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Sensitivity of spherical gravitational-wave detectors to a stochastic background of non-relativistic scalar radiation

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We analyze the signal-to-noise ratio for a relic background of scalar gravitational radiation composed of massive, non-relativistic particles, interacting with the monopole mode of two resonant spherical detectors. We find that the possible signal is enhanced with respect to the differential mode of the interferometric detectors. This enhancement is due to: (a) the absence of the signal suppression, for non-relativistic scalars, with respect to a background of massless particles, and (b) for flat enough spectra, a growth of the signal with the observation time faster than for a massless stochastic background.

The detection of a background of relic gravitational radiation is undoubtedly one of the main challenges of the physics of the XXI century. The detection of, or even a direct experimental bound on, a stochastic gravitational background of primordial origin would considerably improve our knowledge of the very early state of the Universe and our understanding of physical processes at a Planckian energy scale (see for instance [1] and references therein).

In view of the present and forthcoming data, soon available from the cross-correlation of all existing (resonant and interferometric) gravitational antennas, the signal-to-noise ratio (SNR) due to a stochastic background has been analyzed in full details (see [2] an references therein), in particular for a background of (ultra)relativistic particles, whose spectral distribution in frequency is identical to their momentum distribution.

As far as the tensor sector of gravitational radiation is concerned, the relativistic assumption is certainly appropriate and self-consistent. However, it could not be fully appropriate in order to analyze the contribution of a possible scalar sector, which could correspond to a stochastic background of massive particles (for instance, dilatons [3]), possibly non-relativistic at the time of present observations.

A non-relativistic background of massive scalar waves requires indeed a generalization of the standard SNR expression which takes into account the relevant mass parameter of the energy spectrum in momentum space [4]. In that case it has been recently shown that, in spite of the induced suppression factor (with respect to massless particles) $(p/E)^4 \ll 1$, it is possible to obtain a resonant response also from the non-relativistic part of the scalar wave spectrum, provided the mass lies within the sensitivity band of the detectors [4,5]. Also, if the energy density of the non-relativistic scalar background is a significant fraction of the critical energy density, the signal could be detectable by future planned detector configurations, such as "Advanced LIGO" [6].

The analysis of [4,5] refers however to the SNR produced by a massive scalar background in the cross-correlation of two interferometric detectors. In particular, for a non-relativistic scalar wave, the suppression factor which appears in the pattern function is due to the interaction of the scalar wave with the differential mode of the interferometers, which is characterized by a 3×3 traceless response tensor. As already pointed out in [5], such a suppression may be absent in the monopole mode of resonant spherical detectors, whose response tensor is proportional to the 3×3 identity matrix [7]. In addition, in the resonant response of a sphere to a non-relativistic scalar background, and for flat enough spectra, the infrared cut-off of the SNR integral is provided *not* by the noise power spectrum of the detector (as usual in the case massless particles), but by a minimal frequency scale determined by the mass of the background field and by the total observation time, T. As a consequence, the corresponding SNR may rise with T much faster than usual (depending on the power of the scalar spectrum).

The aim of this paper is to present, and briefly discuss, these two effects (i.e., the absence of non-relativistic suppression, and the unconventional T-dependence of SNR)

which are new (to the best of our knowledge), and which could greatly favour the detection of a non-relativistic scalar background through the correlation of two (or more) resonant spheres.

We start by recalling the general expression for the optimized SNR obtained through the cross-correlation of two detetctors, with one-sided noise spectral density $P_1(|f|), P_2(|f|)$, and generated by a stochastic background of scalar particles with mass m and spectral energy density $\Omega(p)$ in momentum space [4,5]:

$$SNR = \left(\frac{H_0^2}{5\pi^2}\right)\left[2T\int_0^\infty \frac{dp}{p^3\,(p^2+\tilde{m}^2)^{3/2}}\right.$$
$$\left.\times\frac{\Omega^2(p)\,\gamma^2(p)}{P_1(\sqrt{p^2+\tilde{m}^2})\,P_2(\sqrt{p^2+\tilde{m}^2})}\right]^{1/2}. \qquad (1)$$

