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Dilatonic Interpretation of the Quintessence?

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We discuss the possibility that "quintessential effects", recently displayed by large scale observations, may be consistently described in the context of the low-energy string effective action, and we suggest a possible approach to the problem of the cosmic coincidence based on the link between the strength of the dilaton couplings and the cosmological state of our Universe.

In a string theory context [1], the macroscopic and large scale gravitational interactions are described by a low-energy effective action which necessarily contains a scalar field – the dilaton – and which can be written, in general, as the action of a non-minimally coupled and self-interacting scalar-tensor theory.

The dilaton ϕ controls the strength of the gravitational coupling and – in superstring models of unification – also the strength of all other interactions, since the expectation value of the dilaton should fix the fundamental ratio between string and Planck mass, and the gauge coupling constant of GUT theories [2]. At the tree-level,

$$g_s^2 = \exp\langle\phi\rangle \simeq (M_S/M_P)^2 \simeq \alpha_{\rm GUT}. \qquad (1)$$

It may be possible (and even auspicated, in certain string-inspired cosmological scenarios [3]) that in the very early past of our Universe the dilaton was varying very rapidly in time, and the effective gravitational interactions were very different from those described by the Einstein equations. At present, however, the dilaton has to be frozen, to be consistent with the observed values of the coupling constants. The string effective action should thus contain an appropriate potential to allow the solution $\phi = \phi_0 =$ const, or a regime of slow-enough dilaton variation. In such a regime, in addition, the dilatonic interactions must become weak enough and/or short-range, so that the action may provide a correct description of present macroscopic gravity.

The potential energy of a nearly constant dilaton, on the other hand, introduces into the gravitational equations a cosmological term, which might lead the Universe to a phase of cosmic repulsion and accelerated expansion. It seems thus natural to wonder whether the dilaton could simulate consistently, in a string theory context, the effects of the so-called "quintessence" [4], the phantomatic scalar field introduced to fit recent cosmological observations [5], and to (possibly) alleviate some problems posed by the phenomenological need for a vacuum source with negative pressure and very small energy density, which seems to dominate our present Universe. If the answer would be positive, there would be no need to assume the existence of new exotic scalar fields, and/or to invent *ad hoc* models of scalar interactions.

In this paper we shall assume that the dilaton is frozen, for some mechanism, already in the radiation era, with a potential energy small enough to avoid disturbing the standard cosmological evolution down to the equilibrium epoch. It could be argued that a frozen scalar field with, a very small but non-vanishing potential energy, is a model of quintessence indistinguishable from adding a "pure" cosmological constant to the action. This is certanly true for a minimally coupled field, but is not true, in general, for scalars non-minimally coupled to the geometry and to the other matter fields, like the dilaton.

For non-minimally coupled fields, in fact, a cosmological transition (i.e. a change in the equation of state of the dominant cosmological sources) tends to shift the field away from the initial equilibrium position, and a subsequent transition to the frozen, potential-dominated regime is not at all guaranteed. The dilaton, in particular, is coupled to the trace of the matter stress tensor, and a constant solution is possible in the (traceless) radiation era, as we shall see, but not in the era of matter domination.

Thus, even if the dilaton is "sleeping" in the radiation era, it necessarily "wakes up" and starts rolling down (or up) the potential after the equilibrium time, when the Universe enters the matter-dominated regime. However, if the dilaton mass is not too small, the dilaton may bounce back, and approach again the freezing position, just when its potential energy starts to become critical. It thus becomes a non-trivial consequence of the potential and of the "stringy" dilaton dynamics if the decelerated, matter-dominated era is followed by a phase of potential domination and accelerated expansion.

The main purpose of this paper is to discuss, in the above dilatonic scenario, the problem of the "cosmic coincidence" [6], which seems to affect the so-called "trackers solutions" [7] arising in models with power-law or exponential [8] potentials, as well as in models of quintessential inflation [9]. Our effort, in particular, is to understand whether or not such a problem may be avoided, or relaxed, in a dilatonic scenario in which the ratio of the matter to scalar energy density goes asymptotically to zero, and not to a constant like in recent attempts to solve the coincidence problem based on bulk viscosity [10] and on non-minimal scalar-tensor couplings [11]. The scenario discussed in this paper is also different from other, non-minimally coupled scalar models of quintessence [12], because the dilaton potential and couplings are not *ad hoc*, but (in principle) prescribed by string theory, and because the present value of the dilaton field is not an arbitrary parameter, but has to be determined so as to fix a realistic set of GUT coupling constants, according to eq. (1) (the tree-level relation between the dilaton and the fundamental constants could be non-trivially modified, however, by loop corrections [13]).

