BA-TH/01-415
CERN-TH/2001-149
hep-th/0106019

Localization of Scalar Fluctuations in a Dilatonic Brane-World Scenario

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Abstract

We derive and solve the full set of scalar perturbation equations for a class of Z_2-symmetric five-dimensional geometries generated by a bulk cosmological constant and by a 3-brane non-minimally coupled to a bulk dilaton field. The massless scalar modes, like their tensor analogues, are localized on the brane, and provide long-range four-dimensional dilatonic interactions, which are generically present even when matter on the brane carries no dilatonic charge. The shorter-range corrections induced by the continuum of massive scalar modes are always present: they persist even in the case of a trivial dilaton background (the standard Randall–Sundrum configuration) and vanishing dilatonic charges.

To appear in **Nucl. Phys. B**

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1 Introduction

The possibility, first discovered in the context of Horava–Witten (HW) heterotic M-theory [1, 2], that our Universe could lie on a hypersurface (a "brane") embedded in some higher-dimensional "bulk" space-time –the so-called brane-world scenario– has recently attracted considerable attention. In the original HW paper, our world is a 9-brane sitting at one of the boundaries of an eleven-dimensional bulk, while in a large class of M-theory models [3, 4], in which six dimensions are compactified in a more traditional Kaluza–Klein (KK) way, one can envisage constructing fully consistent four-dimensional brane-world scenarios with, effectively, a five-dimensional bulk. Unfortunately, finding brane configurations which are consistent with all stringy constraints has proved to be a very hard, if not impossible, task.

At a more phenomenological level, i.e. when problems with quantization of the higher-dimensional gravity theory are ignored, one can consider the dimensions orthogonal to the 3-brane as either compact and large [5, 6, 7], or as having infinite proper size [8, 9, 10]. In the latter (so-called Randall–Sundrum (RS)) case, the bulk geometry is bent by an appropriate "warp-factor", providing a crucial difference with respect to the "old" KK scenario, where the bulk geometry is simply the direct product of an "internal" and an "external" manifold. In particular, unlike the KK models, RS models are able to reproduce the four-dimensional Newton law at large distances on the brane by *dynamically* binding the massless gravitons to it [10].

As a consequence of the non-factorized structure of the metric in RS-type scenarios, previous approaches to the study of metric fluctuations in higher-dimensional backgrounds [11]–[16], based on the isometries of a factorizable geometry, cannot be applied directly to the brane-world scenario. A new gauge-invariant formalism is required, like the one developed in [17]. The classical and quantum analysis of metric fluctuations is of primary importance for understanding the possible localization of massless modes on the brane, as well as the nature of the corrections to the long-range interaction due to the continuum of massive modes that are typically living in the bulk. Until now, the study of this problem has been mainly focused on the structure of tensor (i.e. transverse-traceless) perturbations of the bulk geometry (see [18] for a general discussion).

In all string/M-theory models, however, the graviton enjoys the company of (perturbatively) massless scalar partners (the dilaton, compactification moduli, etc.). These typically induce long-range interactions of gravitational strength [19], and are therefore dangerous in view of the existing experimental tests (see for instance [20]). The standard way to solve this problem is to assume that these scalars get a SUSY-breaking non-perturbative mass (for alternatives see e.g. [21]). However, one may ask whether RS-type scenarios can offer an alternative solution to (or an alleviation of) this problem, e.g. by *not* confining scalar fluctuations on the brane. If this were the case, scalar interactions on the brane would be suppressed, or possibly become short-ranged, and brane-world scenarios would naturally solve one of the most serious potential problems with higher-dimensional or stringy extensions of the Standard Model and General Relativity. This particular aspect of brane-world scenarios has never been completely addressed, to the best of our knowledge, in spite of many studies recently appeared in the literature, and devoted to the perturbations of a brane-world background [22]–[30].

The main purpose of this paper is to directly address, and to discuss in detail, the possible localization (or de-localization) of scalar fluctuations in a typical brane-world scenario of the RS type, taking into account the possible existence of scalar sources and scalar fields living in the bulk. To this purpose, we will take a phenomenological approach, considering a scalar-tensor model of gravity in which the scalar fluctuations of the metric are also non-trivially coupled to the scalar fluctuations of the matter sources (unlike tensor fluctuations, i.e. gravitons, which are decoupled from matter). We shall consider, in particular, a non-compact, Z_2-symmetric, five-dimensional background, generated by a positive tension 3-brane and by a bulk dilaton field coupled to the brane and to the (negative) bulk energy density. We shall restrict ourselves to the gravi-dilaton solutions discussed in [31, 32] (hereafter called CLP backgrounds, for short), which generalize the AdS_5 RS scenario [10] in the presence of a bulk scalar field, and which are already known to guarantee the localization of tensor metric fluctuations [31, 32] and of bulk fields of various spin [33] (the localization of scalar and higher spin fields in the AdS_5 background was previously discussed in [34]).

By extending the analysis of the perturbations, and by using an appropriate gauge-invariant formalism [17], we find that the same class of CLP backgrounds that localize massless tensor perturbations on the four-dimensional brane also localize massless scalar perturbations (i.e. the dilatonic interaction). However, the short-range corrections to the scalar interactions, due to the massive modes propagating in the extra-dimension, are in general different from the higher-dimensional corrections affecting the pure tensor part of the gravitational interaction. We also find that, in general, scalar metric fluctuations exhibit a non-trivial, "self-sustained" spectrum of solutions, even for a trivial dilaton background. This implies that long- and short-range gravitational interactions on the brane are effectively of the scalar-tensor type, in agreement with previous results [35], and are therefore subject to strong phenomenological constraints.

The paper is organized as follows. In Section 2 we present the action and the classical equations of motion for a 3-brane non-minimally coupled to a five-dimensional gravi-dilaton background, and we retrieve the whole class of CLP solutions [31]. In Section 3 we give the full set of scalar perturbation equations in the so-called "generalized longitudinal gauge" [17], and we find the four independent canonical variables diagonalizing them. In Section 4 we discuss the localization of massless modes, and determine the class of backgrounds admitting long-range dilatonic interactions confined on the brane. In Section 5 we present the general spectrum of solutions for the massive modes that propagate throughout the bulk, and determine the relative magnitude of their amplitudes. In Section 6 we evaluate, in the weak field limit, the leading-order corrections to the effective gravitational potential generated by a static source with a point-like mass and dilatonic charge, confined on the brane. The main results of this paper are finally summarized in Section 7.

2 Background equations

We shall consider a five-dimensional scalar-tensor background $\{g_{AB}, \phi\}$, possibly arising from the bosonic sector of a dimensionally reduced string/supergravity theory, and non-trivially coupled to a negative cosmological constant Λ and to a 3-brane of positive tension T_3:

$$\begin{aligned} S = S_{\text{bulk}} + S_{\text{brane}} &= M_5^3 \int d^5x \sqrt{|g|} \left(-R + \frac{1}{2} g^{AB} \partial_A \phi \partial_B \phi - 2\Lambda e^{\alpha_1 \phi} \right) \\ &- \frac{T_3}{2} \int d^4\xi \sqrt{|\gamma|} \left[\gamma^{\alpha\beta} \partial_\alpha X^A \partial_\beta X^B g_{AB} e^{\alpha_2 \phi} - 2 \right]. \end{aligned} \qquad (2.1)$$

Here M_5 is the fundamental mass scale of the five-dimensional bulk space-time, and the parameters α_1, α_2 control the coupling of the bulk dilaton to Λ and to the brane. Particular values of these parameters may simply correspond to the rescaling of the minimally coupled σ-model action in the canonical Einstein frame [36], but here we take a phenomenological approach, allowing in general for non-minimal couplings (a single exponential potential for the dilaton can also be derived from the dimensional reduction of a suitable higher-dimensional model [31]). The brane action (see for instance [37]) is parametrized by the coordinates $X^A(\xi)$ describing the embedding of the brane in the bulk manifold, and by the auxiliary metric tensor $\gamma_{\alpha\beta}(\xi)$ defined on the four-dimensional world-volume of the brane, spanned by the coordinates ξ^α. Consequently, $\partial_\alpha X^A$ is a short-cut notation for $\partial X^A(\xi)/\partial \xi^\alpha$.

