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Low-energy graceful exit in anisotropic string cosmology backgrounds

G. De Risi[(1)] and M. Gasperini[(2,3)]

[(1)] *Dipartimento di Fisica, Università di Perugia, Via A. Pascoli, 06123 Perugia, Italy*
[(2)] *Dipartimento di Fisica, Università di Bari, Via G. Amendola 173, 70126 Bari, Italy*
[(3)] *Istituto Nazionale di Fisica Nucleare, Sezione di Bari, Bari, Italy*

Abstract

We discuss the possibility of a smooth transition from the pre- to the post-big bang regime, in the context of the lowest-order string effective action (without higher-derivative corrections), taking into account with a phenomenological model of source the repulsive gravitational effects due to the back-reaction of the quantum fluctuations outside the horizon. We determine a set of necessary conditions for a successful and realistic transition, and we find that such conditions can be satisfied (by an appropriate model of source), provided the background is higher-dimensional and anisotropic.

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According to the pre-big bang cosmological scenario [1], inspired by the duality symmetries of the string effective action [2], and also recently motivated by models of brane-world dynamics [3], the present Universe is assumed to emerge from an inital state of very low curvature and small couplings (in string units), asymptotically approaching the string perturbative vacuum. The "birth " of our Universe, in this context, may thus be represented as a process of decay of the string perturbative vacuum, and described in the language of quantum string cosmology as a transition between the pre- and post-big bang regimes [4,5] associated to a tunelling (or anti-tunnelling [6]) of the Wheeler-De Witt wave function in minisuperspace.

At a classical level the representation of this transition process is problematic, as it requires a smooth evolution of the background from an initial accelerated configuration in which the curvature and the string coupling (i.e. the dilaton) are growing, to a final decelerated configuration in which the curvature is decreasing, and the dilaton is constant or decreasing – the so-called "graceful exit". This requires, in particular, the regularization of the curvature singularities which in general affect the cosmological solutions of the string effective action and which disconnect, classically, the duality-related pre- and post-big bang regimes. This also implies that the growth of the dilaton has to be stopped, to avoid that the curvature is regular in a frame but blows up in a different, conformally related frame [7].

For the lowest order gravi-dilaton string effective action there are indeed "no-go theorems" [8], excluding a smooth transition even in the presence of a (local) dilaton potential and of matter sources in the form of perfect fluids and/or Kalb-Ramond axions. For such a reason, it has been repeatedly stressed, in the literature, the need for including higher-order (quantum loops [9,10] and higher-derivative [11] - [13]) corrections in the string effective action, in order to smooth out the background singularities, and to implement a graceful exit from the phase of pre-big bang inflation to the subsequent phase of standard, decelerated evolution.

The higher-derivative terms, in particular, can efficiently stop the growth of the curvature during a phase of linear dilaton evolution [11], thus preparing the background to the action of the loop corrections, which in turn provide the necessary "repulsive gravity" effects [10] needed to evade the classical singularity theorems (see for instance [14]), and to regularize the transition.

The loop corrections, in fact, are physically induced by the "back-reaction" of the quantum fluctuations against the classical solution, which describes initially a pre-big bang phase of growing curvature and shrinking horizons. As the curvature is growing, the quantum fluctuations are stretched outside the horizon, and it is known that in this regime they are characterized by an effective gravitational energy density which is negative [15], and which may favour the transition to the post-big bang branch of the classical solution [16]. Such a negative back-reaction is eventually damped to zero when the curvature start decreasing, the horizon blows up again, and all the fluctuations re-enter inside the horizon and in the regime of positive energy density. It is important to notice, indeed, that all successful examples of graceful exit (either with a non-local potential [1,4], higher-derivatives [10,13], or different mechanisms [17]) always contain repulsive-gravity effects, directly or indirectly related to the quantum back-reaction of the loop corrections.

