BA-TH/00-378
February 2000
gr-qc/0002080

Birth of the Universe as anti-tunnelling from the string perturbative vacuum

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Abstract

The decay of the string perturbative vacuum, if triggered by a suitable, duality-breaking dilaton potential, can efficiently proceed via the parametric amplification of the Wheeler-De Witt wave function in superspace, and can appropriately describe the birth of our Universe as a quantum process of pair production from the vacuum.

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Essay written for the 2000 Awards of the Gravity Research Foundation, and selected for Honorable Mention.
To appear in **Int. J. Mod. Phys. D**

Typeset using REVTEX

A consistent and quantitative description of the birth of our Universe is one of the main goals of the quantum approach to cosmology. In the context of the standard scenario, in particular, quantum effects are expected to stimulate the birth of the Universe in a state approaching the de Sitter geometric configuration, appropriate to inflation [1]. The initial cosmological state is unknown, however, and has to be fixed through some "ad-hoc" prescription. It follows that there are various possible choices for the initial boundary conditions [2–4], leading in general to different quantum pictures of the early cosmological evolution.

In the context of the pre-big bang scenario [5], typical of string cosmology, the initial state on the contrary is fixed, and has to approach the string perturbative vacuum. The quantum decay of this initial state necessarily crosses the high-curvature, Planckian regime, and can be appropriately described by a Wheeler-de Witt (WDW) wave function [6], evolving in superspace. The birth of the Universe may then be represented as a process of scattering and reflection [7,8], in an appropriate minisuperspace parametrized by the metric and by the dilaton. In that case the pre-big bang initial state – emerging from the string perturbative vacuum – simulates the boundary conditions prescribed for a process of "tunnelling from nothing", in the context of the standard scenario [4]. It seems thus appropriate to say that the above scattering process describes the birth of the Universe as a *"tunnelling from the string perturbative vacuum"* [7,9].

In a process of tunnelling, or quantum reflection, the WDW wave function corresponding to our present cosmological configuration turns out to be exponentially damped: the birth of the Universe from the string perturbative vacuum would thus appear to be a very unlikely (i.e., highly suppressed) quantum effect, according to the above representation. In the string cosmology minisuperspace, however, there are also other, more efficient "channels" of vacuum decay. The main purpose of this paper is to show that, with an appropriate model of dilaton potential, the transition from the pre-big bang to the post-big bang regime may correspond to a parametric amplification of the wave function, in such a way that the birth of the Universe can be represented as a process of *"anti-tunneling from the string perturbative vacuum"*. The name "anti-tunnelling", which is synonymous of parametric amplification (a well known effect in the theory of cosmological perturbations [10]) follows from the fact that the transition probability in that case is controlled by the inverse of the quantum-mechanical transmission coefficient in superspace.

In order to illustrate this possibility we shall use a quantum cosmology model based on the lowest order, gravi-dilaton string effective action, which in $d+1$ dimensions can be written as:

$$S = -\frac{1}{2\,\lambda_s^{d-1}} \int d^{d+1}x \sqrt{|g|}\, e^{-\phi} \left[R + (\nabla_\mu \phi)^2 + V(\phi, g_{\mu\nu})\right]. \qquad (1)$$

Here λ_s is the fundamental string length parameter, and V is the (possibly non-local and non-perturbative) dilaton potential. Considering an isotropic, spatially flat cosmological background,

$$\phi = \phi(t), \qquad g_{\mu\nu} = \mathrm{diag}\left(g_{00}(t), -a^2(t)\delta_{ij}\right), \qquad (2)$$

with spatial sections of finite volume (a toroidal space, for instance), it is convenient to introduce the variables

$$\beta = \sqrt{d}\ln a, \qquad \overline{\phi} = \phi - \sqrt{d}\,\beta - \ln \int d^dx/\lambda_s^d, \qquad (3)$$

and the corresponding canonical momenta, defined in the cosmic time gauge by:

$$\Pi_\beta = \left(\frac{\delta S}{\delta \dot\beta}\right)_{g_{00}=1} = \lambda_s\,\dot\beta\, e^{-\overline{\phi}}, \qquad \Pi_{\overline{\phi}} = \left(\frac{\delta S}{\delta \dot{\overline{\phi}}}\right)_{g_{00}=1} = -\lambda_s\,\dot{\overline{\phi}}\, e^{-\overline{\phi}}. \qquad (4)$$

