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Superstrings in the Early Universe

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We investigate some aspects of thermodynamics and cosmology for superstrings. By a rather delicate computation using the microcanonical ensemble we show that the thermodynamic description of strings is sound (specific heat is positive at large energies) only for strings propagating in spaces where all the spatial directions are compact. Using this result and by considering a simple model, we show how strings resolve the initial singularity of the Big Bang. We also discuss a cosmological scenario which has the potential of explaining the space-time dimensionality.

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With the emergence of string theory as a potential candidate for a quantum theory of gravity, we should begin to answer questions which in the past had been set aside because of a lack of a satisfactory quantum theory of gravity. We have a long list of such questions. One major area which we shall begin to address in this paper is the question of superstring cosmology. There are a number of unacceptable features of the standard Big Bang scenario: The initial singularity of space is a major issue which has not been totally believable from a physical point of view. We do not trust Einstein's equation in the very early universe. We believe it (at best) to a few orders of magnitude away from the Planck scale. The infinity of the temperature in the standard scenario (as time approaches the time of the Big Bang) has been similarly not totally trustable. Can string theory shed some light on these questions? Another question is the origin of the dimension of space-time. In general relativity this is set to four by hand. Can string theory explain the dimension of extended space (superstrings seem to naively predict the wrong dimension for space-time (namely ten))?

To answer these questions it is appropriate to review two aspects of strings which are relevant to us: One is how strings behave when they are put in small spaces, spaces which are smaller than the string scale. The other question is the behaviour of strings in thermal equilibrium. We shall address these two questions in sections 1 and 2 respectively. In section 3 we present a simple setup for a superstring cosmology, and compute the behaviour of temperature as a function of the size of the universe. We also present some elements of an 'oscillatory' cosmology in the context of superstrings. In section 4 we present a mechanism which could potentially explain the dimensionality of extended space. In section 5 we make some concluding remarks. In appendix A we compute the entropy as a function of energy (for large energies) using the microcanonical ensemble, and show that thermodynamical equilibrium makes sense for strings in compact spaces (we obtain positive specific heat).

1. Duality: Strings in tiny boxes

We will consider strings put in a box of length R in each side (the dimension of the box is irrelevant for the considerations of this section). We impose periodic boundary conditions for both bosonic and fermionic degrees of freedom (so in mathematical language we are considering string propagation on tori). There are various string states in such a box: there are oscillator modes which correspond to stationary vibrating strings. The energy of these modes is independent of the size of the box. There are momentum modes which describe strings moving in the box. A string moving in one direction with Fourier mode n has energy $p = n/R$. There are also winding modes, i.e, strings which go from one end of the box to the other. These have energies of the form $w = mR$ where m denotes how many times the string winds around the box. There are of course string states which are combination of above states, i.e., vibrating, moving wound strings. Just by looking at the energetics we see that the spectrum of states is unchanged if we replace R by $1/R$: If we exchange the momentum modes with the winding modes $n \leftrightarrow m$, we obtain the same spectrum. This symmetry is not just a symmetry of the spectrum, rather it is the symmetry of the whole string theory (this duality in the context of strings seems to have been noted first in [1] and subsequently studied and generalized in a number of more recent works [2]). This means that any physical process computed for the strings in a box of size R, is identical to a dual physical process computed for the strings in a box of size $1/R$. This is done by showing that not only the spectrum is invariant but also all the scattering amplitudes for dual processes are equal. One is at first sight tempted to disbelieve this duality: How is it possible that we cannot distinguish the small from the large? After all the notion of size seems to be an invariant concept in general relativity. We can measure the size of the box by measuring the time it takes to send a light ray from one side to the other. How could this possibly fail? Well, the problem is that the thought experiment fails to take into account the quantum nature of light, as well as the stringy nature of the winding modes. For a large sized box we can ignore these subtleties and deal with light classically. If the size of the box gets smaller and smaller, the Fourier modes of the light waves get heavier and heavier (which go like $1/R$). It becomes energetically more and more difficult to prepare a localized light signal to send from one side to another. On the other hand the energy of the winding strings decreases (proportional to R) and it becomes easier and easier to create winding modes. It becomes easier to construct light signals using the superposition of winding modes. But how can such a superposition be localized in space, as the winding modes are highly non-local (since they wind around the whole space)?

The point is that the *notion of position is a derived concept.* In ordinary quantum field theory we define the position to be the Fourier transform of momentum states, namely

$$|x\rangle = \sum_p exp(ix.p)|p\rangle$$

In the string theory under consideration we have, in addition, the option of choosing the Fourier transform of the winding modes to define a new position,

$$|\tilde{x}\rangle = \sum_w exp(i\tilde{x}.w)|w\rangle$$

It is clear that

$$|x\rangle = |x + 2\pi R\rangle \qquad |\tilde{x}\rangle = |\tilde{x} + 2\pi/R\rangle \qquad (1.1)$$

So in one definition of position the radius is R and in the other the radius is $\tilde{R} = 1/R$. The original momentum states wind around in the $\tilde{x}$ space. In the $\tilde{x}$ space one can make localised wave packets by taking the superposition of winding modes.

There is no physical principle which tells us which definition of position is more fundamental, and therefore they are equally valid. It is interesting to note, in connection with this, that there is no physical experiment which tells us whether today we live in a Universe of size 10^{62} Planck lengths (10^{10} light years), or in a tiny Universe of size 10^{-62} in Planck units[1]. Of course the more useful definition of position for large R is $|x\rangle$, and for small R is $|\tilde{x}\rangle$, which are

[1] Throughout this paper we identify Planck scale with the string scale.

defined as the Fourier transform relative to the light states. With this definition of position we see that the effective minimum size of the box is $R = 1$, which is the Planck length. For $R < 1$ the position $|\tilde{x}\rangle$ defines a radius $\tilde{R} = 1/R > 1$.

