CERN-TH.5893/90
MEUDON-90091

HIGHLY UNSTABLE FUNDAMENTAL STRINGS IN INFLATIONARY COSMOLOGIES

M.Gasperini[1], N.Sánchez[2] and G.Veneziano[3]

[1] Dipartimento di Fisica Teorica dell'Università, Via P. Giuria 1, 10125 Torino, Italy, and INFN, Sezione di Torino, Italy

[2] Observatoire de Paris-Meudon, DEMIRM UA 336, Laboratoire associé au CNRS, Observatoire de Meudon et Ecole Normale Supérieure, 5 Place Jules Janssen, 92195 Meudon Cedex, France

[3] CERN, Theory Division, CH-1211 Geneva 23, Switzerland

Abstract

We characterize a regime of extreme (Jeans-like) instability, for strings evolving in cosmological backgrounds, in which the string's proper size grows asymptotically like the scale factor of the expanding Universe.We develop a new approximation scheme, based on the asymptotic proportionality of world-sheet and conformal times, for the systematic, quantitative description of such non-linear regime. We find that only inflationary geometries (accelerated expansion) are compatible with this instability, and we derive an equation of state for a perfect fluid of unstable strings. The effective pressure is negative, but not large enough to sustain by itself a phase of accelerated expansion.

CERN-TH.5893/90
MEUDON-90091
October 1990

1. Introduction

Recent studies [1,2] of string propagation in cosmological backgrounds have revealed some interesting, perhaps unexpected, features:

i) For non-accelerated expansions of the background geometry (e.g. for ordinary matter or radiation dominated cosmologies) strings behave very much like point particles: the centre of mass of the string follows a geodesic path while the harmonic-oscillator amplitudes shrink in such a way as to keep the string's proper size constant [2]. As expected, the distance between two strings increases with time, relative to their own size, just like the background scale factor $R(t)$.

ii) The above picture only holds [2] if the expansion of the cosmological background is introduced through the σ-model metric $G_{\mu\nu}$. If, instead, one tries to achieve a cosmological background through a time dependent dilaton field ϕ [3], the string proper size itself appears to grow like the scale factor [2] (while the oscillatory behaviour persists) so that the distance among strings stays constant relative to their sizes. The energy density ρ decreases with time, in D dimensions, like $R^{-(D-2)}$. This can be interpreted, alternatively, as a constant ρ and a time dependent Newton constant, $G \propto R^{-(D-2)} \propto e^{-2\phi}$ (recall that only the product $G\rho$ enters in the right-hand side of Einstein's equations), in agreement with expectations from the low energy string effective action.

iii) For a positive and sufficiently large acceleration d^2R/dt^2 (e.g. de Sitter with a large Hubble constant) the expansion of the exact solutions to the string equations around the point-particle motion [1] turns out to break down [2]. The harmonic oscillators develop imaginary frequencies [1] and stop oscillating, while their proper amplitudes start to grow. This was described in Ref.[2] as the onset of a Jeans-like instability, in analogy with the well known phenomenon of exponential grow of long-wavelength perturbations [4]. The phenomenon is certainly related to what has been called the stretching of cosmic strings [5] during a period of fast

inflation.

Although the methods of Refs.[2,5] allow to detect the onset of instabilities, they are not adequate for a quantitative description of the phenomenon (e.g. the effective mass of the string becomes time-dependent and this increases the instability in an apparently complicated, non-linear way). This prevents one from studying some interesting questions like the suggestion, advocated in Refs.[6,7], that highly "unstable" or "stretched" strings might lead to a regime of self-sustained inflation, which does not require a cosmological constant or the dominance of the vacuum configuration of some scalar field (inflaton).

In this paper we develop a new quantitative description of what we call the "highly unstable regime" (we stress that this instability has to be distinguished from the usual one due to the decay, or break up, of an excited string). We shall be able to construct a solution to both the non-linear equations of motion and the constraints in the form of a systematic asymptotic expansion in the large R limit, and to classify the (spatially flat) Friedman-Robertson-Walker (FRW) geometries according to their compatibility with this unstable regime.

