rev. D **58**, 44017 (1998); D. Clancy, A. Feinstein, J. E. Lidsey and R. Tavakol, Phys. Lett. B **451**, 303 (1999).
[9] A. Gosh, G. Pollifrone and G. Veneziano, Phys. Lett. B **440**, 20 (1998).
[10] W. Fischler and L. Susskind, *Holography and cosmology*, hep-th/9806039; D. Bak and S. J. Rey, *Holographic principle and string cosmology*, hep-th/9811008; A. K. Biswas, J. Maharana and R. K. Pradhan, *The holography*

hypothesis and pre-big bang cosmology hep-th/9811051; G. Veneziano, Phys. Lett. B **454**, 22 (1999).
[11] M. Gasperini, Int. J. Mod. Phys. A **13**, 4779 (1998).
[12] M. Gasperini, J. Maharana and G. Veneziano, Nucl. Phys. B **472**, 394 (1996); M. Gasperini and G. Veneziano, Gen. Rel. Grav. **28**, 1301 (1996).
[13] R. Brustein and G. Veneziano, Phys. Lett. B **329**, 429 (1994); N. Kaloper, R. Madden and K. A. Olive, Nucl. Phys. B **452**, 677 (1995); R. Easther, K. Maeda and D. Wands, Phys. Rev. D **53**, 4247 (1996); I. Antoniadis, J. Rizos and K. Tamvakis, Nucl. Phys. B **415**, 497 (1994); S. J. Rey, Phys. Rev. Lett. **77**, 1929 (1996); M. Gasperini and G. Veneziano, Phys. Lett. B **387**, 715 (1996); M. Gasperini, M. Maggiore and G. Veneziano, Nucl. Phys. B **494**, 315 (1997); M. Maggiore, Nucl. Phys. B **525**, 413 (1998); R. Brustein and R. Madden, Phys. Lett. B. **410**, 110 (1997); Phys. Rev. D **57**, 712 (1998); R. Brandenberger, R. Easther and J. Maia, JHEP **9808**, 007 (1998); M. Maggiore and A. Riotto, Nucl. Phys. B **548**, 427 (1999) ; S. Foffa, M. Maggiore and R. Sturani, Nucl. Phys. B **552**, 395 (1999).
[14] M. Gasperini, M. Maggiore and G. Veneziano, Nucl. Phys. B **494**, 315 (1997); M. Maggiore and R. Sturani, Phys. Lett. B **415**, 335 (1997).
[15] See for instance R. Brustein, M. Gasperini, M. Giovannini and G. Veneziano, Phys. Lett. B **361**, 45 (1995); M. Gasperini, M. Giovannini and G. Veneziano, Phys. Rev. Lett. **75**, 3796 (1995); R. Durrer, M . Gasperini, M. Sakellariadou and G. Veneziano, Phys. Rev. D **59**, 043511 (1999).