Before closing this section let us emphasize one important lesson that we have learnt: The invariant notions of general relativity such as distance may not be invariant notions for string theory for short distance scales.

2. String Thermodynamics

String thermodynamics has been studied in a number of works [3]. Some of the basic ideas relevant for string thermodynamics have been worked out long ago [4][5]. Let us briefly recapitulate the main results, emphasizing those aspects most relevant to the present work.

The main simplifying assumption we will make in order to study thermodynamics in a theory of gravity, is to ignore the back reaction of thermodynamical condensates on the curvature of space. In other words we assume that the string coupling is sufficiently small for us to ignore gravitational effects due to the non-zero expectation value for the energy (of course we cannot set the string coupling completely to zero, as there would be no thermal equilibrium).

The most important aspect of string thermodynamics is the existence of a maximum temperature called the Hagedorn temperature (T_H) above which the canonical ensemble approach to thermodynamic is invalid as it leads to a divergent partition function. This is due to the large degeneracy of string states which is exponentially increasing with energy[6]:

$$d(E) \propto E^{-p} exp(\beta_H E)$$

where p is a positive number. Therefore

$$Z = \sum_i exp(-\beta E_i)$$

diverges for $\beta < \beta_H$, or $T > T_H$. This divergent partition function in the case of ten dimensional uncompactified strings does not necessarily mean that the Hagedorn temperature is physically unsurpassable: All the physical quantities, such as energy (per unit volume) and specific heat are finite at the Hagedorn temperature. However, canonical ensemble computations suggest that for strings propagating in spaces whose uncompactified space-time dimension is less than or equal to 3, the Hagedorn temperature is the maximally attainable temperature, as in such cases the energy is easily seen to diverge as we approach the Hagedorn temperature[2] (for a stringy derivation see the work of Axenides et. al. [3]). This is a very strange stringy phenomena: just by putting things in a box, the thermodynamical structure changes drastically. This is due to the winding modes which are not present in the infinite volume limit. No matter how large the box, for sufficiently large energies, we will excite the winding modes and they will have to be taken into account. Therefore taking the two limits of large energy and large radius in different orders gives different physics. It seems more physical to consider arbitrarily large but finite boxes, rather than infinite size boxes. In an infinite box, having a thermal bath with as small a temperature as one pleases requires finite energy per volume, but infinite total energy. So we should always put things in finite volumes.

There is even a stronger reason to consider strings propagating in compact spaces: As it turns out even below but close to the Hagedorn temperature the energy fluctuations are too large, and one does not trust the canonical ensemble computation[3] (see for example the work of Mitchell and Turok [3]). Therefore we must use the more fundamental microcanonical ensemble even for temperatures just below Hagedorn temperature in order to check the conclusions we arrived at using the canonical ensemble. It turns out that when some of the spatial directions are not compact the microcanonical ensemble leads to negative specific heat for sufficiently high energies [3], a result which once again emphasizes the unphysicality of thermal equilibrium for non-compact spaces.

[2] This is assuming a very small coupling. For larger couplings there are indications of a first order phase transition before reaching the Hagedorn temperature (pointed out by Atick and Witten [3]).

[3] We would like to thank N. Turok for a discussion on this point.

When we compactify all spatial directions the story is quite different. Using the asymptotic density of states for large energy in a nine dimensional compact space

$$d(E) = c\frac{exp(\beta_H E)}{E}dE, \tag{2.1}$$

we have checked the microcanonical ensemble at high energies and we find that it does not lead to negative specific heat (see the appendix). This is a very delicate computation. In particular it crucially depends on the proportionality constant c in equation (2.1). If $c < 1$ we obtain negative specific heat. If $c > 1$ we obtain positive specific heat. If $c = 1$ the leading term does not suffice to determine the sign of the specific heat at large energies and a more detailed computation is necessary. As it turns out $c = 1$ for superstrings (independently of which nine dimensional compact space (or conformal theory) we choose, and is equally valid for both type II superstrings and heterotic strings). We have performed a careful computation for the subleading contribution to the entropy in the appendix and we find that the specific heat is positive. This means that we can trust thermodynamical concepts for strings only in compact spaces.

In summary, if we allow any of the spatial directions to be non-compact the microcanonical ensemble computation leads to negative specific heat at large energies. This suggests that thermodynamics in the infinite volume is inappropriate and it could be at the root of the unphysical negative specific heat in such cases[4]. Therefore, the *Hagedorn temperature is the maximum allowed physical temperature.*

The computations using the microcanonical ensemble for all spatial directions compactified (see appendix) show that for large E,

$$S = \beta_H E - \frac{const.}{E} + const. \tag{2.2}$$

where β_H is the inverse Hagedorn temperature, and S is the entropy. This is not in agreement with the canonical ensemble computation which is not valid for large energies and for which one finds that

$$S = \beta_H E + cLnE \tag{2.3}$$

[4] This view point has also been expressed by Salomonson and Skagerstam [3].

We shall make a brief remark about the existence of a duality in temperature which is true only for heterotic strings. This is not crucial for the rest of this paper, and the readers uninterested in it could skip the next paragraph.