The WDW equation, which implements the Hamiltonian constraint $H = \delta S/\delta g_{00} = 0$ in the two-dimensional minisuperspace spanned by β and $\overline{\phi}$, takes then the form [7–9]:

$$\left[\partial_{\overline{\phi}}^2 - \partial_\beta^2 + \lambda_s^2\, V(\beta, \overline{\phi})\, e^{-2\overline{\phi}}\,\right] \Psi(\beta, \overline{\phi}) = 0 \qquad (5)$$

(we have used the differential representation $\Pi^2 = -\nabla^2$).

As is well known from low-energy, perturbative theorems, the dilaton potential is strongly suppressed (with an istantonic law) in the small coupling regime, so that the effective WDW potential appearing in eq.(5) goes to zero as we approach the flat, zero-coupling, string perturbative vacuum, $\beta \to -\infty$, $\overline{\phi} \to -\infty$. In the opposite regime of arbitrarily large coupling the dilaton potential is unknown, but we shall assume in this paper that a possible growth of V is not strong enough to prevent the effective WDW potential from going to zero also at large positive values of β and $\overline{\phi}$, so that $V \exp(-2\overline{\phi}) \to 0$ for $\beta, \overline{\phi} \to \pm\infty$. In this case, the asymptotic solutions of the WDW equations (5) can be factorized in the form of plane waves, representing free energy and momentum eigenstates:

$$\Psi(\beta, \overline{\phi}) = \psi_k^\pm(\beta)\psi_k^\pm(\overline{\phi}) \sim e^{\pm ik\beta \pm ik\overline{\phi}}, \qquad (6)$$

where ($k > 0$):

$$\Pi_\beta \psi_k^\pm(\beta) = \pm k \psi_k^\pm(\beta), \qquad \qquad \Pi_{\overline{\phi}} \psi_k^\pm(\overline{\phi}) = \pm k \psi_k^\pm(\overline{\phi}) \tag{7}$$

From a geometric point of view they represent, in minisuperspace, the four branches of the classical, low-energy string cosmology solutions [5], defined by the condition $\Pi_\beta = \pm \Pi_{\overline{\phi}}$, and corresponding to [7–9]:

- expansion, $\Pi_\beta > 0$, contraction, $\Pi_\beta < 0$,
- pre-big bang, $\Pi_{\overline{\phi}} < 0$, post-big bang, $\Pi_{\overline{\phi}} > 0$.

We now recall that, for an isotropic string cosmology solution [5], the dilaton is growing ($\dot{\phi} > 0$) only if the metric is expanding ($\dot{\beta} > 0$), see eq. (3). If we impose, as our physical boundary condition, that the Universe emerges from the string perturbative vacuum (corresponding, asymptotically, to $\beta \to -\infty$, $\phi \to -\infty$), then the initial state Ψ_{in} must represent a configuration which is expanding and with growing dilaton, i.e. $\Psi_{in} \sim \psi^+(\beta)\psi^-(\overline{\phi})$. The quantum evolution of the initial pre-big bang state is thus represented in this minisuperspace as the scattering, induced by the effective WDW potential, of an incoming wave travelling from $-\infty$ along the positive direction of the axes β and $\overline{\phi}$.