Here T is the total observation time, $\tilde{m} = m/2\pi$, and H_0 is the present value of the Hubble parameter, which appears in the above equation because the energy spectrum $\rho(p)$ of the background is expressed in units of critical energy density [1,2]:

$$\Omega(p) = \frac{1}{\rho_c}\frac{d\rho}{d\ln p}, \qquad \rho_c = \frac{3H_0^2\,M_P^2}{8\pi} \qquad (2)$$

where M_P is the Planck mass. Finally, in eq. (1), $\gamma(p)$ is the overlap reduction function [1,2], which accounts for the relative orientation and location of the two gravitational antennas, and determines their efficiency in the detection of a given background. For a plane wave of momentum $\vec{p} = \hat{n}p$, where $\hat{n}$ is a unit vector specifying the propagation direction, $\gamma(p)$ is defined by

$$\gamma(p) = \frac{15}{4\pi}\int d^2\hat{n}e^{2\pi i p\hat{n}\cdot(\vec{x}_1-\vec{x}_2)}F^1(\hat{n})F^2(\hat{n}) \qquad (3)$$

where the integration is extended over the full solid angle and the normalisation constant has been chosen so that – in the massless case – one obtains $\gamma(p) = 1$ for the differential mode of two coincident and coaligned interferometers. Here $\vec{x}_1, \vec{x}_2$ are the positions of the centers of mass of the two detectors, and F^1, F^2 are the so-called antenna pattern functions [1,2], determined by the wave polarization tensor, $e_{ab}(\hat{n})$ (which depends on the spin-content of the background), and by the detector response tensor D_{ab} (which depends on the geometrical shape of the detector). In general:

$$F^i = q_i e_{ab}(\hat{n})D^{ab} \qquad (4)$$

where q_i is the effective coupling of the background to the detector, normalized to 1 for the geodesic coupling to metric fluctuations [4,5].

Note that, for a better comparison with experimental observables, we are choosing "unconventional" units $h = 1$, so that energy and frequency simply coincide, $E = (p^2+\tilde{m}^2)^{1/2} = f$. Note also that, for $m = 0$, $p = f$ and one recovers the standard form of SNR appropriate to a background of massless particles [1,2]; the usual result for a background of relic gravitons then follows by inserting into eqs. (3), (4) the appropriate spin-two polarization tensor, with $q = 1$.

For a massive scalar wave, however, one should in general distinguish two different pattern functions, one related to the geodesic coupling of the detector to the scalar component of metric oscillations [8], the other related to the direct, non-geodesic coupling of the scalar charge q_i of the detector to the spatial gradients of the scalar background [9]. By using the transverse and longitudinal decomposition of the polarization tensor of the massive scalar wave with respect to the propagation direction $\hat{n}$, and defining

$$T_{ab} = (\delta_{ab} - \hat{n}_a\,\hat{n}_b), \qquad L_{ab} = \hat{n}_a\,\hat{n}_b\,, \qquad (5)$$

the two patter functions can be written, respectively, as [5]

$$F(\hat{n}) = D^{ab}\left(T_{ab} + \frac{\tilde{m}^2}{E^2}L_{ab}\right),\; F_q(\hat{n}) = q\frac{p^2}{E^2}D^{ab}\,L_{ab}. \qquad (6)$$

The first one applies when considering the response of the detetctor to the spectrum of scalar metric fluctuations induced by the scalar background, the second one when considering the direct response of the detector to the fluctuations of the background itself. For the differential mode of an interferometric antenna, in particular, $D^{ab}\delta_{ab} = 0$ and, in both cases, the pattern function of a massive scalar wave turns out to be proportional to the massless one, but with the strong suppression factor $(p/E)^2$ for non-relativistic modes [5].

A resonant sphere, on the contrary, has a monopole mode characterized by the response tensor $D^{ab} = \delta^{ab}$ [7], so that $D^{ab}L_{ab} = 1$, $D^{ab}T_{ab} = 2$. The non-geodesic part of the corresponding pattern function, $F_q(\hat{n})$, is still suppressed for all non-relativistic modes. For the geodesic part, however, the angular dependence completely disappears,

$$F(\hat{n}) = \frac{3\tilde{m}^2 + 2p^2}{\tilde{m}^2 + p^2}, \qquad (7)$$

and the overlap function (3), for the monopole modes of two spheres, takes the simple form

$$\gamma(p) = \frac{15}{2\pi}\left(\frac{3\tilde{m}^2+2p^2}{\tilde{m}^2+p^2}\right)^2\frac{\sin(2\pi pd)}{pd}, \qquad (8)$$

where d is the spatial distance between the detectors. The response to non-relativistic modes ($p \ll m$) is no longer suppressed (actually, their response is enhanced by the factor 9/4). This is a first interesting result, which already at this level shows that a resonant sphere is a promising device for the detection of a non-relativistic background of massive scalar particles.