Conventions: Greek indices run from 0 to 3, capital Latin indices from 0 to 4, lower-case Latin indices from 1 to 3. For the bulk coordinates we use the notation $x^A = (t, x^i, z)$. The metric signature is $(+,-,-,-,-)$, and the curvature tensor is defined by $R_{MNA}{}^B = \partial_M \Gamma_{NA}{}^B + \Gamma_{MP}{}^B \Gamma_{NA}{}^P - (M \longleftrightarrow N)$, $R_{NA} = R_{MNA}{}^M$.

The variation of the action with respect to g_{AB}, ϕ, X^A and $\gamma_{\alpha\beta}$ gives, respectively, the Einstein equations (in units such that $M_5^3 = 1$):

$$\begin{aligned} G^A{}_B &= \frac{1}{2}\left(\partial^A\phi\partial_B\phi - \frac{1}{2}\delta^A_B\partial_C\phi\partial^C\phi\right) + \Lambda e^{\alpha_1\phi}\delta^A_B + \\ &+ \frac{T_3}{2}\frac{1}{\sqrt{|g|}}g_{BC}\int d^4\xi\sqrt{|\gamma|}\delta^5(x-X)\gamma^{\alpha\beta}\partial_\alpha X^A\partial_\beta X^C e^{\alpha_2\phi}, \end{aligned} \tag{2.2}$$

the dilaton equation:

$$\nabla_M\nabla^M\phi + 2\alpha_1\Lambda e^{\alpha_1\phi} + \frac{\alpha_2 T_3}{2}\frac{1}{\sqrt{|g|}}\int d^4\xi\sqrt{|\gamma|}\delta^5(x-X)\gamma^{\alpha\beta}\partial_\alpha X^A\partial_\beta X^B g_{AB}e^{\alpha_2\phi} = 0, \tag{2.3}$$

the equation governing the evolution of the brane in the bulk space-time:

$$\partial_\alpha\left(\sqrt{|\gamma|}\gamma^{\alpha\beta}\partial_\beta X^B g_{AB}e^{\alpha_2\phi}\right) = \frac{1}{2}\sqrt{|\gamma|}\gamma^{\alpha\beta}\partial_\alpha X^B\partial_\beta X^C\ \partial_A\left(g_{BC}e^{\alpha_2\phi}\right)\Big|_{x=X(\xi)}, \tag{2.4}$$

and the induced metric on the brane:

$$\gamma_{\alpha\beta} = \partial_\alpha X^A\partial_\beta X^B g_{AB}e^{\alpha_2\phi}. \tag{2.5}$$

We now specialize these equations to the case of a conformally flat bulk metric, with a warp factor a and a dilaton ϕ, both of which only depend on the fifth coordinate z. Also, we shall look for Z_2-symmetric solutions, describing a flat brane rigidly located at $z=0$, and we set

$$g_{AB} = a^2(z)\eta_{AB}, \qquad \phi = \phi(z), \qquad X^A = \delta^A_\mu\xi^\mu. \tag{2.6}$$

where η_{AB} is the five-dimensional Minkowski metric. The induced metric thus reduces to

$$\gamma_{\alpha\beta} = \delta^A_\alpha\delta^B_\beta g_{AB}\ e^{\alpha_2\phi}, \tag{2.7}$$

while the brane equations (2.4) are identically satisfied thanks to the Z_2 symmetry.

The dynamical equations are obtained from the dilaton equation (2.3), which becomes

$$3\frac{a'}{a}\phi' + \phi'' - 2\alpha_1\Lambda a^2 e^{\alpha_1\phi} - 2\alpha_2 T_3 a e^{2\alpha_2\phi}\delta(z) = 0, \tag{2.8}$$

and from the (α,β) and $(4,4)$ components of the Einstein equations (2.2), which give, respectively,

$$-3\frac{a''}{a} = \frac{\phi'^2}{4} + \Lambda a^2 e^{\alpha_1\phi} + \frac{T_3}{2}a e^{2\alpha_2\phi}\delta(z), \tag{2.9}$$

$$-6\frac{a'^2}{a^2} = -\frac{\phi'^2}{4} + \Lambda a^2 e^{\alpha_1\phi} \tag{2.10}$$

(a prime denotes differentiation with respect to z). The last three equations are not independent: the dilaton equation, for instance, can be obtained by differentiating eqs. (2.9), (2.10), as a consequence of the Bianchi identities.

No general solution is known for arbitrary values of α_1, α_2, T_3 and Λ. However, if we fine-tune these parameters by choosing:

$$\alpha_1 = 4\alpha_2 \ , \quad T_3 = 8\sqrt{\Lambda/\Delta} \ , \quad \alpha_1^2 = \Delta + \frac{8}{3} \ , \tag{2.11}$$

where the last equation defines Δ, we recover the four-dimensional sector of a known, one-parameter family of exact domain wall solutions [38], which can be written in an explicitly Z_2-symmetric form. For $\Delta = -2$ the solution is:

$$a(z) = e^{-\frac{k|z|}{3}}, \qquad \phi(z) = \sqrt{\frac{2}{3}}k|z|, \qquad k^2 = -2\Lambda, \tag{2.12}$$

otherwise ($\Delta \neq -2$):

$$a(z) = (1 + k|z|)^{\frac{2}{3(\Delta+2)}} \ , \qquad e^{\phi(z)} = (1 + k|z|)^{-\frac{2\alpha_1}{\Delta+2}} \ , \qquad k^2 = \frac{(\Delta+2)^2\Lambda}{\Delta}. \tag{2.13}$$

We shall choose $k > 0$ so that the z coordinate, transverse to the brane, may run from $-\infty$ to $+\infty$ (the proper size of the transverse dimension is finite, however, unless $\Delta = -8/3$). In that case, the solution corresponds to a brane of positive tension, $T_3 > 0$, provided $\Delta \leq -2$. This range of Δ guarantees a positive tension and also avoids the presence of naked singularities [31]. On the other hand, the reality of α_1 requires $\Delta \geq -8/3$. In the rest of this paper we shall thus assume

$$k > 0, \qquad\qquad -\frac{8}{3} \leq \Delta \leq -2. \tag{2.14}$$

We may note that in the limit $\Delta = -8/3$ the dilaton decouples and becomes trivial, and the solution reduces to the well studied pure AdS_5 background , originally introduced to localize gravity on a 3-brane [10].

In the following section we will obtain the canonical equations governing the evolution of scalar (metric + dilaton) fluctuations around the above CLP background solutions.

3 Scalar perturbations

We now perturb to first order the full set of bulk equations (2.2)–(2.5), keeping the position of the brane fixed, $\delta X^A = 0$. It is known, indeed, that the brane location can be consistently assumed to remain unchanged, to the relevant linear order, when perturbing a background of the type we are considering [17] (see [25] for a study that includes, instead, a non-vanishing deformation of the brane). We thus set

$$\delta g_{AB} = h_{AB}, \qquad \delta g^{AB} = -h^{AB}, \qquad \delta\phi = \chi, \qquad \delta X^A = 0, \tag{3.1}$$

where the indices of the perturbed fields are raised and lowered by the unperturbed metric, and the background fluctuations h_{AB}, χ are assumed to be inhomogeneous.