It follows that, in general, there are four different types of evolution, depending on whether the asymptotic outgoing state Ψ_{out} is a superposition of waves with the same Π_β and opposite $\Pi_{\overline{\phi}}$, or with the same $\Pi_{\overline{\phi}}$ and opposite Π_β, and also depending on the identification of the time-like coordinate in minisuperspace [8]. These four possibilities are illustrated in Fig. 1, where cases (a) and (b) correspond to $\Psi_{out}^\pm \sim \psi^+(\beta)\psi^\pm(\overline{\phi})$, while cases ($c$) and ($d$) correspond to $\Psi_{out}^\pm \sim \psi^-(\overline{\phi})\psi^\pm(\beta)$.

The two cases (a) and (c) represent scattering and reflection along the spacelike axes $\overline{\phi}$ and β, respectively. In case (a) the evolution along β is monotonic, so that the Universe always keeps expanding. The incident wave is partially transmitted towards the pre-big bang singularity (unbounded growth of the curvature and of the dilaton, $\beta \to +\infty$, $\overline{\phi} \to +\infty$), and partially reflected back towards the low-energy, expanding, post-big bang regime ($\beta \to +\infty$, $\overline{\phi} \to -\infty$). In case ($c$) the evolution is monotonic along the time axis $\overline{\phi}$, but not along β. So, the incident wave is totally transmitted towards the singularity ($\overline{\phi} \to +\infty$), but in part as an expanding and in part as a contracting configuration.

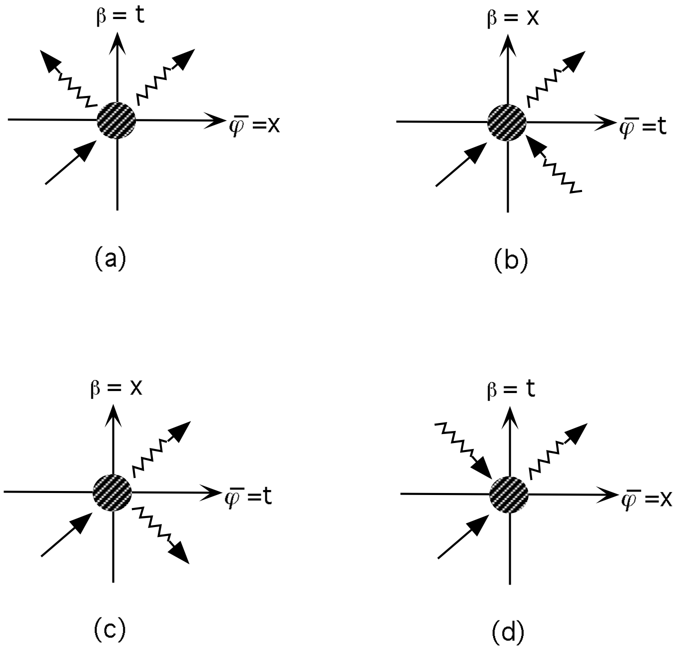

FIG. 1. *Four different classes of scattering for the initial string perturbative vacuum (solid line). The two spatial reflections (a) and (c) describe the transition from an expanding pre-big bang configuration to an expanding post-big bang and contracting pre-big bang configuration, respectively. The two Bogoliubov processes (b) and (d) represent the production of universe-antiuniverse pairs from the vacuum. In case (b) one universe is expanding, the other contracting, but they both fall inside the pre-big bang singularity. In case (d) both universes are expanding, but only one falls inside the singularity, while the other one survives in the post-big bang regime.*

In the language of third quantization [11] (i.e., second quantization of the WDW wave function in superspace) we can say that in case (a) we have the production of expanding post-big bang states from the string perturbative vacuum; in case (c), instead, we have the production of contracting pre-big bang states. In both cases, however, such a production is exponentially suppressed, and the suppression is proportional to the proper spatial volume of the portion of Universe emerging from the string perturbative vacuum [7].