It should be recalled, at this point, that the mentioned no-go theorems, formulated in the context of the lowest-order string effective action, are all referred to a homogeneous and isotropic four-dimensional background. If the isotropy and homogeneity assumptions are relaxed, however, it is known that some singularities can be eliminated (technically, "boosted away") through an appropriate $O(d,d)$ duality transformation, effective also at the tree-level [18]. In that case, the repulsive effects regularizing the singularities are due to the antisymmetric tensor field introduced by the boost-transformation. Such examples of regular backgrounds are not usually regarded as successful models of graceful exit, however, because they describe a Universe that after the transition is too inhomogeneous (see however [19]), or even contracting in all its dynamical dimensions [20], to be realistic.

The aim of this paper is to show, with an explicit example, that the higher-derivative corrections are not at all necessary to formulate a realistic model of graceful exit, which is homogeneous and which contains, in its final configuration, three expanding dimensions. The low-energy dynamics of the string effective action is enough, to this purpose, provided the metric background is anisotropic, and provided we take into account, with a phenomenological source term, the repulsive gravitational effects due to the back-reaction of the quantum fluctuations outside the horizon.

We shall consider, in particular, a D-dimensional Bianchi I-type metric background, with a time-dependent dilaton ϕ,

$$g_{\mu\nu} = \mathrm{diag}(1, -a_i^2\delta_{ij}), \qquad a_i = a_i(t), \qquad \phi = \phi(t), \qquad i = 1, 2, \dots D-1, \tag{1}$$

whose dynamical evolution is controlled by the low-energy gravi-dilaton effective action:

$$S = -\int d^Dx\sqrt{|g|}\; e^{-\phi}\left[R + (\nabla\phi)^2\right] + \Gamma(\phi, g, \mathrm{matter}) \tag{2}$$

(we are working in the string frame, and in units in which the string tension $4\pi\alpha'$ is set to unity). Here Γ is the effective action for the matter fields, including the contribution of all the quantum fluctuations, assumed to be subleading unless they are outside the horizon.

The variation of the action with respect to $g_{\mu\nu}$ and ϕ leads to the equations of motion:

$$
\begin{aligned}
&R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R+\nabla_\mu\nabla_\nu\phi+\frac{1}{2}g_{\mu\nu}\left[(\nabla\phi)^2-2\nabla^2\phi\right]=\frac{1}{2}e^{\phi}T_{\mu\nu},\\
&(\nabla\phi)^2-2\nabla^2\phi-R=e^{\phi}\sigma,
\end{aligned}
\tag{3}
$$

containing two source terms,

$$
T_{\mu\nu}=\frac{2}{\sqrt{-g}}\frac{\delta\Gamma}{\delta g^{\mu\nu}},\qquad\qquad \sigma=\frac{1}{\sqrt{-g}}\frac{\delta\Gamma}{\delta\phi}
\tag{4}
$$

(i.e., the gravitational and dilatonic "charge densities"). They are assumed to be compatible with the isometries of the background (1), so that we can set

$$
T_\mu{}^\nu=\mathrm{diag}(\rho,-p_i^2\delta_i^j),\qquad \rho=\rho(t),\qquad p_i=p_i(t),\qquad \sigma=\sigma(t).
\tag{5}
$$

We have thus $D+1$ independent equations, that can be cast in the form (see for instance [1,2]):

$$
\begin{aligned}
&\dot{\overline{\phi}}^2-\sum_i H_i^2=\overline{\rho}e^{\overline{\phi}},\\
&\dot{H}_i-H_i\dot{\overline{\phi}}=\frac{1}{2}(\overline{p}_i+\overline{\sigma})e^{\overline{\phi}},\\
&\dot{\overline{\phi}}^2-2\ddot{\overline{\phi}}+\sum_i H_i^2=\overline{\sigma}e^{\overline{\phi}},
\end{aligned}
\tag{6}
$$

where $H_i=d(\ln a_i)/dt$, t is the cosmic time, and we have introduced the convenient "shifted" variables

$$
\overline{\phi}=\phi-\ln\sqrt{-g},\qquad \overline{\rho}=\rho\sqrt{-g},\qquad \overline{p}_i=p_i\sqrt{-g},\qquad \overline{\sigma}=\sigma\sqrt{-g},\qquad \sqrt{-g}=\prod_i a_i.
\tag{7}
$$