The perturbation of the background equations (2.2)–(2.5) gives, respectively, the linearized equations for the Einstein tensor:

$$\delta G^A{}_B = \frac{1}{2}\left(-h^{AC}\partial_C\phi\partial_B\phi + \partial^A\phi\partial_B\chi + \partial^A\chi\partial_B\phi\right) - \frac{1}{4}\delta^A_B\left(2\partial^M\partial_M\chi - h^{MN}\partial_M\phi\partial_N\phi\right)$$
$$+\Lambda\alpha_1\chi e^{\alpha_1\phi}\delta^A_B + \frac{T_3}{2}\frac{1}{\sqrt{|g|}}\int d^4\xi\sqrt{|\gamma|}\delta^5(x-X)\gamma^{\alpha\beta}\partial_\alpha X^A\partial_\beta X^C e^{\alpha_2\phi}$$
$$\times\left[h_{BC} + g_{BC}\left(-\frac{1}{2}g^{MN}h_{MN} + \frac{1}{2}\gamma^{\mu\nu}\delta\gamma_{\mu\nu} + \alpha_2\chi\right)\right]$$
$$+\frac{T_3}{2}\frac{1}{\sqrt{|g|}}g_{BC}\int d^4\xi\sqrt{|\gamma|}\delta^5(x-X)\partial_\alpha X^A\partial_\beta X^C e^{\alpha_2\phi}\delta\gamma^{\alpha\beta}, \tag{3.2}$$

for the dilaton:

$$\nabla_M\nabla^M\chi - h^{MN}\nabla_M\nabla_N\phi - g^{MN}\delta\Gamma^B_{MN}\partial_B\phi + 2\alpha_1^2\Lambda\chi e^{\alpha_1\phi}$$
$$+\frac{\alpha_2 T_3}{2}\frac{1}{\sqrt{|g|}}\int d^4\xi\sqrt{|\gamma|}\delta^5(x-X)\gamma^{\alpha\beta}\partial_\alpha X^A\partial_\beta X^B g_{AB}e^{\alpha_2\phi}$$
$$\times\left(-\frac{1}{2}g^{MN}h_{MN} + \frac{1}{2}\gamma^{\mu\nu}\delta\gamma_{\mu\nu} + \alpha_2\chi\right)$$
$$+\frac{\alpha_2 T_3}{2}\frac{1}{\sqrt{|g|}}\int d^4\xi\sqrt{|\gamma|}\delta^5(x-X)\partial_\alpha X^A\partial_\beta X^B e^{\alpha_2\phi}\left(g_{AB}\delta\gamma^{\alpha\beta} + h_{AB}\gamma^{\alpha\beta}\right) = 0, \tag{3.3}$$

for the brane:

$$\partial_\alpha\left\{\sqrt{|\gamma|}\partial_\beta X^B e^{\alpha_2\phi}\left[\delta\gamma^{\alpha\beta}g_{AB} + \gamma^{\alpha\beta}h_{AB} + \gamma^{\alpha\beta}g_{AB}\left(\alpha_2\chi + \frac{1}{2}\gamma^{\mu\nu}\delta\gamma_{\mu\nu}\right)\right]\right\} =$$
$$= \frac{1}{2}\sqrt{|\gamma|}\partial_\alpha X^B\partial_\beta X^C\left[\left(\delta\gamma^{\alpha\beta} + \frac{1}{2}\gamma^{\alpha\beta}\gamma^{\mu\nu}\delta\gamma_{\mu\nu}\right)\frac{\partial}{\partial x^A}\bigg|_{x=X(\xi)}\left(g_{BC}e^{\alpha_2\phi}\right)\right.$$
$$\left.+\gamma^{\alpha\beta}\,\partial_A\left(h_{BC}e^{\alpha_2\phi} + \alpha_2\chi g_{BC}e^{\alpha_2\phi}\right)\bigg|_{x=X(\xi)}\right], \tag{3.4}$$

and for the induced metric:

$$\delta\gamma_{\alpha\beta} = \partial_\alpha X^A\partial_\beta X^B e^{\alpha_2\phi}\left(h_{AB} + \alpha_2\chi g_{AB}\right). \tag{3.5}$$

Here all geometrical quantities, such as the perturbed connection $\delta\Gamma_{AB}{}^C$, the perturbed scalar curvature $\delta R = -h^{AB}R_{AB} + g^{AB}\delta R_{AB}$, and so on, are computed to first order in h_{AB}.

By expanding around the background (2.6), it is now easy to study the propagation of the spin-2 physical degrees of freedom on the brane, represented by the transverse and traceless perturbations $\overline{h}_{ij}$,

$$h_{AB} = a^2\delta^i_A\delta^j_B\overline{h}_{ij}, \qquad \overline{h}^i{}_i = 0, \qquad \nabla^i\overline{h}_{ij} = 0. \tag{3.6}$$

In the linear approximation, the tensor fluctuations $\overline{h}_{ij}$ are decoupled from the scalar and matter fluctuations. We can consistently set $\chi = 0$, and find that the dilaton and the brane equations are trivially satisfied; in addition, the right-hand side of the Einstein equations (3.2) is identically vanishing, and the linearized Ricci tensor leads to the well known covariant wave equation for gravitons:

$$\Box_5 \overline{h}_{ij} \equiv \left(\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^{i2}} - \frac{\partial^2}{\partial z^2} - \frac{3a'}{a}\frac{\partial}{\partial z} \right) \overline{h}_{ij} = 0, \tag{3.7}$$

where $\Box_5 \equiv \nabla_M \nabla^M$ is the five-dimensional covariant d'Alembert operator, describing free propagation in the warped bulk geometry.

In this paper (also in preparation of future cosmological applications) we are primarily interested in the scalar fluctuations of the bulk metric, which are coupled to the dilaton fluctuations. Thus, we shall keep $\delta\phi = \chi \neq 0$, and we shall expand around the background (2.6) in the so-called "generalized longitudinal gauge" [17], which extends the longitudinal gauge of standard cosmology [39] to the brane-world scenario. As discussed in [17], in five dimensions there are four independent degrees of freedom for the scalar metric fluctuations: in the generalized longitudinal gauge they are described by the four variables $\{\varphi, \psi, \Gamma, W\}$, defined by

$$\begin{aligned} h_{00} &= 2\varphi a^2, & h_{ij} &= 2\psi a^2 \delta_{ij}, \\ h_{44} &= 2\Gamma a^2, & h_{04} &= -W a^2. \end{aligned} \tag{3.8}$$

(Off-diagonal metric fluctuations have been taken into account also in a recent study of linearized gravity in a brane-world background [40]: in that case, however, there are no scalar sources in the bulk, and no long-range scalar interactions).

By inserting the explicit form of the background (2.6) and of the metric fluctuations (3.8) into the perturbed equations (3.2)–(3.5), we obtain the full set of constraints and dynamical equations governing the linearized evolution of the five scalar variables $\{\varphi, \psi, \Gamma, W, \chi\}$. Let us give them in components, starting from the Einstein equations, and using eq. (3.5) for the perturbations of the induced metric.

Equation $(0,0)$ gives:

$$\begin{aligned} & 2\nabla^2\psi + 3\psi'' + \nabla^2\Gamma + 9\frac{a'}{a}\psi' - 3\frac{a'}{a}\Gamma' - \frac{\phi'}{2}\chi' - 6\frac{a''}{a}\Gamma - \frac{\phi'^2}{2}\Gamma \\ & -a^2 e^{\alpha_1\phi}\Lambda\alpha_1\chi - \frac{1}{2} a e^{2\alpha_2\phi} T_3 \left(\Gamma + 2\alpha_2\chi\right)\delta(z) = 0. \end{aligned} \tag{3.9}$$

Equation (i,i) gives:

$$-\nabla^2\varphi - \varphi'' - 2\ddot{\psi} + \nabla^2\psi + 2\psi'' - \ddot{\Gamma} + \nabla^2\Gamma$$

$$
\begin{aligned}
&-3\frac{a'}{a}\varphi' - \dot{W}' - 3\frac{a'}{a}\dot{W} + 6\frac{a'}{a}\psi' - 3\frac{a'}{a}\Gamma' - \frac{\phi'}{2}\chi' - 6\frac{a''}{a}\Gamma - \frac{\phi'^2}{2}\Gamma \\
&-a^2 e^{\alpha_1\phi}\Lambda\alpha_1\chi - \frac{1}{2}ae^{2\alpha_2\phi}T_3\left(\Gamma + 2\alpha_2\chi\right)\delta(z) = 0.
\end{aligned} \tag{3.10}
$$