The other two cases, (b) and (d), are qualitatively different, as the final state is a superposition of positive and negative energy eigenstates, i.e. of modes of positive and negative frequency with respect to time axes chosen in minisuperspace. In a third quantization context they represent a "Bogoliubov mixing", describing the production of pairs of universes from the vacuum. The mode moving backwards in time has to be "re-interpreted", like in quantum field theory, as an "antiuniverse" of positive energy and opposite momentum (in superspace). Since the inversion of momentum, in superspace, corresponds to a reflection of $\dot{\beta}$, the re-interpretation principle in this context changes expansion into contraction, and vice-versa.

Case (b), in particular, describes the production of universe-antiuniverse pairs – one expanding, the other contractiong – from the string perturbative vacuum. The pairs evolve towards the strong coupling regime $\overline{\phi} \to +\infty$, so both the members of the pair fall inside the pre-big bang singularity. Case (d) is more interesting, in our context, since in that case the universe-antiuniverse of the pair are both expanding: one falls inside the pre-big bang singularity, the other expands towards the low-energy, post-big bang regime, and may expand to infinity, representing the birth of a Universe like ours in a standard Friedman-like configuration.

Case (b) was discussed in a previous paper [12]: with a simple, duality-invariant model of potential, it was shown to represent an efficient conversion of expanding into contracting internal dimensions, associated to a parametric amplification of the wave function of the pre-big bang state. In this paper we shall concentrate on the process illustrated in case (d), already conjectured [9] to represent a promising candidate for an efficient transition from the pre- to the post-big bang regime, but never discussed in previous papers. To confirm this conjecture, we will provide here an explicit example in which the production of pairs of universes containing an expanding post-big bang configuration may be associated to a parametric amplification of the WDW wave function.

To this purpose we should note, first of all, that for a duality-invariant dilaton potential the string cosmology Hamiltonian associated to the action (1) is translationally invariant along the β axis, $[H, \Pi_\beta] = 0$: in this case, an initial expanding configuration keeps expanding, and the *out* state cannot be a mixture of states with positive and negative eigenvalues of Π_β. In order to implement the process (d) of Fig. 1 we thus need a non-local, duality-breaking potential, that contains both the metric and the dilaton, but *not* in the combination $\overline{\phi}$ of eq. (3).

We shall use, in particular, a two-loop dilaton potential induced by an effective cosmological constant Λ, i.e. $V \sim \Lambda \exp(2\phi)$ (two-loop potentials are known to favour the transition to the post-big bang regime already at the classical level [5,13], but only for appropriate repulsive self-interactions with $\Lambda < 0$). We shall assume, in addition, that such a potential is rapidly damped in the large radius limit $\beta \to +\infty$, and we shall approximate such a damping, for simplicity, by the Heaviside step function $V \sim \theta(-\beta)$. With this damping we represent the effective suppression of the cosmological constant, required for the transition to a realistic post-big bang configuration, and induced by some physical mechanism that does not need to be specified explicitly, for the purpose of this paper. Also, the choice of the cut-off function $\theta(-\beta)$ is not crucial, in our context, and other, smoother functions would be equally appropriate.

With these assumptions, the WDW equation (5) reduces to

$$\left[\partial_{\overline{\phi}}^2 - \partial_\beta^2 + \lambda_s^2 \Lambda\, \theta(-\beta)\, e^{2\sqrt{d}\beta}\right] \Psi(\beta, \overline{\phi}) = 0, \tag{8}$$

and the general solution can be factorized in terms of the eigenstates of the momentum $\Pi_{\overline{\phi}}$, by setting

$$\Psi(\beta, \overline{\phi}) = \Psi_k(\beta) e^{ik\overline{\phi}}, \qquad \left[\partial_\beta^2 + k^2 - \lambda_s^2 \Lambda\, \theta(-\beta)\, e^{2\sqrt{d}\beta}\right] \Psi_k(\beta) = 0. \tag{9}$$

In the region $\beta > 0$ the potential is vanishing, so that the general outgoing solution is a superposition of eigenstates of Π_β corresponding to the positive and negative frequency modes $\psi_k^\pm$, as in case (d) of Fig. 1. In the opposite region $\beta < 0$ the general solution is a combination of Bessel functions $J_\nu(z)$, of imaginary index $\nu = \pm ik/\sqrt{d}$ and argument $z = i\lambda_s\sqrt{\Lambda/d}\, e^{\sqrt{d}\beta}$.