The second interesting result follows from the study of the integral (1) for the SNR associated to the monopole modes of two spheres, and induced by a cosmic background whose spectrum $\Omega(p)$ is dominated by the non-relativistic sector $p \lesssim m$. Such a relic scalar spectrum may be expected in string cosmology models of the early Universe [10] and, as discussed also in [5], it can be simply approximated by the power-law behaviour

$$\Omega(p) = \delta\ \Omega_0\, (p/\tilde{m})^{\delta}\,, \quad p \lesssim \tilde{m}, \quad \delta > 0, \tag{9}$$

where Ω_0 is the present fraction of critical energy density stored in the scalar background. The low energy branch of the spectrum is assumed to be non-decreasing ($\delta > 0$), to guarantee a finite energy density,

$$\int_0^{\tilde{m}} d\ln p\ \Omega(p) = \Omega_0. \tag{10}$$

More complicated distributions are also possible, with $\Omega(p)$ peaked at a scale $p_m < \tilde{m}$ (see [5,10]), depending on the details of the cosmological mechanism of production. But the simple monotonic spectrum (9) is already appropriate to the illustrative purpose of this paper, and will be adopted in what follows for all our estimates of the sensitivity.

Inserting into eq. (1) the expression for the overlap reduction function given by eq. (8) and the spectrum (9), and assuming (for a resonant response) that $\tilde{m}$ lies within the detector sensitivity band, so that $P_i(\tilde{m})$ are finite, the resulting SNR would seem infrared divergent for $\delta < 1$. Indeed, for $p \to 0$, $\gamma(p) \to$ const, $P_i \to$ const, and

$$(SNR)^2 \sim \int_0^{\tilde{m}} dp\ p^{2\delta-3} \sim \left[p^{2\delta-2}\right]_0^{\tilde{m}}. \tag{11}$$

Unlike for massless particles, the infrared part of the integral is not killed by the instrumental noises, whose power spectra $P_i(|f|)$ (appearing in the denominator of eq. (1)) blow up to infinity when their argument approaches zero. In our case, when $p \to 0$, P_i keep frozen at the frequency scale fixed by the mass of the background.

It must be noticed, however, that the presence of a lower bound fixed by m in the allowed range of frequencies, $f = (p^2 + \tilde{m}^2)^{1/2} \geq \tilde{m}$, toghether with the existence of a "maximal" finite time scale, determined by the observation time T, necesssarily implies an intrinsic infrared cut-off for the momentum variable in the SNR integral. Indeed, for a given T, the minum observable frequency interval

$$\Delta f = (p^2 + \tilde{m}^2)^{1/2} - \tilde{m} > T^{-1}, \tag{12}$$

defines, in the limit $p \to 0$, the minimum momentum scale

$$p_{\min} = (2\tilde{m}/T)^{1/2}\,, \tag{13}$$

to be used as the effective lower bound in the SNR integral. Modes with $p < p_{\min}$ cannot be resolved by an instrument working for a finite time-interval T, and may be included in a constant background (over which scalar fluctuations are propagating) without affecting the SNR ratio. This instrinsic cut-off may thus introduce an important T-dependence in the final value of SNR.

In order to illustrate this effect, and to provide some estimates, let us assume for simplicity that the instrumental noises P_1 and P_2 are equal, and nearly constant, for p ranging from 0 to $\tilde{m}$ (i.e., for frequencies between $\tilde{m}$ and $\sqrt{2}\tilde{m}$): $P_1(\tilde{m}) = P_2(\tilde{m}) \equiv S_h(\tilde{m})$. Also, we shall assume that the distance between the two spheres is negligible. The SNR, for a background of scalar metric fluctuations charaterized by the spectral distribution (9), interacting with the monopole modes of two spheres, then reads

$$SNR = \frac{3\delta H_0^2\Omega_0}{\pi^2 S_h(\tilde{m})}\sqrt{2T} \times \left[\int_{p_{\min}}^{\tilde{m}} \frac{dp}{p^3(p^2+\tilde{m}^2)^{3/2}} \left(\frac{p}{\tilde{m}}\right)^{2\delta} \left(\frac{3\tilde{m}^2+2p^2}{\tilde{m}^2+p^2}\right)^4\right]^{1/2} \tag{14}$$