In order to solve the above system of $D+1$ equations, for the $2D+1$ variables $\{a_i,\phi,\rho,p_i,\sigma\}$, we now need D "equations of state" relating p_i and σ to the energy density of the sources. In a complete, and fully realistic scenario, including all the relevant matter fields, p_i and σ are in general complicated functions of ρ, with time-dependent coefficients. However, since we are mainly interested in the graceful exit, here we shall restrict our discussion to the transition regime, where the back-reaction of the quantum fluctuations is expected to give the dominant contribution to Γ, and we shall assume a simple "barotropic" equation of state,

$$
p_i=\gamma_i\rho,\qquad\qquad \sigma=\gamma_0\rho,
\tag{8}
$$

where γ_i,γ_0 are D constant parameters specific to the given model of matter fields and of their quantum fluctuations.

In that case the system of equations (6) can be integrated exactly, following the method developed in [1] and already applied to various classes of homogeneous backgrounds [21]. By introducing a new (dimensionless) time-coordinate x, such that

$$\frac{1}{2}\overline{\rho} = \frac{1}{L}\frac{dx}{dt} \tag{9}$$

(L is a constant parameter, with dimension of length), the equations (6) can be integrated a first time to give:

$$\overline{\phi}' = -2(1+\gamma_0)\frac{(x+x_0)}{D(x)}, \qquad (1+\gamma_0) \neq 0, \tag{10}$$

$$\frac{a_i'}{a_i} = 2(\gamma_i+\gamma_0)\frac{(x+x_i)}{D(x)}, \qquad (\gamma_i+\gamma_0) \neq 0 \tag{11}$$

(a prime denotes differentiation with respect to x). Here x_i and x_0 are D integration constants, and $D(x)$ is a quadratic form related to $\overline{\rho}$ by

$$L^2\overline{\rho}e^{-\overline{\phi}} = D(x) \equiv (1+\gamma_0)^2(x+x_0)^2 - \sum_i (\gamma_i+\gamma_0)^2(x+x_i)^2. \tag{12}$$

The above equations hold for $(1+\gamma_0) \neq 0$, and $(\gamma_i+\gamma_0) \neq 0$. If $(1+\gamma_0) = 0$, however, eq. (10) is to be replaced by

$$\overline{\phi}' = -2\frac{x_0}{D(x)}, \qquad (1+\gamma_0) = 0, \tag{13}$$

and the quadratic form becomes

$$D(x) = x_0^2 - \sum_i (\gamma_i+\gamma_0)^2(x+x_i)^2. \tag{14}$$

If instead $(1+\gamma_0) \neq 0$, but $(\gamma_i+\gamma_0) = 0$ for $i = 1, 2, \ldots n$, then the first n equations in (11) are to be replaced by

$$\frac{a_i'}{a_i} = 2\frac{x_i}{D(x)}, \qquad (\gamma_i+\gamma_0) = 0, \qquad i = 1, 2 \ldots n, \tag{15}$$

and the quadratic form becomes

$$D(x) = (1+\gamma_0)^2(x+x_0)^2 - \sum_{i=1}^{n} x_i^2 - \sum_{i=n+1}^{D} (\gamma_i+\gamma_0)^2(x+x_i)^2. \tag{16}$$

In both cases, $L^2\overline{\rho}e^{-\overline{\phi}} = D(x)$.

To discuss the possibility of graceful exit, we should now separately consider the various possibilities for the values of $(1+\gamma_0)$ and $(\gamma_i+\gamma_0)$. However, as shown by a detailed analysis, in the case $(1+\gamma_0)=0$ the curvature cannot be regular everywhere: even if $D(x)$ is always non-zero, the curvature necessarily blows up at $x\rightarrow\pm\infty$. On the other hand, if $(\gamma_i+\gamma_0)=0$, the condition of smooth curvature turns out to be incompatible with the condition of smooth energy density, $|\rho|<\infty$. We shall thus concentrate, in the following discussion, on the set of equations (10–12), and we shall introduce the convenient definitions:

$$
\begin{aligned}
& D(x)=\alpha x^2+bx+c, && \alpha=(1+\gamma_0)^2-\sum_i(\gamma_i+\gamma_0)^2, \\
& b=2(1+\gamma_0)^2x_0-2\sum_i(\gamma_i+\gamma_0)^2x_i, && c=x_0^2(1+\gamma_0)^2-\sum_i(\gamma_i+\gamma_0)^2x_i^2.
\end{aligned}
\tag{17}
$$