An interesting feature of our solution is that it implies an asymptotic proportionality between the world-sheet time τ and the conformal time coordinate η of the background manifold. This is to be contrasted with the stable (point-like) regime which was characterized [1] by a proportionality between τ and the cosmic time t. Indeed, the conformal time η (or τ) will be the small expansion parameter of the solution: the asymptotic regime (small τ limit) thus corresponds to the large R limit only if the background geometry is of the inflationary type.

Moreover, by exploiting the general propertties of the solution, we obtain an equation of state for a gas of highly unstable strings, in the perfect fluid approximation. In such way we can show, through the use of the constraints, that there is an explicit connection between instability and the occurrence of an effective

negative pressure.

The content of the paper is as follows. In Section 2 we recall the system of equations and constraints to be solved, present our leading order approximation for various inflationary cosmologies, and discuss the expected form of the corrections. In Section 3 we derive a very simple equation of state obeyed by a perfect gas of highly-unstable strings and compare it with what would be needed in order to provide a scenario of self-sustained inflation. Section 4 contains a discussion of our results (partially summarized in Table 1) and some conclusions.

2. Inflation and highly-unstable string configurations

The action for the bosonic string, coupled to a curved D-dimensional metric background G_{AB}, is given by

$$S = -\frac{1}{4\pi\alpha'}\int d^2\xi\sqrt{-g}g^{\alpha\beta}\partial_\alpha X^A\partial_\beta X^B G_{AB} \tag{2.1}$$

(conventions:$A, B = 0, 1, ..., D-1$, $i, j = 1, .., D-1$, $\alpha, \beta = 0, 1$; a dot and a prime denote, respectively, differentiation with respect to the world-sheet time and space variables, τ and σ). The variation with respect to the world-sheet metric $g_{\alpha\beta}$ (using the gauge in which $g_{\alpha\beta}$ is conformally flat) provides the constraints

$$\begin{aligned} G_{AB}(\dot{X}^A\dot{X}^B + X'^A X'^B) &= 0 \\ G_{AB}\dot{X}^A X'^B &= 0 \end{aligned} \tag{2.2}$$

while the variation with respect to X^A gives the string equation of motion in the external gravitational field

$$\ddot{X}^A - X''^A + \Gamma_{BC}{}^A(\dot{X}^B + X'^B)(\dot{X}^C - X'^C) = 0 \tag{2.3}$$

where Γ is the Christoffel connection for the metric G_{AB}.

In this paper we are interested in the case in which the background corresponds to an isotropic, homogeneous cosmological model, with flat spatial sections,

described by the FRW scale factor $R(t)$ (see [2] for a discussion of the fact that this manifold does not provide a conformally invariant σ model, at the quantum level, and hence it is not a candidate string vacuum). Choosing a frame in which the target time X^0 coincides with the cosmic time t, the background metric takes the form

$$G_{AB}(t) = diag(1, -R^2(t)\delta_{ij}) \tag{2.4}$$

so that the constraints become

$$(\dot{X}^0)^2 + (X'^0)^2 = R^2 \sum_i [(X'^i)^2 + (\dot{X}^i)^2] \tag{2.5a}$$

$$X'^0 \dot{X}^0 = R^2 \sum_i \dot{X}^i X'^i \tag{2.5b}$$

and the time and space components of the string equations of motion are ($t = X^0$)

$$\ddot{X}^0 - X''^0 = R\frac{dR}{dt} \sum_i [(X'^i)^2 - (\dot{X}^i)^2] \tag{2.6}$$

$$\ddot{X}^i - X''^i = \frac{2}{R}\frac{dR}{dt}(X'^0 X'^i - \dot{X}^0 \dot{X}^i) \tag{2.7}$$

In refs.[1,2] the solution of the string equations and constraints has been expanded around the geodesic $q^A(\tau)$ of a point particle. A typical feature of this expansion is that, for large R, the cosmic and world-sheet time become proportional,

$$X^0 \to \alpha' M \tau \quad , \qquad \tau \to \infty \ , \ R \to \infty \tag{2.8}$$