The duality in temperature for heterotic strings can be seen by noting that thermodynamical quantities could be obtained by considering an imaginary periodic time with period β, i.e., by compactifying time on a circle of circumference β, with anti-periodic boundary conditions for fermions. Heterotic string enjoys a duality $\beta \to 1/\beta$ in appropriate units (which is not shared by type II superstrings). This duality is unphysical (see for example the work of Atick and Witten [3]). In addition it is not physically observable as we cannot cross the Hagedorn temperature. This duality, even though it has a mathematically similar structure to the duality mentioned for R in the previous section, is *not* to be confused with it. The main difference is that in the duality for R we are putting periodic boundary conditions for both bosons and fermions in the spatial directions, whereas for thermodynamical duality we break supersymmetry and put anti-periodic boundary conditions for fermions in the Euclidean time direction. In addition there is no physical barrier to go from $R > 1$ to $R < 1$, and the spatial duality is therefore physically observable. Furthermore, in contrast to the thermal duality, the duality $R \to 1/R$ is true for both the type II strings and heterotic strings.

3. Superstrings in the early universe

Having discussed the two aspects of large temperatures, and small boxes in the context of strings, we return to our main goal of understanding string cosmology. Cosmology in a stringy setup has been considered in some recent works [7][8][9][10]. It is clear that we have to make a number of assumptions before even getting started.

The main assumption we shall make is that the coupling constant is small enough such that thermal equilibrium computations of the free string are applicable, without any back reactions on curvature of space.

Next, we assume that the evolution of the universe is adiabatic. It seems safe, in our opinion, to make this assumption (except for heterotic strings at the self dual radius which we shall discuss below).

Our last assumption is about the size and shape of space. Ordinarily one speaks of 'spontaneous compactification' of six of the ten dimensions of superstrings to obtain a $3 + 1$ uncompactified space-time. In other words one starts from an uncompactified ten dimensional space-time, and assumes that somehow, by unknown physical mechanisms, six of the directions curl up. In our opinion, this is not a natural way to look at the structure of space time. It seems to us that apriori all the dimensions should be small and compact, of the order of Planck length. The issue to be explained physically should not be the spontaneous compactification of six of the dimensions, but the *decompactification* of three of the spatial directions. In other words we have to explain why three of the spatial directions have grown so large compared to the other directions which are Planckian in size. In this way of thinking, the dimensionality of the extended space-time (the decompactified directions) becomes very much related to a cosmological scenario, which one may hope to explain by a plausible scenario. In the next section, we will present a cosmological scenario which could potentially explain why only three of the directions decompactify.

We shall assume that space consists of a nine dimensional box with periodic boundary conditions. This is just a simplifying assumption and is certainly not totally realistic (in particular it gives rise to a real fermion representation). We hope to learn some physical lessons from this model, without necessarily trusting all the details. The assumption of the universe being a nine dimensional box (torus) is not in contradiction with the observed three dimensionality of the extended space, as one can assume that in three of the directions the size of the box is huge, equal at least to the size of the observed universe. Of course we wish to derive this asymmetry between six and three of the spatial directions, so far as their size is concerned. Therefore to begin with we shall assume equal size R for all nine directions of the box.

In the standard Big Bang scenario, using Einstein's equations one can obtain a curve of R versus t, the proper time since the Big Bang. In the radiation dominated era (for four dimensional space-time), we have $R \propto t^{1/2}$ (see fig.1). One can also plot the temperature T versus R for which $T \propto 1/R$ (fig.2). This curve is a thermodynamical curve, in the sense that it can be determined from the thermodynamical entities, without any reference to the dynamics of gravity. Adiabaticity dictates the form of the $T - R$ curve. This is quite important for strings where we do not yet have a full understanding of the dynamics of the theory, but we know the full spectrum of string states. We do not know at the present the analogue of Einstein's equation for string theory, so we cannot reliably obtain the analogue of the R vs. t curve (fig. 1) for string theory. However, we *can* obtain the analogue of the $T - R$ curve for string theory, and we shall proceed to do so.

Before discussing the shape of the $T-R$ curve, it is clear that the shape of fig. 2 (or fig. 3) should be drastically modified in string theory: We do not have a temperature beyond the Hagedorn temperature T_H in string theory, so the infinitely rising temperature for smaller radii is going to be modified in string theory. Moreover the shape of the $T-R$ curve for point particle theories has no symmetry between the large and small, i.e., it does not respect the $R \to 1/R$ symmetry (the $lnR \to -lnR$ symmetry is absent from fig.3). So these two aspects suggest that we are going to see a major difference in the case of string theory.

Indeed we have computed numerically the shape of the $T - R$ curve for type II superstrings (fig.4) with some values for the entropy (the heterotic string modulo a minor modification to be mentioned gives a similar curve). This is easily done because we know the spectrum of type II superstring theory, and we can impose constant entropy condition to get a $T - R$ plot. In other words given some initial R and T we compute the entropy, and then we vary R, and adjust T to get the same entropy. For the low lying states we have used the canonical ensemble approach, and for the higher energy states we have used the microcanonical ensemble computation discussed in the appendix which is based on the asymptotic degeneracy of states. For large R the behaviour $T \propto 1/R$ is valid, as is to be expected, because the winding modes will become irrelevant and we are back to the point particle theory whose spectrum includes some

massless particles. However, as we go to smaller radii the temperature does not continue to rise in inverse proportion to R (fig.4). The curve suddenly flattens out at a temperature very close to the Hagedorn temperature, and as we go to even smaller radii, the temperature drops (in proportion to radius)! The fact that the temperature drops for smaller radii, is a consequence of the duality that we discussed before. It is very striking from the point of view of point particle theory which would predict a continually rising temperature. The reason for the drop in temperature is that the winding modes (which are absent for point particle theories) are becoming lighter and lighter as we decrease the radius, and so with a constant entropy most of the thermal bath consists of winding modes. Therefore we will have smaller and smaller temperatures with further decrease of the radius. A consequence of this duality is that fig. 4 enjoy the symmetry $lnR \to -lnR$.