A necessary condition for for the existence of smooth solutions is the absence of zeros in the quadratic form $D(x)$. When the background is isotropic, i.e. γ_i and x_i have the same values for all the $D-1$ spatial directions, then the discriminant of $D(x)$ is always non-negative,

$$
\Delta=b^2-4\alpha c=4(D-1)(1+\gamma_0)^2(\gamma_i+\gamma_0)^2(x_i-x_0)^2\geq 0,
\tag{18}
$$

and $D(x)$ necessarily has zeros on the real axis, correponding to singularities both in the curvature and in the dilaton kinetic energy. A negative value of Δ can be obtained, however, when γ_i and x_i have different values in different directions. Here is why anisotropy is needed, for a graceful exit.

To illustrate this possibility we shall consider a simple example of background, in which the spatial geometry is factorizable as the direct product of two conformally flat manifolds with d and n dimensions, respectively, so that we can set:

$$
\begin{aligned}
& a_i=a_1, && \gamma_i=\gamma_1, && x_i=x_1, && i=1,\dots d, \\
& a_i=a_2, && \gamma_i=\gamma_2, && x_i=x_2, && i=d+1,\dots d+n.
\end{aligned}
\tag{19}
$$

Also, we shall choose a convenient set of integration constants, such that the linear term in the quadratic form (17) disappears. For instance:

$$
x_0=0, \qquad\qquad x_1=-x_2\frac{n(\gamma_2+\gamma_0)^2}{d(\gamma_1+\gamma_0)^2}.
\tag{20}
$$

It turns out that $c<0$, and that the absence of zeros in $D(x)$ can be avoided, $\Delta=-4\alpha c<0$, provided

$$\alpha = (1+\gamma_0)^2 - d(\gamma_1+\gamma_0)^2 - n(\gamma_2+\gamma_0)^2 < 0. \tag{21}$$

If this condition is satisfied then $D(x) < 0$ everywhere, and this implies, through eq. (12), $\rho < 0$ (note that the result $D(x) < 0$ in the absence of zeros is independent from the particular choice $b = 0$). As discussed before, this agrees with our expectation that during the exit the dominant contribution to the gravitational sources should come from the back-reaction of the quantum fluctuations outside the horizon, when their effective energy density is indeed negative [15,16]. We stress again that such a negative energy density goes to zero at large times (well inside the post-big bang regime), when the horizon becomes larger and larger and all modes of the quantum fluctuations re-enter inside the horizon, giving rise to the well known phenomenon of cosmological particle production. The energy density thus asymptotically switches to a positive regime, dominated by the contribution of the effective stress tensor of the produced radiation [20]. Such an asymptotic regime will not be considered in this paper, as here we are mainly interested in the discussion of the exit, and we shall concentrate our attention on the transition regime where the backreaction of particle production is negligible.

When the conditions (19–21) are satisfied, the integration of eqs. (10,11) leads to the exact solution

$$\begin{aligned}
&e^{\overline{\phi}} = e^{\phi_0}\,|D(x)|^{-\frac{1+\gamma_0}{\alpha}}, \qquad \overline{\rho} = -\frac{e^{\phi_0}}{L^2}\,|D(x)|^{1-\frac{1+\gamma_0}{\alpha}},\\
&a_i = a_{i0}E_i(x)\,|D(x)|^{\frac{\gamma_i+\gamma_0}{\alpha}}, \qquad E_i(x) = \exp\left[\frac{2x_i(\gamma_i+\gamma_0)}{\sqrt{\alpha c}}\tan^{-1}\left(\frac{\alpha x}{\sqrt{\alpha c}}\right)\right], \qquad i=1,2,
\end{aligned} \tag{22}$$