This is consistent with eqs.(2.5),(2.6) provided that, in this limit,

$$\sum_i (X'^i)^2 = \sum_i (\dot{X}^i)^2 \tag{2.9}$$

and

$$(\alpha' M)^2 = R^2 \sum_i [(X'^i)^2 + (\dot{X}^i)^2] \tag{2.10}$$

The study in refs.[1,2] reveals, however, that only in some cases (for example radiation or matter-dominated FRW backgrounds) the expansion of eq.(2.7) around a geodesic path $q^i(\tau)$ is consistent, in the sense that fluctuations around it do not grow in time. Moreover, the r.h.s. of eq.(2.10) remains approximatively constant, allowing the identification of the parameter M appearing in eq.(2.8) with the (time-independent) mass of the system. We may call "stable" these situations.

By contrast, if the FRW background is of the de Sitter type with large enough Hubble constant H (i.e. $\alpha' MH > 1$, where $H = R^{-1}dR/dt$), it has been shown [1,2] that the fluctuations around $q^i(\tau)$ develop a Jeans-like instability and their proper amplitude start growing exponentially (like proper distances in a de Sitter metric) while the co-moving amplitudes become "frozen". Obviously this regime cannot be described by perturbations around the path of a point particle (M itself must grow with time). This suggests to call "unstable" (in a Jeans sense) a situation in which $\chi^i = RX^i$ essentially grows like the cosmological scale factor, so that X^i tends to remain constant in time as the universe expands, i.e. $X^i \simeq X^i(\sigma)$.

Since, according to the discussion of the X^0 fluctuations [1,2], we may expect that $|X'^0| << |\dot{X}^0|$ both for the stable and unstable case, we are thus led to characterize unstable string configurations, at large R, by the properties

$$|X'^0| << |\dot{X}^0|, \qquad |\dot{X}^i| << |X'^i| \tag{2.11}$$

We shall now determine the class of geometries for which the string equations and constraints (2.5-2.7) can be consistently solved by a configuration satisfying these properties.

First of all, by using the conditions (2.11), we find that eqs.(2.5a) and (2.6) can be simultaneously satisfied, asymptotically, by

$$\dot{X}^0(\sigma, \tau) = RL(\sigma) \tag{2.12}$$

where $L(\sigma) = (\delta_{ij}X'^iX'^j)^{1/2}$ (in the large R limit X^i has to approach a τ-independent function, $X^i \to A^i(\sigma)$). As a consequence, the world-sheet time

variable of unstable strings turns out to be proportional, asymptotically, to the conformal time coordinate η, defined by $R = dt/d\eta$. Eq.(2.12) indeed implies the relation

$$\eta(\sigma, \tau) = \tau L(\sigma) \tag{2.13}$$

in contrast with the proportionality between τ and cosmic time (see eq.(2.8)), typical of a stable string configuration. By using (2.13) we have

$$X'^0 = \eta' R = \tau R L' \tag{2.14}$$

which leads to the ratio

$$|X'^0|/|\dot{X}^0| = |\tau L'/L| \tag{2.15}$$

The solution (2.13) is asymptotically consistent with the instability conditions (2.11), therefore, only in the $\tau \to 0$ limit.

On the other hand, the configurations characterized by the properties (2 .11) can describe stretched strings, with growing proper size, provided the regime in which (2.11) are satisfied corresponds to an expanding phase, so that the scale factor, asymptotically, goes to infinity. When expressed in terms of the conformal time, the $R \to \infty$ limit corresponds to $\eta \to 0$ or $\eta \to \infty$ according to whether the background is, respectively, of the inflationary type ($d^2R/dt^2 > 0$) or not. Because of eq.(2.13) and of the fact that the asymptotic consistence of unstable configurations is achieved for $\tau \to 0$, we can thus conclude that only inflationary backgrounds are compatible with the presence of highly-unstable strings, with a proper size that grows proportionally to the scale factor.