Equation (i,j), with $i \neq j$, gives:

$$
\partial_i\partial_j\left(\varphi - \psi - \Gamma\right) = 0. \tag{3.11}
$$

Equation $(4,4)$ gives:

$$
\begin{aligned}
&-\nabla^2\varphi - 3\ddot{\psi} + 2\nabla^2\psi - 3\frac{a'}{a}\varphi' - 3\frac{a'}{a}\dot{W} + 9\frac{a'}{a}\psi' + \frac{\phi'}{2}\chi' \\
&-12\frac{a'^2}{a^2}\Gamma + \frac{\phi'^2}{2}\Gamma - a^2 e^{\alpha_1\phi}\Lambda\alpha_1\chi = 0.
\end{aligned} \tag{3.12}
$$

Equation $(i,0)$ gives:

$$
\partial_i\left(\frac{W'}{2} + \frac{3}{2}\frac{a'}{a}W + 2\dot{\psi} + \dot{\Gamma}\right) = 0. \tag{3.13}
$$

Equation $(4,0)$ gives:

$$
\frac{1}{2}\nabla^2 W - 3\dot{\psi}' + 3\frac{a'}{a}\dot{\Gamma} + \frac{\phi'}{2}\dot{\chi} = 0. \tag{3.14}
$$

Equation $(4,i)$ gives:

$$
\partial_i\left(\frac{\dot{W}}{2} + \varphi' - 2\psi' + 3\frac{a'}{a}\Gamma + \frac{\phi'}{2}\chi\right) = 0. \tag{3.15}
$$

The dilaton equation gives:

$$
\begin{aligned}
&\Box_5\chi - \phi'\varphi' - \phi'\dot{W} + 3\phi'\psi' - \phi'\Gamma' - 6\frac{a'\phi'}{a}\Gamma - 2\phi''\Gamma \\
&+2a^2 e^{\alpha_1\phi}\Lambda\alpha_1^2\chi + 2ae^{2\alpha_2\phi}T_3\alpha_2\left(\Gamma + 2\alpha_2\chi\right)\delta(z) = 0
\end{aligned} \tag{3.16}
$$

(the dots denote differentiation with respect to Minkowski time on the brane). Finally, the brane perturbation equation gives a constraint at $z = 0$ which is always satisfied because of the Z_2 symmetry. A similar set of equations was already derived in [17]. We have no contributions from the time derivatives of the background, since our background is static.

In the absence of bulk sources with anisotropic stresses, we can now eliminate φ from eq. $(i \neq j)$, thus reducing to four scalar degrees of freedom, by setting:

$$
\varphi = \psi + \Gamma. \tag{3.17}
$$

As a consequence, we immediately find that the variable W decouples from the other fluctuations. The combination of the time derivative of eq. $(4,i)$ with the z-derivative of eq. $(i,0)$ and with eq. $(4,0)$ leads in fact to the equation

$$\Box_5 W = 3\left(\frac{a''}{a} - \frac{a'^2}{a^2}\right) W. \tag{3.18}$$

Thus W is decoupled but, because of the non-trivial self-interactions, it does not freely propagate in the background geometry like the graviton, eq. (3.7).

In order to discuss the dynamics of the remaining variables ψ, Γ and χ, it is now convenient to recombine their differential equations in an explicitly covariant way, to obtain a canonical evolution equation. To this aim, we can use eq. (3.17) and eq. $(4,i)$ to eliminate φ and $\dot{W}$ in eqs. (i,i), $(4,4)$ and in the dilaton equation (3.16). We then combine the simplified version of eqs. $(4,4)$, (i,i) with eq. $(0,0)$, and obtain the following system of coupled equations, where the source terms depend only on Γ and χ:

$$\begin{aligned}
\Box_5 \psi &= -2\frac{a''}{a}\Gamma + 2\frac{a'^2}{a^2}\Gamma + \frac{a'\phi'}{a}\chi - \frac{2}{3}a^2 e^{\alpha_1\phi}\Lambda\alpha_1\chi \\
&- \frac{1}{6}ae^{2\alpha_2\phi}T_3\left(\Gamma + 2\alpha_2\chi\right)\delta(z), \qquad (3.19)\\
\Box_5 \Gamma &= -2\frac{a''}{a}\Gamma + 8\frac{a'^2}{a^2}\Gamma - \phi'^2\Gamma + \phi''\chi + \frac{a'\phi'}{a}\chi - \frac{2}{3}a^2 e^{\alpha_1\phi}\Lambda\alpha_1\chi \\
&- \frac{2}{3}ae^{2\alpha_2\phi}T_3\left(\Gamma + 2\alpha_2\chi\right)\delta(z), \qquad (3.20)\\
\Box_5 \chi &= 2\phi''\Gamma - \phi'^2\chi - 2a^2 e^{\alpha_1\phi}\Lambda\alpha_1^2\chi \\
&- 2ae^{2\alpha_2\phi}T_3\alpha_2\left(\Gamma + 2\alpha_2\chi\right)\delta(z). \qquad (3.21)
\end{aligned}$$

This system can be diagonalized by introducing the fields

$$\omega_1 = 2\psi + \Gamma, \qquad \omega_2 = 6\alpha_2\Gamma + \chi, \qquad \omega_3 = \Gamma - 2\alpha_2\chi, \tag{3.22}$$

relations that can be inverted as:

$$\begin{aligned}
&\psi = \frac{\omega_1}{2} - \frac{2\alpha_2\omega_2 + \omega_3}{2\left(1 + 12\alpha_2^2\right)}, \qquad \Gamma = \frac{2\alpha_2\omega_2 + \omega_3}{\left(1 + 12\alpha_2^2\right)} \\
&\chi = \frac{\omega_2 - 6\alpha_2\omega_3}{\left(1 + 12\alpha_2^2\right)}. \qquad (3.23)
\end{aligned}$$

In terms of these new variables, the perturbation equations (3.19)–(3.21) reduce to

$$\begin{aligned}
\Box_5 \omega_1 &= 0, \qquad (3.24)\\
\Box_5 \omega_2 &= 0, \qquad (3.25)\\
\Box_5 \omega_3 &= \left[-\frac{\phi''}{2\alpha_2} - \phi'^2 - \frac{a'\phi'}{2\alpha_2 a} + \left(\frac{\alpha_1}{3\alpha_2} - 2\alpha_1^2\right)a^2 e^{\alpha_1\phi}\Lambda\right]\omega_3 \\
&+ \left[\left(\frac{2}{3} - 4\alpha_2^2\right)ae^{2\alpha_2\phi}T\delta(z)\right]\omega_3. \qquad (3.26)
\end{aligned}$$

Together with eq. (3.18), and the constraints (3.13)–(3.15), the above decoupled equations describe the complete evolution of the scalar (metric + dilaton) fluctuations in the CLP brane-world background (2.12), (2.13). Two variables (ω_1, ω_2) are (covariantly) free on the background like the graviton, while the other two variables (ω_3, W) have non-trivial self-interactions.

In all cases, we can introduce the corresponding "canonical variables" $\widehat{W}$, $\widehat{\omega}_i$ $(i = 1, 2, 3)$, which have canonically normalized kinetic terms [39] in the action, simply by absorbing the geometric warp factor as follows:

$$W = \widehat{W} a^{-3/2}, \qquad \omega_i = \widehat{\omega}_i a^{-3/2}. \tag{3.27}$$

The variables $\widehat{W}$, $\widehat{\omega}_i$ are required for a correct normalization of the scalar perturbations to a quantum fluctuation spectrum, as they satisfy canonical Poisson (or commutation) brackets. When the general solution is written as a superposition of free, factorized plane-waves modes on the brane,

$$\widehat{W} = \Psi_w(z) e^{-ip_\mu x^\mu}, \qquad \widehat{\omega}_i = \Psi_i(z) e^{-ip_\mu x^\mu}, \tag{3.28}$$

they define the inner product of states with measure dz [18], as in conventional one-dimensional quantum mechanics, $\int dz |\Psi(z)|^2$. Such a product is required for an appropriate definition of normalizable solutions.