Let us start our discussion with the tree-level, lowest order in α', gravi-dilaton string effective action, minimally coupled to perfect fluid sources:

$$S = -\frac{1}{2\,\lambda_s^2}\int d^4x\,\sqrt{|g|}\,e^{-\phi}\,\left[R + (\nabla_\mu\phi)^2 + V(\phi)\right] + S_m \tag{2}$$

($\lambda_s = M_s^{-1}$ is the fundamental string length parameter). Consider a homogeneous, isotropic and spatially flat background. By varying the action with respect to g_{00}, g_{ij} and ϕ, and using the dilaton equations to simplify the g_{ij} equation, we get, respectively:

$$\dot\phi^2 + 6H^2 - 6H\dot\phi - V = e^{\phi}\rho, \tag{3}$$

$$\dot H - H\dot\phi + 3H^2 + \frac{V'}{2} = \frac{1}{2}e^{\phi}p, \tag{4}$$

$$\dot\phi^2 + 12H^2 - 6H\dot\phi - 2\ddot\phi + 6\dot H + V' - V = 0, \tag{5}$$

where $V' = \partial V/\partial\phi$, and we have chosen the cosmic time gauge. Note also that we have chosen units in which $2\lambda_s^2 = 1$, so that e^{ϕ} represents, in string units, the effective four-dimensional Newton constant $16\pi G$. The combination of the above equations leads to the usual covariant conservation of the energy density:

$$\dot\rho + 3H(\rho + p) = 0. \tag{6}$$

It is useful, at this point, to rewrite the dilaton equation by eliminating H^2 and $\dot H$. This gives the condition

$$\ddot\phi + 3H\dot\phi - \dot\phi^2 + \frac{1}{2}e^{\phi}(\rho - 3p) + V' + V = 0. \tag{7}$$

A stable solution $\phi = \phi_0 =$ const is thus possible only if

$$3p - \rho = 2e^{-\phi}(V + V') = \text{const}, \tag{8}$$

which, combined with eq. (6), leaves only three possibilities: *i)* vacuum, $\rho = p = 0$, $V + V' = 0$; *ii)* cosmological constant, $\rho = -p = \rho_0 =$ const, $V + V' = -2e^{\phi_0}\rho_0 =$ const; *iii)* radiation, $\rho = 3p$, $V + V' = 0$.

The first two cases corresponds to an accelerated de Sitter solution, $H^2 =$ constant. Only in the third case we can obtain a decelerated background, which coincides with the standard radiation-dominated solution provided $V(\phi_0), V'(\phi_0) \ll e^{\phi_0}\rho$. Let us thus suppose that, for some mechanism (to be discussed elsewhere), the dilaton is attracted to the equilibrium position $V + V' = 0$ early enough in the radiation era, and that the potential energy is always subdominant during the whole radiation epoch, $V(\phi_0) \ll H_{\rm eq}^2$, in such a way as to avoid any conflict with standard big bang nucleosynthesis (unlike other models of extended quintessence, see [14]).

It should be noted that such a dilaton configuration extremizes the effective (canonical) potential when transformed to the Einstein frame. In the (tilded) Einstein frame variables, defined by the conformal transformation

$$\tilde g_{\mu\nu} = g_{\mu\nu}e^{-\phi}, \qquad \tilde\phi = \phi, \tag{9}$$

the dilaton in fact is minimally coupled to the metric, but non-minimally coupled to the fluid sources, and the cosmological equations (3-5) become (in units $16\pi G = 1$):

$$6\tilde H^2 = \tilde\rho + \tilde V + \frac{\dot\phi^2}{2}, \tag{10}$$

$$4\dot{\tilde H} + 6\tilde H^2 = -\tilde p + \tilde V - \frac{\dot\phi^2}{2}, \tag{11}$$

$$\ddot\phi + 3\tilde H\dot\phi + \frac{1}{2}(\tilde\rho - 3\tilde p) + \tilde V' = 0, \tag{12}$$

where $\tilde V = e^{\phi}V$, $\tilde\rho = e^{2\phi}\rho$, $\tilde p = e^{2\phi}p$, and the dot denotes the derivative with respect to the Einstein cosmic time, $d\tilde t = dte^{-\phi/2}$. Thus, in the radiation era, $\tilde V' = 0$ is the necessary condition for $\phi =$ const.