In order to check this conclusion, we shall now verify that for the geometries describing accelerated expansions the conditions (2.11) are consistent, when $\tau \to 0$, also with the second constraint (2.5b) and with the other equation (2.7).

We start by noting that, by using (2.5b) to eliminate X'^0, the equation (2.7)

for X^i becomes

$$\ddot{X}^i - X''^i + 2\frac{\dot{R}}{R}[\dot{X}^i - \frac{R^2}{(\dot{X}^0)^2}X'^i X'^j \dot{X}^j] = 0 \tag{2.16}$$

In the $\tau \to 0$ limit, where $X^i \to A^i(\sigma)$ and eqs.(2.12-2.14) are valid, the above equation linearizes in the form

$$\ddot{X}^i - X''^i + 2\frac{\dot{R}}{R}(\delta^{ij} - \frac{A'^i A'^j}{L^2})\dot{X}^j = 0 \tag{2.17}$$

and the constraint (2.5b) reduces to

$$\tau L L' = A'^i \dot{X}^i \tag{2.18}$$

We shall consider, in particular, three classes of expanding geometries, in which the scale factor has the following asymptotic behaviour:

a) Power-law inflation

The conformal scale factor

$$R(\eta) = (k\eta)^{-\alpha} , \qquad 1 < \alpha < \infty \tag{2.19}$$

(where k is a constant) can be expressed in terms of the cosmic time as $R(t) \propto t^{\alpha/(\alpha-1)}$, and describes an inflationary background (accelerated expansion, $d^2R/dt^2 > 0$) of the power-law type ($dH/dt < 0$). The large R limit ($\eta \to 0$, $\tau \to 0$) corresponds in this case to $t \to \infty$. In this limit, by using (2.13), we get

$$X^0(\sigma, \tau) = \frac{k^{-\alpha}}{1-\alpha}(\tau L)^{1-\alpha} \tag{2.20}$$

and eq.(2.17) becomes

$$\ddot{X}^i - X''^i - \frac{2\alpha}{\tau}(\delta^{ij} - \frac{A'^i A'^j}{L^2})\dot{X}^j = 0 \tag{2.21}$$

The general solution, to leading order as $\tau \to 0$, depends on two arbitrary functions of σ, $A^i(\sigma)$ and $B^i(\sigma)$, and can be written as

$$X^i(\sigma, \tau) = A^i(\sigma) + \frac{\tau^2}{2}D^i(\sigma) + \tau^{1+2\alpha}F^i(\sigma) \tag{2.22}$$

where

$$D^i(\sigma) = \frac{1}{1-2\alpha}(A''^i - \frac{2\alpha}{L^2}A'^i A'^j A''^j) \tag{2.23}$$

$$F^i(\sigma) = B^i - \frac{A'^i}{L^2}B^j A'^j \tag{2.24}$$

To this order in τ, the solution (2.22) automatically satisfies also the constraint (2.18), because of the identities

$$\begin{aligned} A'^i D^i &= A'^i A''^j \equiv LL' \\ A'^i F^i &= 0 \end{aligned} \tag{2.25}$$

Moreover, for $\tau \to 0$, eq.(2.22) leads to the ratio

$$|\dot{X}^i|/|X'^i| = |\tau D^i + \tau^{2\alpha}(1+2\alpha)F^i|/|A'^i| \tag{2.26}$$

which is asymptotically vanishing, consistently with the instability properties (2.11).

b) de Sitter inflation

Consider now an exponentially expanding background $R(t) = exp(Ht)$ of the de Sitter type ($H = const$), which corresponds to the conformal scale factor

$$R(\eta) = -(H\eta)^{-1} \tag{2.27}$$

By using (2.13) we have, in the large R limit ($\tau \to 0$, $t \to \infty$)

$$X^0(\sigma,\tau) = -H^{-1}ln(-\tau HL) \tag{2.28}$$

In this background the asymptotic expression for X^i is still given by the solution (2.22), with $\alpha = 1$, so that the ratio (2.26) goes to zero , for $\tau \to 0$, again consistently with eq.(2.11).