where ϕ_0 and a_{i0} are integration constants. Using eq. (9) we can then obtain the corresponding Hubble parameters $H_i = (a_i'/a_i)(dx/dt)$, and the dilaton kinetic energy $\dot{\phi} = \dot{\overline{\phi}} + dH_1 + nH_2$:

$$\begin{aligned}
&H_1 = \frac{e^{\phi_0}}{L}(\gamma_1+\gamma_0)(x+x_1)\,|D(x)|^{-\frac{1+\gamma_0}{\alpha}}, \qquad H_2 = \frac{e^{\phi_0}}{L}(\gamma_2+\gamma_0)(x+x_2)\,|D(x)|^{-\frac{1+\gamma_0}{\alpha}},\\
&\dot{\phi} = \frac{e^{\phi_0}}{L}\,|D(x)|^{-\frac{1+\gamma_0}{\alpha}}\left[-(1+\gamma_0)x + d(\gamma_1+\gamma_0)(x+x_1) + n(\gamma_2+\gamma_0)(x+x_2)\right]
\end{aligned} \tag{23}$$

(for $\rho < 0$, it is convenient to choose $L < 0$, so that $dx/dt > 0$). Finally, by rescaling $\overline{\phi}, \overline{\rho}$ through the explicit solutions for the scale factors, we can also obtain the evolution of the non-shifted variables:

$$\begin{aligned}
&e^{\phi} = e^{\phi_0} a_{10}^d a_{20}^n E_1^d(x) E_2^n(x)\,|D(x)|^{-[(1+\gamma_0)-d(\gamma_1+\gamma_0)-n(\gamma_2+\gamma_0)]/\alpha},\\
&\rho = -\frac{e^{\phi_0}}{L^2} a_{10}^{-d} a_{20}^{-n} E_1^{-d}(x) E_2^{-n}(x)\,|D(x)|^{1-[(1+\gamma_0)+d(\gamma_1+\gamma_0)+n(\gamma_2+\gamma_0)]/\alpha}.
\end{aligned} \tag{24}$$

The above exact solution satisfies the condition (21), which is necessary for a model for graceful exit, but non sufficient. In addition, we have to impose that the curvature and the dilaton kinetic energy of eq. (23), toghether with the effective string coupling e^{ϕ}, are bounded everywhere. This requires, respectively:

$$2(1+\gamma_0) < \alpha, \qquad (1+\gamma_0) - d(\gamma_1+\gamma_0) - n(\gamma_2+\gamma_0) < 0. \tag{25}$$

The energy density ρ of eq. (24) also should be bounded and, in particular, should go asymptotically to zero at large times, to be consistently interpreted as the contribution of the quantum back-reaction. This imposes the condition

$$(1+\gamma_0) + d(\gamma_1+\gamma_0) + n(\gamma_2+\gamma_0) < \alpha. \tag{26}$$

Finally, for possible applications to a realistic scenario, our anisotropic background should contain, in its final configuration, d expanding and n contracting dimensions. This requires (see the solutions for a_i in eq. (22)):

$$\gamma_1+\gamma_0 < 0, \qquad \gamma_2+\gamma_0 > 0. \tag{27}$$

A consistent and successful model of graceful exit should satisfy the whole set of conditions (21), (25–27).

A detailed analysis of the above inequalities shows that there is a region of non-zero extension in the space of the parameters γ_i, γ_0 for which all the conditions are satisfied. This means that, if the back-reaction generated by the quantum fluctuations is appropriate, a model of graceful exit can be implemented even in the context of the low-energy string effective action, without higher-derivative corrections.