Let us now approach the problem of the cosmic coincidence considering the Einstein frame equations (10–12), which we rewrite in the matter dominated era $\rho = \rho_m$, $p = 0$ (omitting the tilde, for simplicity), and taking into account the possibility of a more general (loop-induced, see below) matter-dilaton coupling, parametrized by the function $\alpha(\phi)$:

$$6H^2 = \rho_m + V + \frac{\dot\phi^2}{2}, \tag{13}$$

$$4\dot H + 6H^2 = V - \frac{\dot\phi^2}{2}, \tag{14}$$

$$\ddot\phi + 3H\dot\phi + \frac{1}{2}\alpha(\phi)\rho_m + V' = 0 \tag{15}$$

($\alpha = 1$ corresponds to the previous, lowest-order action). Their combination gives

$$\dot{\rho_m} + 3H\rho_m - \frac{1}{2}\alpha(\phi)\rho_m\dot\phi = 0. \tag{16}$$

Assuming that the matter-dominated era starts at the equilibrium time with $\phi = \phi_0$, $\dot\phi = 0 = V'$, $V_0 \equiv V(\phi_0) \ll 6H_{\rm eq}^2$, $\rho_m \simeq 6H_{\rm eq}^2$, we ask the question: is it possible for the phase of matter domination to evolve into a subsequent phase dominated by the potential energy of the scalar field, and corresponding to an effective negative pressure $p_\phi/\rho_\phi \equiv (\dot\phi^2/2 - V)/(\dot\phi^2/2+V) < 0$?

In the absence of the non-minimal coupling to matter (i.e $\alpha = 0$, pure general relativity) the answer would be yes, for all values of $V_0 < 6H_{\rm eq}^2$. In that case, indeed, the dilaton wold keep constant at the minimum, and the Universe would enter the potential-dominated phase as soon as $\rho_m \lesssim V_0$, rapidly approaching a final regime with $p_\phi/\rho_\phi = -1$, and $H^2 = V_0/6 =$ const. However, given the (semi-)infinite range of allowed values for V_0, why $V_0 \sim \rho_m(t_0) \sim H_0^2$ (as suggested by osservations), i.e. why the potential energy of the scalar field is just of the same order as the present matter energy density? (cosmic "coincidence").

The dilaton, however, *is* non-minimally coupled to macroscopic matter ($\alpha \neq 0$, in general), and cannot stay frozen at the minimum when the Universe is driven by ρ_m (see eq. (15)). The above question concerning the possible advent of the quintessential regime thus becomes: for which values of the initial potential $V_0 < H_{\rm eq}^2$ the dilaton may come back to the minimum, and the Universe may enter the potential-dominated phase, with total negative pressure? If the answer would point, only and precisely, at the value $V_0 \sim H_0^2$, the coincidence problem would disappear. If the answer would indicate instead a restricted range of values for V_0, the problem would be nevertheless alleviated.

The answer to the above question obviously depends on the shape of the potential, and on the coupling $\alpha(\phi)$. Let us start assuming, first of all, that α is a constant, or that its dilaton dependence (in the range of interest of our problem) is so weak to be consistently neglected. For the dilaton potential, we can ask the assistance of string theory. Although the full and detailed form of $V(\phi)$ is largely unknown (mainly because of non-perturbative effects), we know, however, that in the weak coupling regime ($\phi \to -\infty$, $g_s \ll 1$), the supersymmetry breaking potential [15] of critical superstring theory has to be strongly suppressed in an istantonic way, i.e. $V \sim \exp(-1/g_s^2) \equiv \exp[-\exp(-\phi)]$. In the regime of moderately strong coupling ($-\phi \sim 1$), on the other hand, the potential must exhibit some structure, leading to a minimum ϕ_0 such that [2]

$$g_s^2 = \exp(\phi_0) \simeq 0.1 - 0.01, \tag{17}$$

(according to eq. (1)). Finally, in the strong coupling regime ($g_s \sim 1$), we may expect an exponential growth of the potential, due to the factor e^ϕ induced by the transformation from the string to the Einstein frame. Such an exponential behaviour could be suppressed, at very strong coupling $g_s \gg 1$, by some non-perturbative effect or by loop corrections, but we are not concerned with this possibility here since the matter coupling is expected to shift the dilaton towards negative values, and then towards the weak coupling regime (see eq. (15)).