It should be stressed that in this paper we are primarily interested in the quantum fluctuations of the metric, and of the other background fields, around their mean values determined by the classical solution. For a consistent quantization of such perturbations we have thus to impose appropriate boundary conditions, in order to eliminate the non-normalizable solutions (blowing up in z). In the case of classical perturbations, the existence of solutions that are initially small enough to justify the linear approximation, and that blow up at late times, would signal an instability of the background. In this paper we have not analyzed this problem in full generality. It looks however that, whenever we have looked at tachyonic modes that grow at late times, we have found that they also blow up at large z (see Section 5), making the linear analysis of this paper inadequate for drawing any strong conclusion.

In the case of quantum fluctuations, discussed in this paper, the allowed mass spectrum of $m^2 = \eta^{\mu\nu} p_\mu p_\nu$, for the scalar fluctuations on the brane, can then be obtained by solving an eigenvalue problem in the Hilbert space $L^2(R)$ for the canonical variables Ψ_w, Ψ_i, satisfying a Schrödinger-like equation in z, which is obtained from the equations (3.18), (3.24)–(3.26) for W and ω_i, and which can be written in the conventional form as:

$$\Psi_w'' + \left(m^2 - \frac{\xi_w''}{\xi_w} \right) \Psi_w = 0, \qquad \Psi_i'' + \left(m^2 - \frac{\xi_i''}{\xi_i} \right) \Psi_i = 0. \tag{3.29}$$

Here, by analogy with cosmological perturbation theory [39], we have introduced four "pump fields" ξ_w, ξ_i, defined as follows:

$$\xi_w = a^{\beta_w}, \qquad \xi_i = a^{\beta_i},$$
$$\beta_w = -\frac{3}{2}, \qquad \beta_1 = \beta_2 = \frac{3}{2}, \qquad \beta_3 = -\frac{1}{2}(1 + 3\alpha_1^2) = -\frac{3}{2}(\Delta + 3). \tag{3.30}$$

The effective potential generated by the derivatives of the pump fields depends on β_w, β_i, and contains in general a smooth part, peaked at $z = 0$, plus a positive or negative δ-function contribution at the origin. We may have, in principle, not only volcano-like potentials, which correspond to the free covariant d'Alembert equation with $\beta = 3/2$ [31] (and which are known to localize gravity [10, 18]), but also potentials that are positive everywhere and admit no bound states. The possible localization of scalar interactions on the 3-brane, for the given background and perturbation equations, will be discussed in the next section.

4 Localization of the massless modes

The general solutions of the canonical perturbation equations (3.29) are labelled by the mass eigenvalue m, by their parity with respect to z-reflections, and by the parameters β_w, β_i, which depend on the type of perturbation. The massless scalar modes corresponding to bound states of the effective potential [10, 18, 31] will describe long-range scalar interactions confined to the four-dimensional brane. Concerning z-reflections, we shall follow the perturbative formalism developed in [17] consistently, restricting ourselves to a perturbed background that is still Z_2-symmetric, namely to Z_2-odd solutions for W, and to Z_2-even solutions for ω_i. To describe a bound state, we shall further restrict such solutions to those with a normalizable canonical variable w.r.t. the measure dz, namely to $\Psi(z) \in L^2(R)$. Among the acceptable solutions, we shall finally select those satisfying the constraints (3.13)–(3.15) derived in the previous section. The above set of conditions will determine the class of brane-world backgrounds allowing the four-dimensional localization of long-range scalar interactions.

For $m = 0$ the exact solution of the canonical equations (3.29), for a generic pump field $\xi(z)$, can be separated into an even and an odd part, Ψ^+ and Ψ^-, as follows:

$$\Psi_0^+(z) = c_+\xi(z), \qquad \Psi_0^-(z) = c_-\xi(z)\int dz'\xi^{-2}(z'), \tag{4.1}$$

where c_+ and c_- are integration constants. In order to parametrize the solutions for different values of Δ and β (and also in view of subsequent applications to the massive mode solutions), it is now convenient to introduce the two indices ν and ν_0, defined by

$$\nu_0 = \frac{\Delta}{2(\Delta + 2)}, \qquad \nu = \frac{1}{2} - \frac{2\beta}{3(\Delta + 2)}. \tag{4.2}$$

For the three possible values of β_w, β_i (see eq. (3.30)), they are related by

$$\begin{aligned} &\nu = 1 - \nu_0, && \beta = \beta_w = -\frac{3}{2}, \\ &\nu = \nu_0, && \beta = \beta_{1,2} = \frac{3}{2}, \\ &\nu = 2 - \nu_0, && \beta = \beta_3 = -\frac{3}{2}(\Delta + 3). \end{aligned} \tag{4.3}$$

We recall that, in the class of backgrounds we are considering, the parameter Δ is constrained in the range $-8/3 \leq \Delta \leq -2$ (because of the conditions of positive tension, absence of naked singularities and reality of the dilaton couplings). As a consequence, the index ν_0 can range from 2 to $+\infty$.

Let us first discuss the limiting case $\nu = \nu_0 = \infty$, corresponding to $\Delta = -2$. For a generic pump field, with exponent β, the massless solutions are:

$$\Psi^+_{0,\infty}(z) = c^+_{0,\infty} e^{-\frac{\beta}{3}k|z|}, \qquad \Psi^-_{0,\infty}(z) = c^-_{0,\infty} \mathrm{sgn}\{z\} \left(e^{\frac{\beta}{3}k|z|} - e^{-\frac{\beta}{3}k|z|} \right). \tag{4.4}$$

In our background $k > 0$, so that the fluctuations W and ω_3, both with $\beta = -3/2$, are not normalizable. The even solutions of the free d'Alembert equation, with $\beta = 3/2$, are instead normalizable, so we have acceptable solutions for ω_1 and ω_2. However, from the constraint $(i, 0)$ of eq. (3.13), ω_1 is forced to vanish when $W = 0$, unless the fluctuations are static, $\dot{\omega}_1 = 0$. All the other constraints (using also the background equations) are instead identically satisfied by ω_2. We can also check, by using the inverted expressions (3.23), that the perturbation equation (0,0), as well as all the original perturbed set of evolution equations, are identically satisfied by the above solutions. It follows that, for $\Delta = -2$, there are two independent massless modes localized on the brane: one, ω_2, is propagating and the other, ω_1, is static.

The same is true for the case $\Delta < -2$, i.e. finite ν_0. In that case the massless solutions can be written in the form

$$\begin{aligned} \Psi^+_{0,\nu}(z) &= c^+_{0,\nu} \left(1 + k|z|\right)^{\frac{1}{2}-\nu}, \\ \Psi^-_{0,\nu}(z) &= c^-_{0,\nu} \mathrm{sgn}\{z\} \left[\left(1 + k|z|\right)^{\frac{1}{2}+\nu} - \left(1 + k|z|\right)^{\frac{1}{2}-\nu} \right]. \end{aligned} \tag{4.5}$$

The solutions for W and ω_3 always correspond to $\nu < 0$, and are not normalizable. The even d'Alembert modes, with $\nu = \nu_0$, are normalizable for $\nu > 1$, i.e. $\Delta > -4$, so that ω_1 and ω_2 are always acceptable in our class of backgrounds, for which $\Delta \geq -8/3$. Again, however, the constraint (3.13) implies $\omega_1 = 0$, unless $\dot{\omega}_1 = 0$, while the other constraints are identically satisfied by ω_2, so we are left with non-trivial massless solutions only for a propagating fluctuation, ω_2, and for a static one, ω_1. Again, we can check that all the perturbed evolution equations (for instance, eq. (0,0)), are satisfied by the above solution.