How does this contraction appear to an observer who goes from large R to small R? As we mentioned previously the convenient variables to use in order to define position for $R < 1$ is the Fourier transform of the winding modes which are now the light 'particles'. If the observer makes such a change of variables (i.e., readjusts his measuring apparatus to measure distance in terms of the light states) he will see the radius contracting to $R = 1$ and then expanding again. This way of looking at things makes one feel more at home with the idea that temperature starts dropping as we go beyond $R = 1$. So in this language we are describing an *oscillation* of the universe. Of course it should be clear to the reader that this oscillation is not the usual kind of oscillation, as we have to relabel the physical variables as we cross $R = 1$. Therefore it seems possible, in the context of string theory, to have a universe expanding and contracting and then again expanding, and continuing this oscillation forever. In this way string theory never encounters the singularity seen in the standard Big Bang scenario.

The temperature of the plateau T_p is determined by how large the entropy is. In fact using the asymptotic form for the density of states (2.1), one finds (see appendix) that $T_p = T_H - c/S^2$, where c is a constant[5]. The radius at which the $T - R$ curve for superstrings (fig. 4) flattens out is also determined by how large the entropy is. The larger the entropy the larger the radius at which the curve flattens out ($R_c \propto S^{1/9}/T_H$). One may wonder whether it is possible to avoid getting so close to the Hagedorn temperature by considering lower entropy. In fact it is possible to generate such curves using the canonical ensemble, and avoid getting to a plateau altogether. However, such curves are not physically relevant, because in such cases one has so little total energy and entropy that a thermodynamical description is not appropriate. So the very assumption of a thermodynamical description necessitates temperatures very close to the Hagedorn temperature at the Planckian era, and this may be viewed as a signature for strings.

We now wish to comment on the shape of the $T - R$ curve for the heterotic string. All the features resemble that of the type II superstrings except the behaviour near $R = 1$. If we assume adiabaticity of the evolution, the temperature for heterotic strings drops to sero at $R = 1$. The reason for this behaviour is that there are some momentum and winding modes ($p = 1, w = 1$) which become massless at $R = 1$. At this point the heterotic string enjoys an enhanced symmetry (an extra $SU(2)^9$ symmetry). This is the same mechanism which is responsible for giving the heterotic string its gauge symmetry. The fact that we have some extra massless states in the heterotic case at $R = 1$, implies that if we wish to keep entropy constant, most of the entropy will be concentrated in the new massless modes, and since they carry little energy the temperature drops to sero. However, with very large entropies, which one needs for a proper thermodynamical description, this drop in temperature takes place in such a narrow region of radius ($\Delta R \propto e^{-aS}$ where a is a constant) that one may disbelieve the assumption of adiabaticity. It is more physically believable that in such cases we have a constant energy as we pass from infinitesimally

[5] The canonical ensemble computation gives $T_p = T_H - c/S$, which is not in agreement with the above result. As we discussed in section 2 this disagreement merely means that the canonical ensemble is not valid due to large energy fluctuations near the Hagedorn temperature.

above $R = 1$, to infinitesimally below it. In such cases the shape of the curve for superstrings and heterotic string will be very similar. The major difference is in the number of degeneracies of the massless particles which dictates the proportionality constant in $T \propto 1/R$ for a fixed entropy, and also the temperature of the plateau would be lower for the heterotic strings, since the Hagedorn temperature is lower.

Before closing this section we will make a few remarks on the possibility of deriving an equation for R as a function of time in the context of string theory. In standard cosmology with vanishing cosmological constant, in flat d-dimensional space-time, the cosmological equation reads (in appropriate units):

$$(\dot{R}/R)^2 = \rho = E(R)/R^{d-1} \qquad (3.1)$$

Unfortunately, this equation cannot be a good equation in string theory; not even in the adiabatic approximation (where we keep the lowest derivative terms). The reason for this is that (3.1) does not satisfy the $R \leftrightarrow 1/R$ duality; the left hand side is invariant under this duality but the right hand side is not ($E(R) = E(1/R)$). So this equation is valid only for $R >> 1$. Roughly speaking we have two types of gravitons in the context of strings in a box. One is the graviton modes associated with the momentum states, and the other with the winding states. Einstein's equation takes into account only the former, and thus does not respect the duality. It is an important and challenging question in the context of string theory to find the modification to (3.1) even in the adiabatic approximation for $R \sim 1$.

4. A stringy attempt for explaining space-time dimensionality

Since the very early days of the development of string theory the incorrect prediction of the dimension of space-time has been a great embarrassment for the theory. Even though to reconcile string theory with observation one could construct models with six tiny dimensions and extended four dimensional space-time, this division has not appeared in a natural way. In other words we do not have a mechanism which explains this division. In this section we shall propose a mechanism for potentially explaining the dimensionality of the observed space-time. The arguments in this section will be unfortunately rather qualitative, and we can not, with our present knowledge of string theory, support our proposal with concrete quantitative evidence. However we shall discuss our proposal with the hope of stimulating discussions which could eventually lead to quantitative progress in understanding string dynamics.

We first make the following observation: The winding states contribute negative pressure in the thermal bath. This is because increasing the volume (by increasing R) increases the energy for the winding modes and thus decreases the phase space available to them[6]. Therefore, as is intuitively clear, the winding modes would 'like' to prevent expansion. In other words it costs a lot of energy to expand with winding modes around. Of course the thermal equilibrium dictates that with increasing radius of the box, the number of winding modes decrease. So we can continue with expansion only as long as we have thermal equilibrium which favors less and less of the winding states with larger and larger radii.