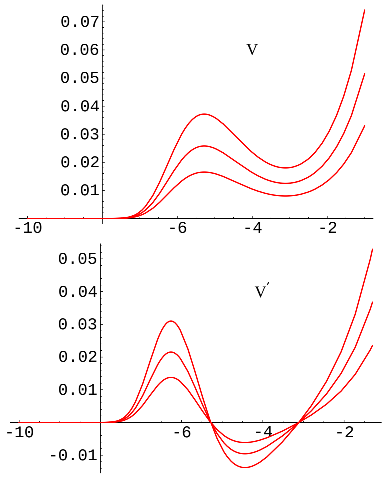

FIG. 1. *Plot of $V(\phi)$ and $V'(\phi)$ from eq. (18), with $k_1 = k_2 = \beta = \gamma = 1$, $\epsilon = 0.1$, $\phi_1 = -3$. The three curves, from top to bottom, correspond respectively to $m = 1/10$, $m = 1/12$, $m = 1/15$, in units of $H_{\rm eq}$.*

What is imporant, in this context, is that the potential barrier separating the minimum ϕ_0 from the perturbaive regime is not infinitely high, and it is possible for the shifted dilaton to escape to $-\infty$, provided its mass is light enough, and/or its coupling to matter (i.e., the initial acceleration) is sufficiently strong. The possible transition to a phase dominated by the potential energy V_0 thus becomes a precise problem of balance between the initial force aiming at moving the dilaton away from the minimum, $-\alpha\rho/2 < 0$, and the restoring force generated by the potential, $-V' > 0$.

In order to illustrate this mechanism, let us consider a "minimal" example of dilaton potential satisfying the above-mentioned string theory requisites, and controlled only by one dimensional parameter m, related to the effective (low-energy) mass of the dilaton. Such a potential can be simply parametrized (in the Einstein frame) as follows:

$$V = m^2 \left[e^{k_1(\phi-\phi_1)} + \beta e^{-k_2(\phi-\phi_1)} \right] e^{-\epsilon \exp[-\gamma(\phi-\phi_1)]} \tag{18}$$

(see also [16]), where $k_1, k_2, \phi_1, \epsilon, \beta, \gamma$ are dimensionless numbers of order one, whose precise values are not crucial for the purpose of this paper, provided they determine a minimum around a value ϕ_0 consistent with a superstring unification scenario (eq. (1)). For our illustrative purpose we will choose the particular values $k_1 = k_2 = \beta = \gamma = 1$, $\epsilon = 0.1$, $\phi_1 = -3$, in such

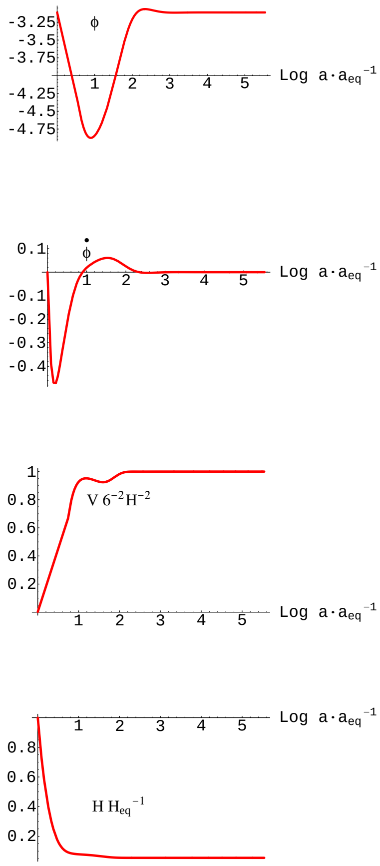

FIG. 2. *Numerical integration of eqs. (13-16) with* $\alpha = 1$, $m = 10^{-1}H_{\rm eq}$, *and initial conditions* $\phi = \phi_0 = -3.112$, $\dot\phi = 0$, $a = a_{\rm eq} = 1$, $H = H_{\rm eq} = 1$, $\rho_m = 6H_{\rm eq}^2 - V_0$.