For $\delta \leq 1$ the integral is dominated by the infrared cut-off $p_{\min}$ (see eq. (11)). We obtain the following estimates:

$$SNR \simeq \frac{27}{\pi^2}\left(\frac{2}{1-\delta}\right)^{\frac{1}{2}} \left(\frac{H_0}{\tilde{m}}\right)^2 \frac{\delta\ \Omega_0}{\tilde{m}\ S_h(\tilde{m})} \left(\frac{\tilde{m}T}{2}\right)^{1-\frac{\delta}{2}}, \quad \delta < 1, \tag{15}$$

$$SNR \simeq \frac{27}{\pi^2}\left(\frac{H_0}{\tilde{m}}\right)^2 \frac{\Omega_0}{\tilde{m}\ S_h(\tilde{m})} \left(\frac{\tilde{m}T}{2}\right)^{\frac{1}{2}} \ln\left(\frac{\tilde{m}T}{2}\right), \quad \delta = 1. \tag{16}$$

For $\delta > 1$ the integral is instead dominated by the end point $p = \tilde{m}$ of the spectrum, and the SNR reads

$$SNR \simeq \frac{10}{\pi^2}\left(\frac{H_0}{\tilde{m}}\right)^2 \frac{\Omega_0}{\tilde{m}\ S_h(\tilde{m})} \left(\frac{\tilde{m}T}{2}\right)^{\frac{1}{2}} F(\delta), \quad \delta > 1, \tag{17}$$

where

$$F(\delta) = \delta\left[\int_0^1 \frac{dx(2x^2+3)^4 x^{2\delta-3}}{(1+x^2)^{11/2}}\right]^{1/2} \tag{18}$$

is a number of order one. The three different cases give results which are very similar, *except for the dependence on the observation time* T, and it is now evident that, for a given T, the sensitivity is strongly enhanced for flat enough spectra ($\delta \leq 1$).

It is important to stress that such a dependence of the SNR on the total observation time yields an effective enhancement - with respect to the usual square root law - of the signal, for a massive stochastic background, only if m is not much smaller than the typical resonant frequency f_0 of the detector. Indeed, if $m \ll f_0$, the infrared cut-off (12) is ineffective, being suppressed by a very high instrumental noise. It is also worth stressing that this enhancement is present only in the correlation of two spheres. Indeed, only because of their particular overlap function (8) the SNR integral may be dominated by the infrared cut-off, and not by the peak of the spectrum (as in the case of interferometers).

For a numerical estimate of the sensitivity we shall now compute the fraction of critical energy density Ω_0 required to have a detectable signal, say $SNR > 5$. We take $T \sim 10^7$ sec as a typical obervation time, and $S_h \sim 10^{-46}\mathrm{Hz}^{-1}$ as a possible realistic noise density for a resonant sphere [11,12] at a frequency $f = 3 \times 10^3$ Hz. The minimal required energy density, for $\delta = 1/2$, $\delta = 1$ and $\delta > 1$ is given, respectively, by

$$h_0^2\,\Omega_0 \gtrsim 10^{-8}\left(\frac{SNR}{5.0}\right)\left(\frac{T}{10^7\mathrm{sec}}\right)^{-\frac{3}{4}}\left(\frac{S_h}{10^{-46}\mathrm{Hz}^{-1}}\right) \times\left(\frac{\tilde{m}}{3\cdot 10^3\mathrm{Hz}}\right)^{\frac{9}{4}}, \qquad \delta = 1/2, \tag{19}$$