We may thus conclude that all backgrounds of the class defined by the conditions (2.14) localize on the brane not only the massless spin-2 degrees of freedom [31] (long-range tensor interactions), but also one propagating massless scalar degree of freedom (ω_2), corresponding to a long-range scalar interaction generated by the dilaton field. In the longitudinal gauge, the canonical representation ω_2 of such a scalar interaction is associated not only with the dilaton fluctuation χ, but also with the four-dimensional "Bardeen potential" ψ, and with the "breathing mode" Γ of the dimension orthogonal to the brane (see eqs. (3.23)). In addition, we have a second independent massless degree of freedom localized on the brane (ω_1), which is not propagating ($\dot{\omega}_1 = 0$), but is essential to reproduce the standard long-range gravitational interaction in the static limit, as we shall discuss in Section 6.

The localization of the scalar interactions does not impose any further constraints on the background, besides those of eq. (2.14). Also, in the limiting case of a pure AdS_5 solution ($\Delta = -8/3$, $\nu_0 = 2$, $\alpha_1 = 0 = \alpha_2$) the dilaton background disappears, and the dilaton fluctuation $\chi = \omega_2$ decouples from the others. The only (static) contribution to the scalar sector of metric fluctuations comes from ω_1, which generates the long-range Newton potential $\varphi = \psi$ on the brane (see Section 6). However, even in the case $\Delta = -8/3$, the propagating dilaton flucutuation ω_2 is non-vanishing, and remains there to describe a (possibly dangerous) long-range scalar interaction. By contrast, no propagating massless scalar modes appear in the "pure-gravity" models without bulk scalar fields in the action, as the AdS_5 brane-world scenario discussed in [10].

5 The massive mode spectrum

The massive part of the spectrum of the canonical equations (3.29) is not localized on the brane; it may induce higher-dimensional (and short-range) corrections to the long-range scalar forces (just as in the case of the pure tensor part of the gravitational interaction [10, 18, 31]). Such corrections thus provide physical effects from the fifth dimension on our brane.

In order to determine the massive modes able to survive the constraints of Section 3, and to evaluate the corresponding short-range corrections, we shall first present the exact solutions of the massive canonical equations (3.29), distinguishing, as before, the even and odd parts under z-reflections. For $m \neq 0$, $\Delta = -2$ (i.e. $\nu_0 = \infty$), and a generic pump field with power β, such solutions can be written as follows

$$\begin{aligned} \Psi^+_{m,\infty}(z) &= \frac{1}{\sqrt{\pi m q}} \left(q \cos q|z| - \frac{\beta}{3} k \sin q|z| \right), \\ \Psi^-_{m,\infty}(z) &= -\sqrt{\frac{m}{\pi q}} \sin qz, \end{aligned} \tag{5.1}$$

where

$$q=\left(m^2-\frac{k^2}{4}\right)^{1/2}. \tag{5.2}$$

For $\Delta < -2$, i.e. finite ν_0, the solution can be written as a combination of first- and second-kind Bessel functions J_ν and Y_ν [41], of index ν given by eq. (4.2):

$$\begin{aligned}\Psi^+_{m,\nu}(z) &= c_{m,\nu-1}\sqrt{1+k|z|}\left[Y_{\nu-1}\left(\frac{m}{k}\right)J_\nu(y)-J_{\nu-1}\left(\frac{m}{k}\right)Y_\nu(y)\right],\\ \Psi^-_{m,\nu}(z) &= c_{m,\nu}\,\mathrm{sgn}\{z\}\sqrt{1+k|z|}\left[Y_{\nu}\left(\frac{m}{k}\right)J_\nu(y)-J_{\nu}\left(\frac{m}{k}\right)Y_\nu(y)\right],\end{aligned} \tag{5.3}$$

where

$$c_{m,\nu}=\sqrt{\frac{m}{2k}}\left[J_\nu^2\left(\frac{m}{k}\right)+Y_\nu^2\left(\frac{m}{k}\right)\right]^{-\frac{1}{2}}, \qquad y=\frac{m}{k}(1+k|z|). \tag{5.4}$$

Note that in the above equations we have adopted the δ-function normalization of the continuum modes, as for plane waves in one-dimensional quantum mechanics. As a consequence, Ψ_m is dimensionless (unlike in [31], where a different normalization is adopted).

It is important to note that modes with negative squared mass (tachyons) are not included in the spectrum, as they would not correspond to a normalizable canonical variable (Ψ would blow up in z). This can be regarded as a direct check of the stability of the given class of CLP backgrounds against quantum scalar perturbations, since tachyonic modes would also blow up in time, and would destroy the assumed homogeneity of the four-dimensional brane. Another consequence of the normalization condition is the mass gap ($m^2 > k^2/4$, see eq. (5.2)) between the localized massless mode and the massive corrections, in the limiting background with $\nu_0=\infty$ (already noticed in [31] for the case of pure tensor interactions).

We shall now impose the constraints (3.13)–(3.15) following from the perturbed background equations. It is convenient to introduce the four amplitudes A_i, A_w, defined by

$$\begin{aligned}\omega_1 &= \frac{a^{-3/2}A_1}{c_{m,\nu_0-1}}\Psi^+_{m,\nu_0}(z)e^{-ip_\mu x^\mu}, & \omega_2 &= \frac{a^{-3/2}A_2}{c_{m,\nu_0-1}}\Psi^+_{m,\nu_0}(z)e^{-ip_\mu x^\mu}\\ \omega_3 &= \frac{a^{-3/2}A_3}{c_{m,1-\nu_0}}\Psi^+_{m,2-\nu_0}(z)e^{-ip_\mu x^\mu}, & W &= \frac{a^{-3/2}A_w}{c_{m,1-\nu_0}}\Psi^-_{m,1-\nu_0}(z)e^{-ip_\mu x^\mu},\end{aligned} \tag{5.5}$$

where $\Psi^\pm$ are the above normalized solutions. Plugging this ansatz into the constraint equations (3.13)–(3.15), we find that, in contrast with the massless case, none of the four scalar fluctuations is forced to vanish. However, only two amplitudes are independent. By taking, for instance, ω_2 and ω_3 as independent variables, we can indeed express A_1 and A_w in terms of A_2 and A_3, for all values of ν_0, in such a way that all the constraints are identically satisfied. For a generic mode of mass m and momentum p, we find, in particular,

$$
\begin{aligned}
A_1 &= \frac{m^2}{9m^2+6p^2}\frac{\sqrt{2\nu_0-1}}{\nu_0-1}\left(\sqrt{3\,(\nu_0-2)}A_2+3\sqrt{2\nu_0-1}A_3\right), \\
A_w &= \frac{2im\sqrt{m^2+p^2}}{9m^2+6p^2}\frac{\sqrt{2\nu_0-1}}{\nu_0-1}\left(\sqrt{3\,(\nu_0-2)}A_2+3\sqrt{2\nu_0-1}A_3\right),
\end{aligned}
\tag{5.6}
$$

which also hold when $\nu_0 \to \infty$. Note that a single combination of A_2 and A_3 determines both A_1 and A_w. We can check that the above constrained solutions satisfy eq. (0,0) and, more generally, the complete set of perturbed evolution equations.

For such backgrounds we thus have four types of higher-dimensional contributions to the scalar interactions on the brane, arising from the massive spectrum of ω_i and W. The exchange of such massive modes generates corrections to the four-dimensional scalar forces. The corrections are in general different from those of tensor interactions, arising from the massive spectrum of a variable satisfying the free d'Alembert equation, like ω_2. In the weak field limit, however, the leading-order contributions to the non-relativistic potential generated by a static scalar sources have the same qualitative behaviour as in the case of tensor interactions, as will be illustrated in the next section.

6 Static limit and leading-order corrections

To make contact with previous works, and for future phenomenological applications, we shall now compute the effective scalar-tensor interaction induced on the brane, in the weak field limit, by a static and point-like source of mass M and dilatonic charge Q.