a way that the minimum is at $\phi_0 = -3.112...$, and $g_s^2 = \exp(\phi_0) \simeq 0.045$, in agreement with eq. (17). (Many other choices are possible, of course. In particular, the qualitative behaviour of the potential remains the same if we increase ϵ, provided the values of k_1 and k_2 are also simultaneously increased, or the value of γ is decreased).

With the above choice of parameters, the dilaton potential (18) and its gradient V' are plotted in Fig. 1, for various values of m. We may note that, as m is decreased, the effective potential barrier is lowered, the restoring force $-V'$ becomes weaker, and it becomes easier for the dilaton to escape from the minimum, as previously anticipated.

With such a potential, we have numerically integrated eqs. (13-16), at fixed α (in particular, $\alpha = 1$), and initial condition at the equilibrium epoch: $\phi = \phi_0 = -3.112$, $\dot\phi = 0$, $H = H_{\rm eq}$, $\rho_m = 6H_{\rm eq}^2 - V_0$, $a = a_{\rm eq}$. We have found that, when the mass parameter $m < H_{\rm eq}$ is not too small (for instance, $m = 10^{-1}H_{\rm eq}$), the cosmological evolution is typically the one illustrated in Fig. 2: the dilaton is shifted from the minimum but is subsequently re-attracted to it, its velocity is damped and goes to zero after some oscillations, the potential energy asymptotically becomes critical, and the Hubble parameter freezes at a constant value determined by V_0 (and corresponding to a de Sitter equation of state $p_\phi/\rho_\phi = -1$).

When the mass parameter m is too small, however, the evolution is qualitatively different, and is illustrated in Fig. 3, where we present the results of a numerical integration with the same initial conditions, but a mass ten times smaller, $m = 10^{-2}H_{\rm eq}$: the velocity $\dot\phi$ is still damped and goes to zero, asymptotically, but it keeps negative for ever, and the dilaton runs monotonically towards the perturbative regime $\phi \to -\infty$, $V \to 0$.

We observe now that, in a realistic model of quintessence, the potential energy of the dilaton should represent an important fraction of the critical energy density (in particular [5], $V_0 \simeq (2/3)\rho_c \simeq 4H^2$) just at the present epoch, i.e. when the radiation energy density ρ_r (which evolves independently from the dilaton) has been reduced by a factor $(\rho_m/\rho_r)_0 = (a_0/a_{\rm eq}) \sim 10^4$ with respect to the matter energy density. This requires $V_0 \sim H_0^2$, where H_0 is the present value of the Hubble radius, and since (from eq. (18)) $V_0 = 2m^2\cosh(0.112)\exp[-0.1\exp(0.112)] \simeq 2m^2$, this implies $m \sim H_0 \sim 10^{-6}H_{\rm eq}$. The qustion is now whether or not, for such a value of m, the solution may become asymptotically dominated by the potential.

The answer is strongly dependent on the value of the dilaton coupling $\alpha(\phi)$. If $\alpha = 1$, for instance, the value $m \sim H_0 \sim 10^{-6}H_{\rm eq}$ is certainly to be excluded because, as shown by a numerical integration, the dilaton returs to the minimum of the potential only if $m \gtrsim 0.077H_{\rm eq}$. In that case, the regime of potential domination occurs too early, as illustrated in Fig. 2 where $V_0 \sim \rho_m$ for $a \sim 10a_{\rm eq}$, i.e. for $\rho_r \sim 10^{-1}\rho_m$. If $\alpha = 10^{-1}$, instead, we find that all the values of m from $H_{\rm eq}$ down to $10^{-7}H_{\rm eq}$ are compatible with a dilaton trapped around the minimum. The realistic value $m \sim 10^{-6}H_{\rm eq}$ is still to be fixed by hand, but the choice is restricted within a limited range and, in this sense, the coincidence problem seems to be alleviated.