We now fix the boundary conditions by imposing that the Universe starts expanding from the string perturbative vacuum: for $\beta \to -\infty$, the solution must then reduce to a plane wave representing a classical, low-energy pre-big bang solution, with $\Pi_\beta = -\Pi_{\overline{\phi}} = k > 0$. In particular, if we use the differential representation $\Pi = i\nabla$ for both β and $\overline{\phi}$:

$$\Psi_{in} = \lim_{\beta \to -\infty} \Psi(\beta, \overline{\phi}) \sim e^{ik(\overline{\phi} - \beta)}. \tag{10}$$

This choice uniquely determines the WDW wave function as:

$$\begin{aligned}
\Psi(\beta, \overline{\phi}) &= N_k J_{-\frac{ik}{\sqrt{d}}}\left(i\lambda_s\sqrt{\frac{\Lambda}{d}}\, e^{\sqrt{d}\beta}\right) e^{ik\overline{\phi}}, \qquad && \beta < 0, \\
&= \left[A_+(k) e^{-ik\beta} + A_-(k) e^{ik\beta}\right] e^{ik\overline{\phi}}, \qquad && \beta > 0.
\end{aligned} \tag{11}$$

With the matching conditions at $\beta = 0$ we can then compute the Bogoliubov coefficients $|c_\pm(k)|^2 = |A_\pm(k)|^2/|N_k|^2$ determining, in the third quantization formalism, the number

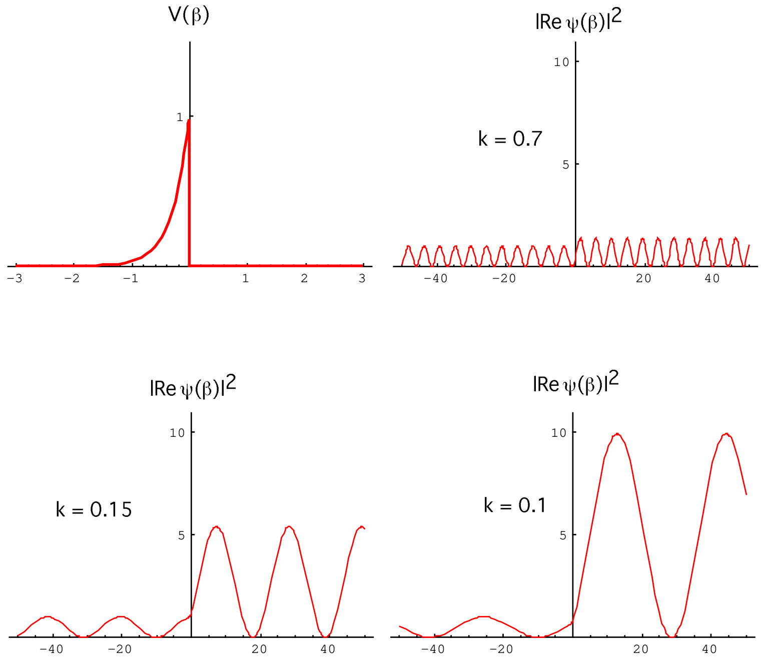

FIG. 2. *The first plot represents the effective potential of the WDW equation (9), for $d = 3$, in units of $\lambda_s^2 \Lambda$. The other plots represent the evolution in superspace of $|\Re\mathrm{e}\Psi_k(\beta)|^2$, obtained by solving eq. (9) with the initial boundary condition (10), for different values of k. We have used for all modes the same normalization, $|\Psi_k|^2 = 1$ at $\beta \to -\infty$, to emphasize that the amplification is more effective at lower frequencies.*

n_k of universes produced from the vacuum, for each mode k (here k represents a given configuration in the space of the initial parameters).