However, suppose for some reason thermal equilibrium cannot be maintained for the winding modes, and we are left with a large number of winding modes. In this case the winding modes will slow down the expansion and eventually put a halt to it[7].

In the previous section we assumed a nine dimensional box with equal size in each direction. Now we shall argue that for an expanding nine dimensional box thermal equilibrium may be impossible to maintain for the winding modes. Suppose the box has expanded somewhat and we still have a large number of winding modes. With increasing sise of the box, we can begin to think of winding modes very much like we think of cosmic strings, i.e., as classical strings

[6] Winding modes for the internal manifold has been considered in a cosmological setting in [8].

[7] It would be important to find the correct stringy analogue of gravitational equation in order to confirm or correct our intuition on this point.

with a width of the order of one in Planck unit, which go from one side of the box to the other (fig. 5).

The processes which keep the winding modes in thermal equilibrium with the rest of the string states are of the form

$$W + \overline{W} \leftrightarrow unwound \quad states \tag{4.1}$$

where W denotes a winding state and $\overline{W}$ denotes the winding state with the opposite winding (see fig.6). In order for the processes (4.1) to take place the winding strings have to come to within a Planck length of one another. As the winding modes move, they span a two dimensional world sheet (with an effective thickness of the order of one in Planck units). For two winding modes to interact, their world sheets will have to intersect. But if the size of the extended space is nine dimensional the world sheets (even with the thickness being taken into account) will generically not intersect one another because $2+2 < 9+1$. In other words when the dimensionality is so high the strings will generically miss one another. With winding strings being unable to find one another in a nine dimensional box, they will fail to annihilate each other and equilibrate. They will thus fall out of equilibrium, and as discussed, will stop the expansion[8].

It is clear that in this mechanism the largest extended space-time dimensionality consistent with maintaining thermal equilibrium is 4 for which $2+2 = 3+1$ and strings will generically find each other and will therefore equilibrate. Therefore in a 4 dimensional space-time there will be no problem with getting rid of the winding modes, and thus there is no obstruction to having a three dimensional spatial expansion (i.e., a box six of whose dimensions are of the order of the Planck length, and three of the dimensions decompactifying to give rise to the observed universe).

There are a number of difficulties and issues that have to be addressed in connection with the above scenario. We will discuss them below. When

[8] This mechanism may be more relevant for heterotic strings where some winding modes are massless at the self-dual point, but pick up mass with increasing (or decreasing) radius.

there is a question of maintaining thermal equilibrium we have to compare the equilibration rate with the expansion rate $\dot{R}/R$. We were not very careful when we stated that winding modes fall out of equilibrium, because we do not know what the expansion rate is. If the rate of expansion is very small, all processes will eventually equilibrate. We assumed in the above that we have a 'reasonable' expansion rate. We managed to avoid discussing string dynamics by not asking what the R vs. t curve is, which would require a full knowledge of a quantum theory of strings. In particular equation (3.1) is invalid at Planckian radii as among other things it fails to take into account duality. The question of equilibration of winding modes will presumably have to be settled when the radius is not too large in Planckian units, and we therefore need the modification of (3.1) at small radii in order to check our scenario. It would be interesting to see if we can model strings in some way (maybe along the lines of [11]) which may give us some approximate scheme to ask questions of a dynamical nature.

Another potential criticism is in regard to our classical description of strings. It is clear, intuitively, that the cross section for winding strings to unwind (the process (4.1)) is of the order of one in Planckian units. This can be checked by a simple scattering cross section computation. If we had looked at processes which involved exchange of energy and momentum between the winding modes, without their unwinding, the cross sections are infinite (due to the existence of long range forces). But the cross section for the processes which are responsible for maintaining equilibrium between the wound states and the rest of the string modes, will be Planckian. This should justify the use of a classical picture for $R >> 1$. However since the question of equilibration of winding modes may have to be settled before the radius is too large in Planck units, it would be worthwhile to make a more careful quantitative study of this intuitive picture. This will involve scattering amplitude computations in all possible channels (similar to the analysis in [12]) and is in our belief an important question for further investigation. In particular it should be of utmost importance for our scenario to compute the decay rate and the branching ratio of very energetic string states to winding modes and the subsequent rate of annihilation of the winding modes (the computations for high energy scattering

amplitudes of strings [13] as well as computations done in the context of cosmic strings [14] may be useful in this context as well).

A question which is left unanswered in the above scenario is why we do not end up with a space with dimension smaller than three. After all the equilibrium can be maintained even more efficiently as we lower the dimension. We do not have an answer to this criticism. However we wish to point out that this may be related to the dynamical question which we left unanswered. Namely, it could be that thermal fluctuations would dynamically result in a maximum allowable space for expansion, and that, as we saw above turns out to be 3.

Another point we should comment on is the model dependence of the above scenario. The main feature that we used from the toroidal model above, was the fact that it has some modes, which were topologically conserved, and could not decay by themselves, and that their mass grows with the size of the universe. The existence of winding modes which are topologically stable and whose energy grows with the size is true for all string compactifications for which there are non-trivial loops ($\pi_1(M) \neq 0$).

Finally we wish to comment on the quantum nature of R. We have been treating R as a classical object. This may be fine for larger radii, but for small R this may not be appropriate, and we may have to treat it quantum mechanically. Even in the classical picture we have been drawing we should think of R as an averaged R, because in order to maintain thermal equilibrium there will be certain processes which will change the radius. So we have a fluctuating radius even in the context of classical thermodynamical equilibrium.