It must be stressed, at this point, that the dilaton potential in the Einstein frame also determines the mass $\tilde m$ of the (canonically normalized) dilaton field, through its expansion around the minimum:

$$\tilde m^2 = V''(\phi_0) = 2m^2\cosh(\phi_0-\phi_1)e^{-\frac{1}{10}\exp(\phi_1-\phi_0)} \times \left[1 - 0.1e^{-(\phi_0-\phi_1)} - 10^{-2}e^{-2(\phi_0-\phi_1)}\right] \simeq 2m^2, \qquad (19)$$

where $(\phi_0 - \phi_1) \simeq -0.112$. A realistic scenario, with $m \sim H_0 \sim 10^{-33}$ eV, thus correspond to a very long (infinite, in practice) range for the dilatonic interactions. It follows that the present value of the dilaton coupling,

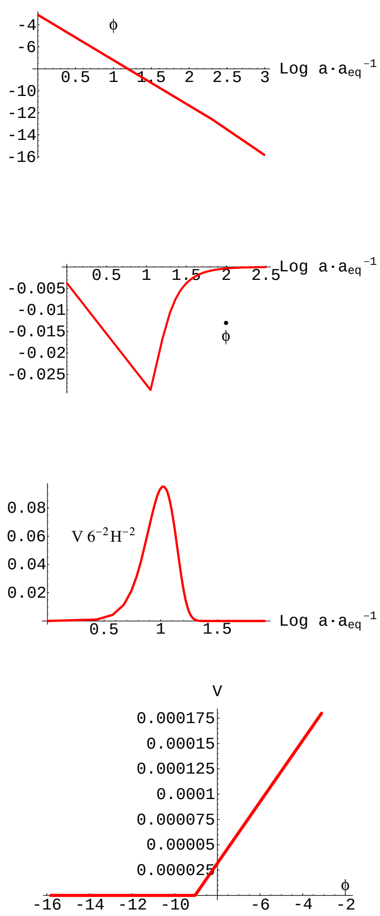

FIG. 3. *Numerical integration of eqs. (13-16) with* $\alpha = 1$, $m = 10^{-2} H_{\rm eq}$, *and the same initial conditions as in Fig. 2.*

$\alpha_0 \equiv \alpha[\phi(t_0)]$, has to be strongly suppressed, to be compatible with the present tests of the gravitational interaction. In particular [17], $\alpha_0 \lesssim 10^{-4}$ for composition-dependent dilatonic couplings, strongly constrained by the precise tests of the equivalence principle; $\alpha_0 \lesssim 10^{-2}$ for universal dilatonic couplings, constrained by tests of post-Newtonian gravity. Such phenomenological constraints can be accounted for, in principle, by including in the string effective action the quantum loop corrections, to all orders [17] (indeed, they are to be included when the string coupling g_s is not negligibly small, like in the case we are considering; higher-curvature α' corrections, on the contrary, can be safely neglected, as the curvature scales we are considering are always small in string units, $\lambda_s^2 H^2 \ll 1$).

The loop corrections modify the dilaton coupling to the matter sources, and can be parametrized, in our case, by three effective dilaton "form factors", $Z_\rho(\phi)$, $Z_p(\phi)$, $Z_\phi(\phi)$, appearing in the string frame action (2), and defined by:

$$\frac{\delta S_m}{\delta g_{00}} = \frac{1}{2} Z_\rho T_{00}, \quad \frac{\delta S_m}{\delta g_{ij}} = \frac{1}{2} Z_p T_{ij}, \quad \frac{\delta S_m}{\delta \phi} = Z_\phi T, \quad (20)$$

(T is the trace of the stress tensor). The transformation to the Einstein frame leads then to cosmological equations in which the canonically rescaled fields $\tilde\rho, \tilde p, \tilde\phi$, are minimally coupled to the metric, according to eqs. (10, 11). The dilaton, however, turns out to be non-minimally coupled to the trace of the fluid stress tensor, as in eqs. (15,16), through the coupling function $\alpha(\phi)$ that depends on Z_ϕ.