In order to check our analytical results, we have numerically integrated the string cosmology equations (6), using directly the cosmic time variable. Such equations, when applied to the factorized configuration (19), are equivalent to a system of four independent equations for the four variables H_i, ϕ, ρ $(i=1,2)$:

$$\begin{aligned}
&\dot{H}_i - H_i\left(\dot{\phi} - dH_1 - nH_2\right) = \frac{1}{2}\rho(\gamma_i+\gamma_0)e^{\phi}, \\
&\left(\dot{\phi} - dH_1 - nH_2\right)^2 - 2\left(\ddot{\phi} - d\dot{H}_1 - n\dot{H}_2\right) + dH_1^2 + nH_2^2 = \gamma_0\rho e^{\phi}, \\
&\dot{\rho} + dH_1(1+\gamma_1)\rho + nH_2(1+\gamma_2)\rho + \gamma_0\rho\dot{\phi} = 0.
\end{aligned} \tag{28}$$

We have used, for the numerical integration, the following set of parameters:

$$d=3, \qquad n=6, \qquad \gamma_0=-3.25, \qquad \gamma_1=2.25, \qquad \gamma_2=3.85, \tag{29}$$

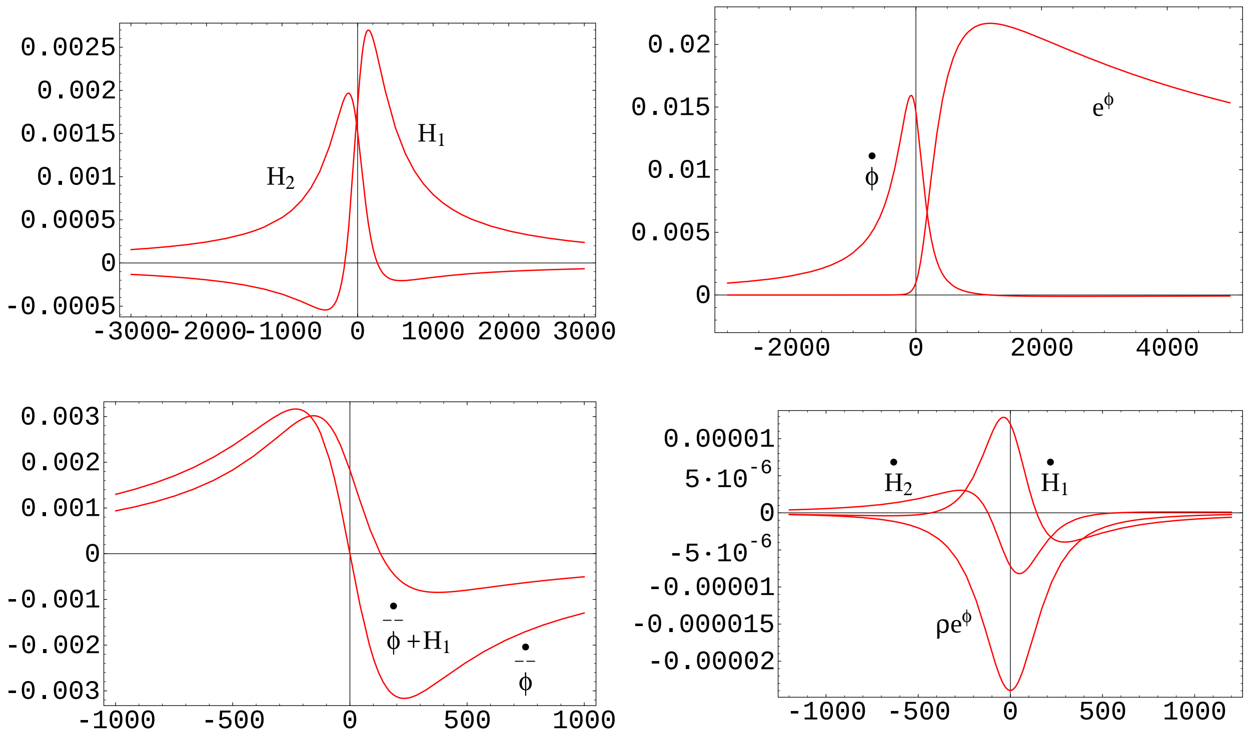

FIG. 1. *The plots show the evolution in cosmic time of* H_i, $\dot{H}_i$, $\dot{\phi}$, $\dot{\bar{\phi}}$, e^{ϕ}, ρe^{ϕ}, *obtained through a numerical integration of eqs. (28), with the set of parameters given in eq. (29), and with the following initial conditions (satisfying eqs. (30), (31)), imposed at* $t=0$: $H_1 = 0.00182772$, $H_2 = 0.0015231$, $\dot{\phi} = 0.0146218$, $\phi = -6.91139$, $\rho = -0.24028$.