In our longitudinal gauge (3.6), (3.8), in which the decomposition of the metric fluctuations is based on the $O(3)$ symmetry of the spatial hypersurfaces of the brane, the energy density of a point-like particle only contributes to the scalar part of the perturbed matter stress tensor (with T_{00} as the only non-vanishing component), and provides a δ-function source to the $(0,0)$ scalar perturbation equation (3.9). Similarly, the charge Q acts as a point-like source in the dilaton perturbation equation (3.16).

As a consequence, we obtain three δ-function sources in the equations for the three ω_i fluctuations, $S_i\delta^3(x-x')\delta(z)$, with three scalar charges S_i, which are "mixtures" of M and Q, while no source term is obtained in the static limit for the W fluctuation (in agreement with the fact that $W=0$, in the static limit, according to the constraint $(i,0)$). The effective sources S_i for the massless and massive ω_i fluctuations are defined by the combination of eqs. $(4,4), (i,i), (0,0)$, and by the dilaton equation (3.16), as follows:

$$
S_1 = M, \qquad S_2 = Q + 2\alpha_2 M, \qquad S_3 = \frac{M}{3} - 2\alpha_2 Q. \tag{6.1}
$$

The exact static solutions of eqs. (3.24)–(3.26), with the above point-like sources, can be easily obtained using the static limit of the retarded Green function evaluated on the brane ($z = 0$), i.e.

$$\omega_i(\nu, x, x') = -S_i G_i(\nu, x, x'), \tag{6.2}$$

where

$$G_i(\nu, \vec{x}, \vec{x}', z = z' = 0) = \int \frac{d^3p}{(2\pi)^3} e^{i\vec{p}\cdot(\vec{x}-\vec{x}')} \left\{ \frac{\left[\psi^+_{0,\nu}(0)\right]^2}{p^2} + \int_{m_0}^\infty dm \frac{\left[\psi^+_{m,\nu}(0)\right]^2}{p^2 + m^2} \right\} \tag{6.3}$$

(see also [31, 35, 42]). Here $\nu = \nu_0$ for $i = 1, 2$, while $\nu = 2 - \nu_0$ for $i = 3$, and $\psi^+_{0,\nu}(z), \psi^+_{m,\nu}(z)$ (including the case $\nu = \infty$) are the exact solutions (4.4), (4.5), (5.1), (5.3), obtained in the previous sections. The first term in the integrand corresponds to the long-range forces generated by the massless modes, the second term to the "short-range" corrections due to the massive modes, and m_0 is the lower bound for the massive spectrum ($m_0 = k/2$ if $\nu_0 = \infty$, while $m_o = 0$ if $\nu_0 < \infty$, see eqs. (5.1), (5.2)).

We should note that in the ω_1, ω_2 case we have to include both the massless and massive contributions, while in the ω_3 case only the massive ones survive (indeed, we recall that for $m = 0$ the ω_3 modes are not normalizable, and that the massless sector of ω_1, in the static limit, is not eliminated by the constraint $(i, 0)$, see Section 4). We also note that the amplitude of the massless solutions (4.4), (4.5) has to be fixed by the correct normalization, i.e.

$$c^+_{0,\infty} = (k\beta/3)^{1/2}, \qquad c^+_{0,\nu} = [k(\nu - 1)]^{1/2}. \tag{6.4}$$

Let us start with ω_1, for which $\nu = \nu_0$, and with the limiting background $\Delta = -2$, i.e. $\nu = \infty$. By setting $\beta = 3/2$, and using eqs. (4.4) and (5.1) for $\psi^+_{0,\infty}$ and $\psi^+_{m,\infty}$, respectively, we obtain

$$\omega_1(\nu = \infty) = -S_1 \frac{k}{8\pi r} \left[1 + \frac{2}{k\pi} \int_{k/2}^\infty \frac{dm}{m} \left(m^2 - \frac{k^2}{4} \right)^{1/2} e^{-mr} \right], \tag{6.5}$$

where $r = |\vec{x} - \vec{x}'|$. The same integral had already been obtained in [31] when discussing the localization of tensor fluctuations, and the associated leading-order corrections (in the large-distance limit) are known to be of the form $e^{-kr/2}(kr)^{-3/2}$.

In the case $\Delta < -2$, i.e. $\nu = \nu_o < \infty$, we shall use instead eqs. (4.5) and (5.3) for ψ^+_{0,ν_0} and ψ^+_{m,ν_0}, respectively. Also, we shall estimate the contribution of the massive modes by expanding the Bessel functions in the small-m regime (which is relevant at the long distances typical of the weak field limit). The small argument limit [41] of J_ν, Y_ν then gives

$$\left[\psi^+_{m,\nu_0}(0)\right]^2 = \frac{1}{\Gamma^2(\nu_0 - 1)} \left(\frac{m}{2k} \right)^{2\nu_0 - 3}, \tag{6.6}$$

and we are led to

$$\omega_1(\nu<\infty) = -S_1 \frac{k(\nu_0-1)}{4\pi r}\left[1+\frac{\Gamma(\nu_0-1/2)}{\sqrt{\pi}\Gamma(\nu_0)}\left(\frac{1}{kr}\right)^{2\nu_0-2}\right] \tag{6.7}$$

(here Γ obviously represents the Euler function, not to be confused with the g_{44} component of the metric fluctuations).

Exactly the same results (6.5), (6.7) are obtained for ω_2, which satisfies the same free d'Alembert equation as ω_1, with the only difference that S_1 has to be replaced by S_2.

Let us then compute ω_3, for which $\nu = 2-\nu_0$, and which has only the massive mode contribution to the Green function. For $\Delta=-2$, i.e $\nu=\infty$, we shall use eq. (5.1) for $\psi^+_{m,\infty}$. At $z=0$ the solution however is β-independent, and we obtain the (massive part) of the result already given in eq. (6.5) (with S_1 replaced by S_3).

In the case $\Delta<-2$, i.e $\nu<\infty$, we shall use $\psi^+_{m,2-\nu_0}$ from eq. (5.3). In the large-distance (small-m) regime, however, the contribution at $z=0$ to the Green function is exactly the same as that of eq. (6.6), so that we obtain

$$\omega_3(\nu<\infty) = -S_3 \frac{k(\nu_0-1)}{4\pi r}\frac{\Gamma(\nu_0-1/2)}{\sqrt{\pi}\Gamma(\nu_0)}\left(\frac{1}{kr}\right)^{2\nu_0-2}. \tag{6.8}$$

We are now ready to estimate, in the static limit, the dilaton and metric fluctuations $\chi, \Gamma, \psi, \varphi$. We shall explicitly consider the case $\nu_0<\infty$, for an easy comparison with previous results relative to an AdS$_5$ background, for which $\nu_0=2$. Defining

$$A_{\nu_0} = \frac{k(\nu_0-1)}{4\pi}, \qquad\qquad B_{\nu_0} = \frac{\Gamma(\nu_0-1/2)}{\sqrt{\pi}\Gamma(\nu_0)}, \tag{6.9}$$

we first rewrite the ω_i solutions in compact form as:

$$\begin{aligned}
\omega_1 &= -\frac{S_1A_{\nu_0}}{r}\left[1+B_{\nu_0}\left(\frac{1}{kr}\right)^{2\nu_0-2}\right],\\
\omega_2 &= -\frac{S_2A_{\nu_0}}{r}\left[1+B_{\nu_0}\left(\frac{1}{kr}\right)^{2\nu_0-2}\right],\\
\omega_3 &= -\frac{S_3A_{\nu_0}}{r}B_{\nu_0}\left(\frac{1}{kr}\right)^{2\nu_0-2}.
\end{aligned} \tag{6.10}$$

We may note, as a check of our previous computations, that the relative amplitude of the massive corrections, in the weak, static limit, is controlled by the correspondig scalar charges, which satisfy the relation (from eq. (6.1)):

$$S_3+2\alpha_2S_2 = \left(\frac{1}{3}+4\alpha_2^2\right)S_1. \tag{6.11}$$

By eliminating α_2 in favour of Δ according to eq. (2.11), and Δ in terms of ν_0, according to eq. (4.2), we obtain

$$S_1 = \frac{1}{\nu_0 - 1}\left[\frac{1}{3}\sqrt{3(\nu_0-2)(2\nu_0-1)}S_2 + (2\nu_0-1)S_3\right], \tag{6.12}$$

which exactly reproduces, in the static limit $m^2+p^2 \to 0$, the general relation (5.6) between the massive amplitudes.