In the absence of a closed and explicit expression for the loops corrections one might assume, in the spirit of [17], that the present value of the coupling is small because the dilaton is attracted towards a local extremum ϕ_m of the coupling function. One can then expand (to first order) $\alpha(\phi) = k(\phi - \phi_m)$, where k is a dimensionless number of order one. In that case, if ϕ_m exactly coincides with ϕ_0, and with the initial position of the dilaton, then the dilaton is decoupled from matter already in the radiation era, it cannot be shifted from the minimum, and the coincidence problem remains.

If, however, the initial dilaton $\phi_{\rm eq}$ at equilibrium is slightly shifted from the extremum, because $\phi_m \neq \phi_0$ and/or $\phi_{\rm eq} \neq \phi_0$, a difference $|\phi_{\rm eq} - \phi_m| \lesssim 10^{-1}$ is already enough (as previously mentioned) to enlarge the allowed values of m to the range $H_{\rm eq} \to 10^{-7} H_{\rm eq}$, which includes $m \sim H_0$. In such a case one can easily match present observations, concerning both the cosmic equation of state and the present coupling of matter to scalar, long-range forces, as illustrated in Fig. 4 for an initial dilaton slightly tilted from the minimum. By choosing the appropriate value of V_0, the quintessential regime with critical potential and frozen Hubble radius may start indeed around a red-shift $a \sim 10^4 a_{\rm eq}$, and just when $H \sim H_0 \sim 10^{-6} H_{\rm eq}$. Note also that, in the example of Fig. 4, $\dot\phi/H$ and $\alpha(\phi)$ tend asymptotically to zero, and remain small enough to satisfy the present phenomenological constraints [17].

The main conclusion of this discussion – quite irrespective of the given particular examples, and of their possible relevance for a complete and fully realistic scenario – is that a string cosmology interpretation of quintessence suggests a close relationship between the problem of the cosmic coincidence, and the problem of fixing the present value of the dilaton coupling to matter (see also [18], for previous discussions of the link between quintessence and possible deviations from Newtonian gravity). In particular, a strong value of the coupling $\alpha(\phi) \sim 1$ – which is excluded by the gravitational phenomenology – would be *indirectly forbidden*, in such a context, *also by the present large scale configuration* of our Universe. Vice-versa, given $\alpha(\phi)$ in agreement with phenomenology, the allowed range of the dilaton potential turns out to be fixed, and the coincidence problem possibly alleviated, once one assumes $V_0 < H_{\rm eq}^2$.

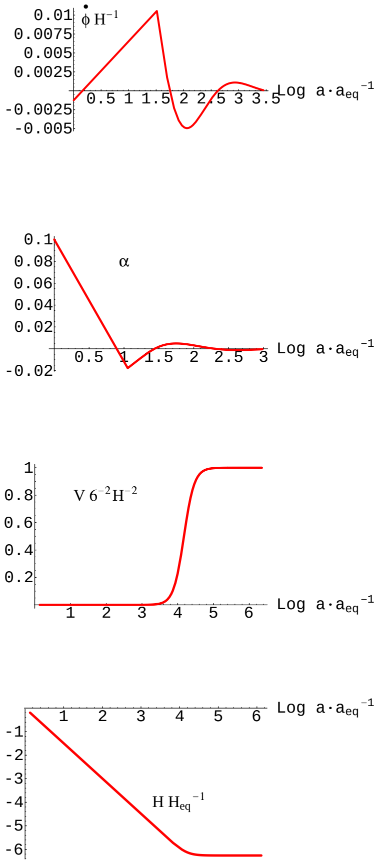

FIG. 4. *Numerical integration of eqs. (13-16) with* $m = 10^{-6} H_{\rm eq}$, $\alpha(\phi) = k(\phi - \phi_m)$, $k = 1$, $\phi_m = \phi_0$, *and an initial dilaton slightly displaced from the minimum,* $\phi_{\rm eq} = \phi_0 + 0.1$. *The other initial conditions are the same as in Fig. 2.*

An important open problem, in this approach, remains however to explain how the dilaton is attracted to the minimum of the potential in the radiation era, and why the vacuum energy is relaxed to a very small value – for instance, according to the mechanism dicussed in [19]. This problem requires a discussion of the initial conditions characterizing the (post-inflationary) cosmological configuration of our Universe, after the re-heating (and possibly pre-heating) phase, and will be addressed in a future paper.

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