5. Concluding Remarks

We have begun a discussion of superstring cosmology, using the very limited tools we have available at the present stage of the development of string theory. In particular, with some technical assumptions, we were able to obtain the behavior of temperature as a function of radius, in a toroidal geometry. Also we have discussed how the duality of certain string models could be viewed as an oscillating solution in a cosmological setting.

We have seen that string thermodynamics makes sense only for strings propagating in spaces where all spatial directions are compact. This is not a mere technicality and seems to indicate a stringy preference for compact spaces. For nine-dimensional compact spaces we found the relation between entropy and energy for large energies. At large energies this relation is independent of which compact space (or more generally conformal theory) we choose for a nine-dimensional space, and is a genuine string prediction.

The main new proposal of this paper is a potential mechanism for explaining the dimension of extended space-time. The basic strategy was to reverse the standard treatment of extra dimensions where one speaks of 'spontaneous compactification' of six dimensions, to a setting where all spatial directions start out compact and with small radii and where one speaks of a 'decompactification scenario'. Due to a lack of techniques in addressing dynamical issues in string theory, the arguments in favour of our proposal are unfortunately rather qualitative at the present time.

The lack of a quantum theory of strings forced us to concentrate mostly on the thermodynamical aspects of cosmology where we are on safer grounds. However, by asking specific dynamical questions, we hope to stimulate research towards the development of methods to address these issues. Also we hope to have raised enough interesting questions about equilibration in string theory to further research in these directions. This is one part of our proposal which is within reach with the present techniques available in string theory. It requires only the knowledge of scattering amplitudes.

We have seen, in the context of a very simple model, how strings could resolve singularities which appear in standard cosmologies. It would be worthwhile to continue this line of thinking and investigate how strings resolve other singularities which appear in general relativity. In particular it should be quite interesting to see how strings resolve the black hole singularity.

A central element in our decompactification proposal was that in high dimensions, strings will generally have a hard time finding one another. Independently of the mechanism we proposed for the explanation of the dimensionality of space-time, we feel that this fact may be at the heart of the unphysicality of

extended space-time dimensionalities bigger than four in the context of string theory.

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Appendix A.

In this appendix we discuss the microcanonical ensemble for superstrings at high energies. The degeneracy of strings compactified on a nine-dimensional box (or more generally for any well defined nine dimensional unitary superconformal theory[9]) gives a density of states of the form

$$d(E) = c\frac{exp(\beta_H E)}{E}dE \qquad (A.1)$$

where β_H is the Hagedorn temperature (for type II superstrings it is 2π and for heterotic string it is $\beta_H = (1+\sqrt{2})\pi$ in units where $\alpha' = 1/2$). In addition $c = 1$ independently of the nine dimensional conformal theory one uses for the compact space. $c = 1$ is the critical value of c above which one obtains a positive specific heat, and below which one obtains negative specific heat. At $c = 1$ the computation of the subleading term shows, as we will see below, that the specific heat is still positive at large E.

For the energy dependence of (A.1) we refer the reader to literature [3]. It is easily derived by employing standard techniques using the the modular properties of the partition function. Here we will show why $c = 1$ independently

[9] We assume, in addition, an unbroken space-time supersymmetry in order to ensure no tachyons.

of the shape or size of the compact space (or the type of conformal theory). One should keep in mind that the asymptotic formula (A.1) is applicable for sufficiently large energies ($E > \Lambda$), and what constitutes large energies will depend on the particular conformal theory in question. For example in a nine dimensional box of size R, the energy should satisfy:

$$E >> R \quad and \quad E >> 1/R$$

for (A.1) to be applicable.

We have checked by three different methods that $c = 1$. We shall briefly discuss them. We will focus on type II superstrings. Similar arguments apply to the heterotic string (as well as to bosonic strings). The most straightforward way is to first note that the degeneracy of states due to the oscillator modes for large level numbers n is given by (for either left-moving or right-moving oscillators):

$$d_n = (2n)^{-11/4} exp(2\pi\sqrt{2n})$$

(the n dependence as well as the proportionality constant have been derived by adapting a method of Hardy and Ramanujan [15])[10]. Then we estimate the number of states taking into account the contribution of winding and momentum modes. Since this has been well described by Salomonson and Skagerstam [3] we won't go any further into it. It is easy to go through the computation keeping track of the normalizations, and finally ending up with (A.1) with $c = 1$.

The second method is to apply the method of Hardy and Ramanujan directly to the partition function of the compactified space. This has the advantage over the previous approach in that it is quite general and is true for arbitrary nine dimensional conformal theories. Even though a naive application of the work of Hardy and Ramanujan gives the right answer, it is subject to the criticism that their method really presupposes holomorphic factorization of the partition function. Even though this can probably be easily relaxed we

[10] For heterotic strings this equation describes the asymptotic degeneracy of states for the right-movers. For the left-movers it is given by $2^{-1/2} n^{-11/4} exp(4\pi\sqrt{n})$.

won't dwell further on it and come to our third method, which we believe is the simplest method of all.

The third method uses the fact that the computation of the expectation value of energy in the canonical ensemble can be done by a one loop computation of strings, in which we take time as a Euclidean parameter with period $\beta = 1/T$ around which fermions are anti-periodic and bosons are periodic. As one raises the temperature the state that winds around the time direction once becomes massless as the temperature reaches the Hagedorn temperature (this interpretation of the Hagedorn temperature has been noted by many physicists independently [3]). As we approach the Hagedorn temperature it is easy to see, using (A.1), that the energy diverges as

$$E \sim \frac{c}{\beta - \beta_H}.$$

Using the one loop computation of strings for which we know the normalizations it is easy to show that the leading divergence which comes about from the contribution of the winding mode which is nearly massless is given by

$$E \sim -\frac{\partial}{\partial\beta}\int^{\infty}\frac{ds}{s}exp(-s(\beta-\beta_H) = \frac{1}{\beta-\beta_H}.$$

Comparing the two results we immediately conclude that $c = 1$. Note that we did not have to assume anything about what the nine dimensional conformal theory is. Indeed we can get the full form of (A.1) (and corrections to it) by this argument[11].