It is convenient, at this point, to explicitly introduce the four-dimensional gravitational constant M_p^{-2}, by noting that

$$A_{\nu_0} = \frac{k(\nu_0-1)}{4\pi} = \frac{1}{4\pi}\left[\psi^+_{0,\nu_0}(0)\right]^2 = \frac{1}{4\pi}\left[\int dz\ a^3(z)\right]^{-1}. \tag{6.13}$$

The above normalization integral, when expressed in terms of a new bulk coordinate y, with $dy = a(z)dz$, represents the warped extra-dimensional volume that controls the ratio between the four- and five-dimensional gravitational constants [9, 10], i.e.

$$M_p^2 = M_5^3 \int dy\ a^2(y) = M_5^3 \int dz\ a^3(z). \tag{6.14}$$

It follows that, in units such that $M_5^3 = 1$,

$$A_{\nu_0} = (4\pi M_p^2)^{-1} = 2G, \tag{6.15}$$

where G is the Newton coupling constant. The scalar and dilaton fluctuations (3.23), using the static solutions (6.10), can then be written in the form:

$$\begin{aligned}
\varphi &= -\frac{GM}{r}\left[1 + \frac{2\alpha_2}{1+12\alpha_2^2}\left(\frac{Q}{M}+2\alpha_2\right) + \frac{4}{3}B_{\nu_0}\left(\frac{1}{kr}\right)^{2\nu_0-2}\right], \\
\psi &= -\frac{GM}{r}\left[1 - \frac{2\alpha_2}{1+12\alpha_2^2}\left(\frac{Q}{M}+2\alpha_2\right) + \frac{2}{3}B_{\nu_0}\left(\frac{1}{kr}\right)^{2\nu_0-2}\right], \\
\Gamma &= -\frac{GM}{r}\left[\frac{4\alpha_2}{1+12\alpha_2^2}\left(\frac{Q}{M}+2\alpha_2\right) + \frac{2}{3}B_{\nu_0}\left(\frac{1}{kr}\right)^{2\nu_0-2}\right], \\
\chi &= -\frac{GQ}{r}\left[\frac{2}{1+12\alpha_2^2}\left(1+2\alpha_2\frac{M}{Q}\right) + 2B_{\nu_0}\left(\frac{1}{kr}\right)^{2\nu_0-2}\right].
\end{aligned} \tag{6.16}$$

It should be noted that the short-range corrections induced by the massive scalar modes have the same qualitative behaviour as in the tensor case, discussed in [31], in spite of the fact that the massive scalar modes have different spectra. The dimensional decoupling (i.e., the suppression of the higher-dimensional corrections) is thus effective for all scales of distance r such that $kr \gg 1$, where $k = (M_5^3/M_p^2)(\nu_0-1)^{-1}$ is the mass scale relating the five- and four-dimensional gravitational constants, in our class of backgrounds.

The limiting case $\Delta = -8/3$, i.e. $\alpha_2 = 0$ and $\nu_0 = 2$, corresponds to a pure AdS_5 background, if there are no scalar charges on the brane. In that case ω_2 exactly corresponds to the dilaton fluctuation χ (see eq. (3.22)), and can be consistently set to zero (toghether with the dilaton background) if we want to match, in particular, the "standard" brane-world configuration originally considered by Randall and Sundrum [10]. In this limit, $B_2 = 1/2$, and we exactly recover previous results for the effective gravitational interaction on the brane [35], i.e.

$$\varphi = -\frac{GM}{r}\left(1 + \frac{2}{3k^2r^2}\right), \qquad \psi = -\frac{GM}{r}\left(1 + \frac{1}{3k^2r^2}\right). \tag{6.17}$$

The massless-mode truncation reproduces in this case the static, weak field limit of linearized general relativity (note, however, that for $Q \neq 0$ there is no way to get rid of the long-range scalar interactions). The massive tower of scalar fluctuations, however, induces deviations from Einstein gravity already in the static limit (as noted in [35]), and is the source of a short-range force due to the "breathing" of the fifth dimension,

$$\Gamma = -\frac{GM}{3r}\frac{1}{(kr)^2}, \tag{6.18}$$

even in the absence of bulk scalar fields, and of scalar charges for the matter on the brane.

In a more general gravi-dilaton background ($\Delta \neq -8/3$, $\alpha_2 \neq 0$), the static expansion (6.16) describes an effective scalar-tensor interaction on the brane, which is potentially dangerous for the brane-world scenario, as it contains not only short-range corrections, but also long-range scalar deviations from general relativity (and, possibly, violations of the Einstein equivalence principle), even in the interaction of ordinary masses, i.e. for $Q = 0$ (similar results have been recently obtained also in the context of a multibrane scenario [43]). This seems to offer an interesting window to investigate the effects of the bulk geometry on the four-dimensional physics of the brane.

7 Conclusions

In this paper we have analysed the full set of coupled equations governing the evolution of scalar fluctuations in a dilatonic brane-world background, supporting a flat 3-brane rigidly located at the fixed point of Z_2 symmetry. We have diagonalized the system of dynamical equations, and found four decoupled but self-interacting variables representing, in a five-dimensional bulk, the four independent degrees of freedom of scalar excitations of the gravi-dilaton background.

We have then restricted our discussion to the class of background solutions characterized by a brane of positive tension, by a decreasing warp-factor as we move away from the brane, and by the absence of naked singularities [31]. Such a class of backgrounds can be characterized by a real parameter Δ ranging from $-8/3$ to -2 or, alternatively, by a real parameter ν_0 ranging from 2 to $+\infty$. The limiting case $\Delta = -8/3$, $\nu_0 = 2$, corresponds to the "pure-gravity" AdS_5 background [10].

We have presented the exact solutions of the canonical perturbation equation for all the scalar degrees of freedom, and we have discussed, in this class of backgrounds, the effects of their massless and massive spectrum for the scalar interactions on the brane, taking into account the appropriate parity under Z_2 symmetry, the normalization condition for the bound states of the spectrum, and the first-order differential constraints arising from the dynamics of scalar perturbations.

We have found that, for all backgrounds, there is one propagating massless mode localized on the brane, associated with a long-range dilatonic interaction in four dimensions. Only very exceptionally (i.e. for a RS background and vanishing dilatonic charges) this "fifth force" disappears. This interaction is always affected by higher-dimensional corrections, due to the scalar massive modes that are not confined on the brane and can freely propagate in the bulk space-time. The amplitudes of such massive modes are constrained by the scalar perturbation equations and, in general, only two amplitudes can be independently assigned. The scalar fluctuation spectra are in general different from the corresponding spectra of the spin-2 degrees of freedom. In the weak and static limit, however, the leading-order short-range corrections to the scalar force have the same radial dependence as in the case of pure tensor interactions.

In particular, we have found a non-trivial massive spectrum of scalar metric fluctuations even in the pure Randall–Sundrum scenario with AdS_5 metric [10]. In more general backgrounds we have found that there are also scalar contributions to the long-range interactions of two massive bodies, even in the absence of specific "dilatonic" charges, with a resulting effective scalar-tensor interaction on the brane. In this sense, the bulk geometry seems to affect not only the radial dependence, but also the spin content of the effective gravitational forces. We believe that this effect is potentially interesting for further applications of (and constraints on) the brane-world scenario.

Acknowledgements

It is a plesure to thank P. Binétruy, M. Giovannini, D. Langlois and C. Ungarelli for helpful conversations. G.V. wishes to acknowledge the support of a "Chaire Internationale Blaise Pascal", administered by the "Fondation de L'Ecole Normale Supérieure", during most of this work.

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