satisfying all the inequalities (21), (25–27). We have imposed, as initial conditions, a small and negative energy density, $\rho_{in} < 0$, and a small but increasing dilaton, $\dot{\phi}_{in} > 0$. We have also restricted the initial conditions to lie on the trajectory of our analytical solution (23), (24), using the fact that, at fixed $x = 0$, the choice of parameters (29) leads to the relations:

$$H_1(0) = 1.2 H_2(0), \qquad \dot{\phi}(0) = 8 H_1(0). \tag{30}$$

The full set of initial conditions is further restricted by the Hamiltonian constraint (first of eqs. (6)) as

$$\left(\dot{\phi} - dH_1 - nH_2\right)^2 - dH_1^2 - nH_2^2 = \rho e^{\phi}. \tag{31}$$

The results of the numerical integration are shown in Fig. 1.

In the example illustrated in Fig. 1 the background undergoes a smooth and homogeneous evolution from a pre-big bang phase in which the curvature and the dilaton are increasing,

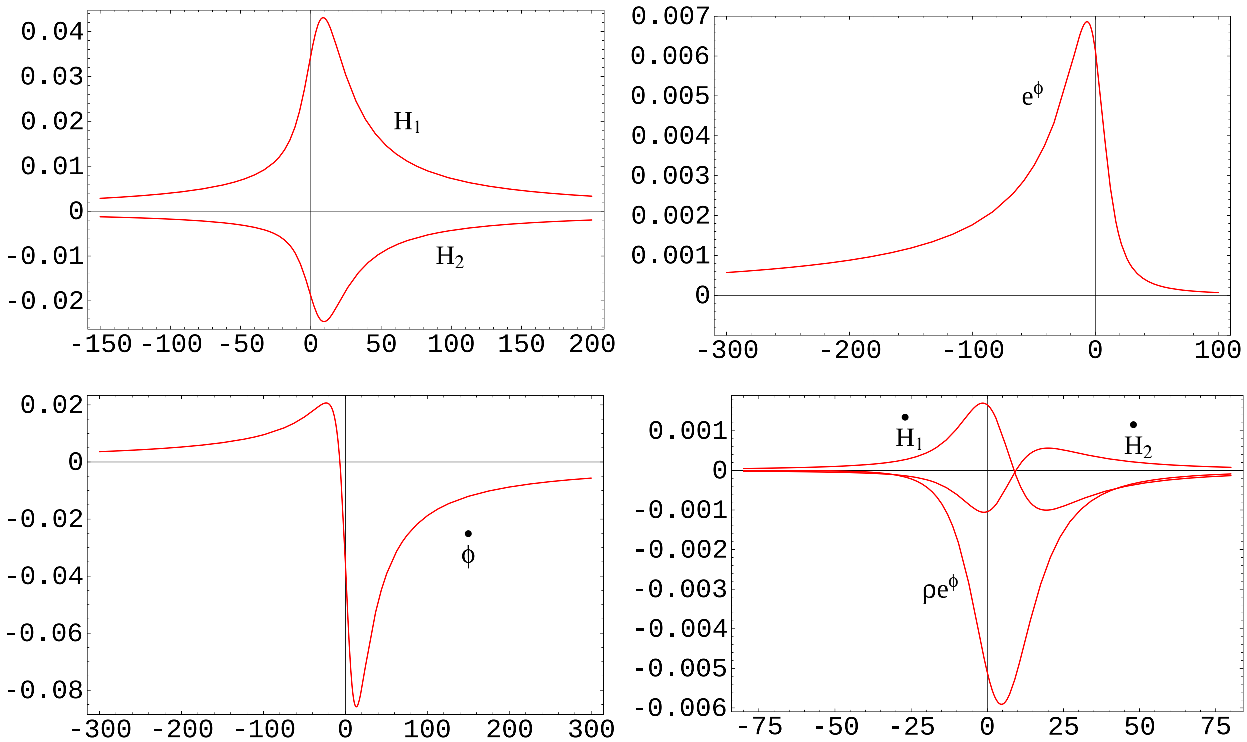

FIG. 2. *The plots show the evolution in cosmic time of H_i, $\dot{H}_i$, $\dot{\phi}$, e^{ϕ}, ρe^{ϕ}, obtained through a numerical integration of eqs. (28), with the set of parameters given in eq. (29), and with the following initial conditions (satisfying eq. (31)) imposed at* $t=-10$: $H_1=0.02$, $H_2=-0.01$, $\dot{\phi}=0.01$, $\phi=-5$, $\rho=-0.25230237$.

to a post-big bang phase in which the curvature and the dilaton are decreasing ($\dot{\phi}\to 0$ from negative values as $t\to+\infty$). The final post-big bang configuration is characterized by $H_1>0$, $H_2<0$ for $t\to+\infty$, and thus describes 3 expanding and 6 contracting spatial dimensions, as appropriate to a phase of dynamical dimensional reduction in a superstring theory context ($D=1+d+n=10$). Also, the final configuration satisfies all the prescribed conditions [10] for a successful exit, i.e. $\dot{\overline{\phi}}<0$, $\dot{\overline{\phi}}<-H_1$ as $t\to+\infty$. The negative energy density of the sources (not shown in the picture) is bounded and goes to zero, far from the transition regime, as appropriate to the back-reaction generated by the quantum fluctuations outside the horizon. Finally, all the curvature terms ($H_i^2,\dot{\phi}^2,\dot{H}_i$) appearing in the equations, including the source term $e^{\phi}\rho$, remains much smaller than one in string units, as appropriate to an action describing low-energy dynamics.