$$h_0^2\,\Omega_0 \gtrsim 10^{-7}\left[1 + 0.04\ln\left(\frac{\tilde{m}}{3\cdot 10^3\mathrm{Hz}}\right)\left(\frac{T}{10^7\mathrm{sec}}\right)\right]^{-1} \times\left(\frac{SNR}{5.0}\right)\left(\frac{T}{10^7\mathrm{sec}}\right)^{-\frac{1}{2}}\left(\frac{S_h}{10^{-46}\mathrm{Hz}^{-1}}\right)\left(\frac{\tilde{m}}{3\cdot 10^3\mathrm{Hz}}\right)^{\frac{5}{2}}, \qquad \delta = 1, \tag{20}$$

$$h_0^2\,\Omega_0 \gtrsim \frac{10^{-5}}{F(\delta)}\left(\frac{SNR}{5.0}\right)\left(\frac{T}{10^7\mathrm{sec}}\right)^{-\frac{1}{2}}\left(\frac{S_h}{10^{-46}\mathrm{Hz}^{-1}}\right) \times\left(\frac{\tilde{m}}{3\cdot 10^3\mathrm{Hz}}\right)^{\frac{5}{2}}, \qquad \delta > 1 \tag{21}$$

where we have used $H_0 = 3.2\,h_0 \times 10^{-18}$ Hz.

It may be appropriate to report here, for comparison, the SNR of two spherical detectors in the case of a relativistic (scalar) background characterized by the same spectral distribution (9), but with $\tilde{m}$ replaced by a typical resonant frequency f_0. By setting $m = 0$ in eq. (14) we obtain, for all values of the spectral index δ,

$$SNR \simeq \frac{12}{\pi^2}\left(\frac{H_0}{f_0}\right)^2 \frac{\Omega_0}{f_0\;S_h(f_0)}\,(2f_0T)^{\frac{1}{2}}\,F_0(\delta), \tag{22}$$

$$F_0(\delta) = \delta\left[\int_{x_{\min}}^{x_{\max}} dx x^{2\delta-6}\right]^{1/2}, \tag{23}$$

where we have assumed, as before, that $P_1 = P_2 = S_h(f_0)$ are nearly constant in the bandwidth of the detector, ranging from $f_{\min} \equiv x_{\min} f_0$ to $f_{\max} \equiv x_{\max} f_0$. We also recall the sensitivity of two coincident and coaligned interferometers, which for the massive spectrum (9) can be expressed in the form [5] (valid for any δ)

$$SNR \simeq \frac{2}{5\pi^2}\left(\frac{H_0}{\tilde{m}}\right)^2 \frac{\Omega_0}{\tilde{m}\;S_h(\tilde{m})}\left(\frac{\tilde{m}T}{2}\right)^{\frac{1}{2}} F_I(\delta), \tag{24}$$

$$F_I(\delta) = \delta\left[\int_0^1 dx \frac{x^{2\delta+5}}{(1+x^2)^{11/2}}\right]^{1/2}. \tag{25}$$

In both cases, the results are very similar to eq. (17), with the important difference that now they are valid for any δ. It follows that eq. (21) (with $\tilde{m}$ and F replaced by f_0 and F_0) also estimates the sensitivity of two spheres to a relativistic background, and (with F replaced by F_I) estimates the sensitivity of two interferometers in the case of a massive scalar background, *for any* δ (even if, for the interferometers, the maximal sensitivity is expected at frequencies lower than 1 kHz). For the interferometers, however, the sensitivity is to be further depressed if the spectrum is peaked at scales smaller than m, because of the non-relativistic suppression factor [5]. For the spherical detectors such a suppression is absent, and the sensitivity remains at the above levels independently from the shape of the spectrum.

We may therefore conclude that the cross-correlation of two resonant spherical detectors seems to be, in principle, particularly appropriate to the search for a possible relic cosmic background of non-reativistic scalar particles, because of two effects: the absence of non-relativistic suppression for the monopole mode of the spheres and, for a flat enough spectrum, the faster increase of the sensitivity with the observation time.

The numerical estimates reported in this paper refer to the case in which the mass of the scalar particle is within the bandwidth of the antennas. The bandwidth depends on the noise performance of the readout system, and is limited to a few Hz in the currently operating resonant bars [13]. By employing a xylophone of advanced hollow spheres [12], or the recently proposed dual spheres [14], it should be possible however to open up the bandwidth and scan a wide mass range at a high level of sensitivity, comparable with the present numerical estimate, thus providing new important information on possible ultra-light scalar partners of the gravitons, and on fundamental string/M-theory models of the early Universe.

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