In contrast to the tunnelling process discussed in preivious papers [7,8], this process may represent an efficient mechanism of vacuum decay since the wave function is parametrically amplified (i.e., $n_k \gg 1$) for all $k < \lambda_s \sqrt{\Lambda}$. To illustrate this point we have numerically integrated eq. (9), and plotted in Fig. 2 the evolution in superspace of the real part of the wave function, for different configurations of initial momentum k (the behaviour of the imaginary part is qualitatively similar).

It may be interesting to note that the amplification is smaller at higher frequencies or – to use the language of cosmological perturbation theory – the pairs of universes are produced with a decreasing spectrum . This result has a quite reasonable interpretation, once we express the momentum k in terms of the physical parameters of the final geometric configuration. Indeed, from the definitons (3) and (4) we find, for a realistic transition occurring at the string scale, $\dot{\beta} \sim \lambda_s$, that $k \sim (\Omega_3/\lambda_s^3) g_s^{-2}$, where $\Omega_3 = a^3 \int d^3x$ is the proper spatial volume emerging from the transition in the post-big bang regime, and $g_s = e^{\phi}/2$ is the string coupling, parametrized by the dilaton. The condition of parametric amplification,

$$k \sim \left(\frac{\Omega_3}{\lambda_s^3}\right) \frac{1}{g_s^2} \lesssim \lambda_s \sqrt{\Lambda}, \tag{12}$$

implies that the transition is strongly favoured for configurations of small enough spatial volume in string units, large enough coupling g_s, and/or large enough cosmological constant Λ, in string units (in agreement with previous results [7,12]).

For $k \gg \lambda_s \sqrt{\Lambda}$ the wave function does not "hit" the barrier, and there is no parametric amplification. The inital state runs almost undisturbed towards the singularity, and only a small, exponentially suppressed fraction is able to emerge in the post big bang regime. In the context of third quantization this process can still be described as the production of pairs of universes, but the number of pairs is now exponentially damped, $n_k \sim \exp(-k/\lambda_s\sqrt{\Lambda})$, with a Boltzmann factor corresponding to a "thermal bath" of universes, at the effective temperature $T \sim \sqrt{\Lambda}$ in superspace.

In view of the above results, we may conclude that the decay of the string perturbative vacuum, if triggered by an appropriate, duality-breaking dilaton potential, can efficiently proceed via the parametric amplification of the WDW wave function in superspace, and can describe the birth of our Universe as a forced production of pairs from the vacuum fluctuations. One member of the pair disappears into the pre-big bang singularity, the other bounces back towards the low-energy region. The resulting effect is a net flux of universes that may escape to infinity in the post-big bang regime (see Fig. 3). This effect is similar to the quantum emission of radiation from a black hole [14], with the difference that the quanta produced in pairs from the vacuum are separated not by the black-hole horizon, but by the "Hubble" horizon associated to the "accelerated" variation of the dilaton in minisuperspace.

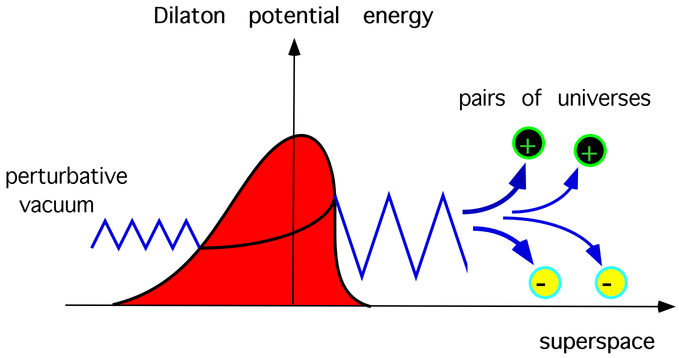

FIG. 3. *Birth of the universe represented as an anti-tunneling effect in superspace, or as a process of pair production from the string perturbative vacuum.*

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