It is important to stress that the exact analytical solution (23), (24), reproduced numerically in Fig. 1, is only a special example of smooth transition corresponding to the particular choice of integration constants given in eq. (20). In general, other smooth configurations are allowed, including also the case of a monotonic evolution of the "external" and "internal" scale factors a_1 and a_2. This possibility is illustrated in Fig. 2, in which we report the results of a numerical integration of eqs. (28), with the same set of parameters given in eq. (29), and with initial conditions satisfying the Hamiltonian constraint (31) but *not* the constraints (30), typical of our particular analytical example. The numerical example of Fig. 2, in particular, describes a smooth transition in which the three external dimensions evolve from accelerated to decelerated expansion, while the six internal dimensions from accelerated to decelerated contraction. The simultaneous flip in sign of $\dot{H}_1$, $\dot{H}_2$, illustrated in the picture, marks the end of the phase of pre-big bang inflation and the beginning of the standard decelerated regime.

In conclusion, the combined effect of anisotropy (physically associated to the dimensional reduction) and of a negative energy density (physically associated to the quantum back-reaction) seem to be able to trigger an efficient and graceful exit from the pre-big bang regime, even at small curvatures, at least for an appropriate range of parameters characterizing the source stress tensor. The toy model that we have presented in this paper, to illustrate the joint effects of anisotropy and back-reaction, is not intended, of course, to represent an exhaustive and fully realistic picture of the complete transition to the post-big bang regime – other effects, like α' corrections, can in principle become important near the transition regime. In addition, at late times, a dilaton potential is expected to be added, and to play a possible significant role for the dilaton evolution. Also, at late times, the (positive) radiation energy density, due to particle production effects, is expected to isotropize the background and possibly contribute to dilaton stabilization, as discussed in [20]. The conditions (21), (25–27) determined in this paper, however, can be applied to various models of (classical or quantum) sources, in the transition regime, to obtain "a priori" indications on the effective back-reaction of their fluctuations outside the horizon, and on their possible ability of driving a smooth evolution from the string perturbative vacuum to our present cosmological configuration.

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