[11] The reader may be puzzled about our using the canonical ensemble near the Hagedorn temperature, as we had previously argued that the canonical ensemble does not describe the physics of thermal equilibrium near the Hagedorn temperature. We have used the canonical ensemble in this case as a formal method to compute $-\partial LogZ/\partial\beta$ near the Hagedorn temperature in two different ways, even though we do not attach any physical significance to it near the Hagedorn temperature. In particular the above formulas that express the relation between the expectation value of energy and the temperature are not physically relevant due to large fluctuations. The correct relation obtained using the microcanonical ensemble is quite different, as we will see below.

Now that we have established that $c = 1$ we move to our main goal which is to compute entropy as a function of energy using the microcanonical ensemble. We assume that the density of states is given by (A.1) for $E > \Lambda$ and ignore the lower energy states (which are not so important for large E). Let $\Omega_n(E)$ denote the degeneracy of states with n strings and total energy E. Then we have

$$\Omega_n(E) = \frac{1}{n!}\int_{\Lambda}^{E}\prod_{i=1}^{n}\frac{dE_i}{E_i}exp(\beta_H E_i)\delta((\sum E_i) - E). \qquad (A.2)$$

by a change of variable and using the constraint imposed by the delta function on the exponent we get

$$\Omega_n(E) = \frac{1}{n!}\frac{exp(\beta_H E)}{E}\int_{\Lambda/E}^{1}\prod_i \frac{dx_i}{x_i}\delta(\sum x_i - 1). \qquad (A.3)$$

Now we use the integral representation of the delta function

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} d\alpha \quad expi(\alpha x) = \delta(x)$$

to transform (A.3) into

$$\Omega_n(E) = \frac{1}{2\pi n!}\frac{exp(\beta_H E)}{E}\int_{-\infty}^{\infty} d\alpha \quad exp(-i\alpha)[\int_{\Lambda/E}^{1}\frac{dx}{x}exp(i\alpha x)]^n. \qquad (A.4)$$

The total density of configurations $\Omega(E)$ is

$$\Omega(E) = \sum \Omega_n(E)$$

Defining

$$F(\alpha, \Lambda/E) = \int_{\alpha\Lambda/E}^{\alpha}\frac{dx}{x}exp(ix)$$

we see that

$$\Omega(E) = \frac{exp(\beta_H E)}{2\pi E}\int_{-\infty}^{\infty} d\alpha \quad exp(-i\alpha)exp(F(\alpha, \Lambda/E)). \qquad (A.5)$$

It is now not too difficult to use (A.5) to arrive at an asymptotic expansion for $\Omega(E)$ for large E. We find

$$\Omega(E) = \frac{exp(\beta_H E)}{E}(\frac{aE}{\Lambda} - b + ...) \qquad (A.6)$$

where

$$a = \frac{1}{\pi}\int_0^{\infty} d\alpha[Cos(\int_0^{\alpha}\frac{x - Sin(x)}{x}dx)]exp(\int_0^{\alpha}\frac{Cos(x) - 1}{x}dx)$$

$$b = \frac{1}{\pi}\int_0^{\infty} \alpha d\alpha[Sin(\int_0^{\alpha}\frac{x - Sin(x)}{x}dx)]exp(\int_0^{\alpha}\frac{Cos(x) - 1}{x}dx).$$

(in the integral for b we have to put an exponentially soft suppression at large α as the integrand goes like $Sin(\alpha)$ at large α). We have numerically evaluated a and b:

$$a = .56 \pm .01 \quad b = .29 \pm .01$$

Using (A.6) we have the desired result

$$S = Ln(\Omega(E)) \sim \beta_H E - \frac{b\Lambda}{aE} + const. \qquad (A.7)$$

It is easily checked that this gives rise to a positive specific heat. If the c in equation (A.1) were not equal to 1 we would have

$$S \sim \beta_H E + (c-1)Ln(E) \qquad (A.8)$$

for which it is easily seen that for $c > 1$ the specific heat is positive, for $c < 1$ it is negative, and at $c = 1$ we have to go to the next leading term which as we have seen gives rise to (A.7). From (A.7) it is easy to see that

$$\beta = dS/dE \sim \beta_H + \frac{b\Lambda}{aE^2} \sim \beta_H + \frac{b\Lambda\beta_H^2}{aS^2}.$$

which implies that

$$T_H - T = \frac{b\Lambda}{aS^2}$$

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Figure Captions

Fig. 1: In the standard Big Bang scenario (for a four dimensional space-time) in the radiation dominated era $R \propto t^{1/2}$.

Fig. 2: In the standard Big Bang in the radiation dominated era $T \propto 1/R$.

Fig. 3: T vs. lnR in the radiation dominated era in standard cosmology.

Fig. 4: T vs. lnR for type II superstrings for some values of entropy ($\sim 10^3, 10^5, 10^7$ respectively). The higher the entropy the larger the plateau. The temperature of the Plateau is very close to T_H. We have used the somewhat unconventional length units so that $lnR = 0$ corresponds to the self-dual point.

Fig. 5: Winding strings going from one side of the box to the other (we are using periodic boundary conditions).

Fig. 6: The process of unwinding two oppositely wound strings.

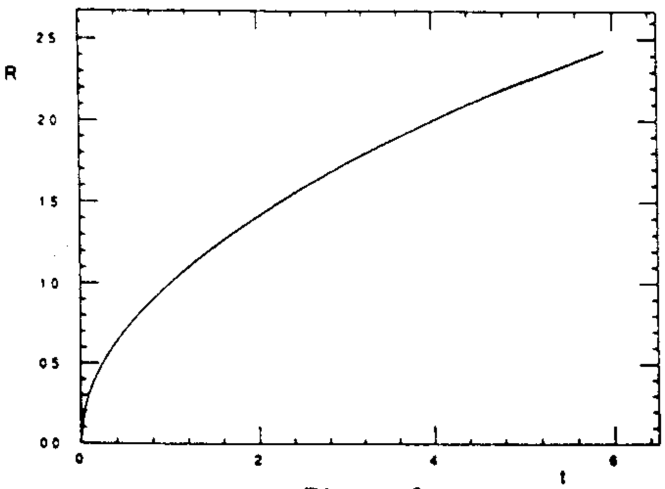

Figure 1

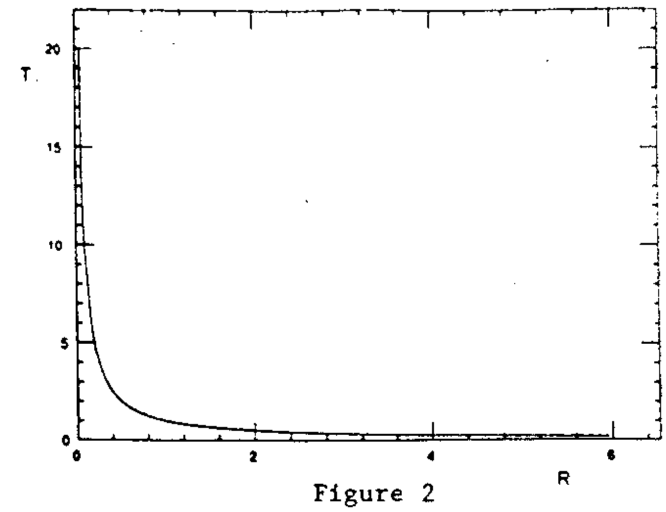

Figure 2

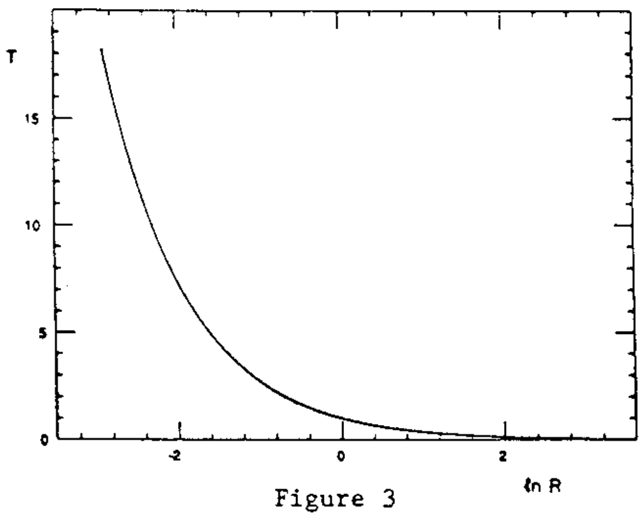

Figure 3

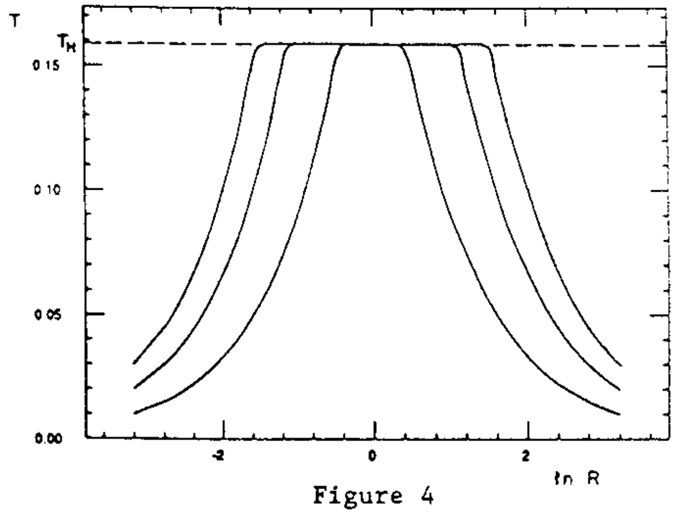

Figure 4

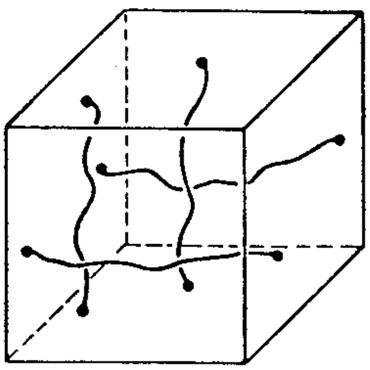
Figure 5

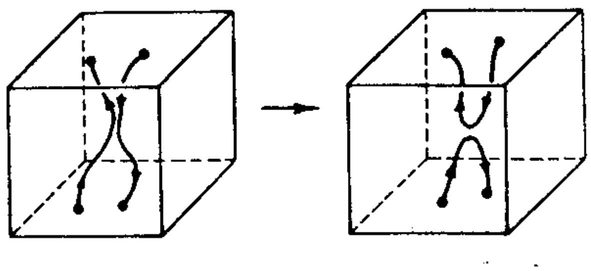
Figure 6