c) Super-inflation

The third possible type of inflationary background, called [8] super- inflation and characterized by $d^2R/dt^2 > 0$, $dH/dt > 0$, is represented by a scale factor of the same type as (2.19), with the exponent α, however, varying in the range $0 < \alpha < 1$. For this background we get, in the $\tau \to 0$ limit,

$$X^0(\sigma,\tau) = t_c + \frac{k^{-\alpha}}{1-\alpha}(\tau L)^{1-\alpha} \tag{2.29}$$

where t_c is an integration constant, determining the value of cosmic time at which R diverges.

Concerning the asymptotic expression for X^i, we have to distinguish two cases: if $\alpha \neq 1/2$, we are led again to the solution (2.22) with arbitrary functions A^i and B^i. If $\alpha = 1/2$, the asymptotic form (2.22) has to be replaced by

$$X^i = A^i + \frac{\tau^2}{2}(F^i + A'^i\frac{A'^jA''^j}{L^2}) + \frac{\tau^2}{2}ln\tau(A''^i - A'^i\frac{A'^jA''^j}{L^2}) \tag{2.30}$$

where F^i is still given by eq.(2.24). In both cases it turns out that the ratio $\dot{X}^i/X'^i$ goes to zero at large R, in agreement with (2.11), so that unstable configurations are compatible also with this scale factor.

It may be noted that , in all cases we have analyzed, our solution appears to depend, at first sight, upon $2D-3$ functions of σ: A^i and F^i $(i = 1,...D-1)$, subject to the constraint $A'^iF^i = 0$. However, by exploiting σ-reparametrization invariance, one can always fix one of the functions, by imposing e.g.

$$(A'^iA'^i) = const \tag{2.31}$$

One is thus left with $2(D-2)$ arbitrary functions only, as it is known to be the case for free strings (in flat spacetime). We stress, therefore, that eqs.(2.22) and (2.30) represent indeed the general solution, to leading order as $\tau \to 0$.

The approach to the asymptotic regime can be studied by constructing a systematic expansion of the solution as a power series in τ. We have been able to compute the first non-leading corrections to eq.(2.22) (which include both integers and α-dependent positive powers of τ), and we have checked that they satisfy, to this order, both the string equations and the constraints*.

The analysis of this Section shows that inflationary backgrounds, whatever their kinematical type, are always compatible, asymptotically, with the presence of highly unstable strings. This is due, in our opinion, to the fact that for a monotonically expanding background the inflation condition $d^2R/dt^2 > 0$ implies the existence of an event horizon at a finite distance from the origin (i.e. the integral $\int^{t_\infty} R^{-1}dt$ converges, where $t_\infty = t_c$ for super-inflation and $t_\infty = \infty$ in the other cases). It becomes thus possible, for the stretched strings, to have a characteristic "proper size" larger than the horizon radius, which is just the typical situation in which co-moving lengths are frozen.

We have to point out, however, that some inflationary backgrounds (like de Sitter [1,2]) can also be compatible with the stable regime, in which the exact solution is expanded around a geodesic. For these backgrounds the development of instabilities will depend on the full history of the string. We will come back to this point in Section 4.

3. Equation of state and self-sustained inflation

In the previous Section we have shown that configurations of highly unstable strings can form and evolve only during a phase of accelerated expansion. In this Section we shall first derive a condition on the energy momentum tensor for such string configurations. This condition will then be used to discuss the possibility of string-driven expansion, for the case in which these unstable configurations are

* We are grateful to Nguyen Suan Han for helping us with these calculations.

the dominant source of gravity.

To this end we consider the D-dimensional Einstein equations for the background geometry (neglecting, in the large R limit, quadratic curvature terms and other quantum corrections)

$$R_A{}^B = 8\pi G(T_A{}^B - \frac{T_C{}^C}{D-2}\delta_A{}^B) \tag{3.1}$$

and we insert, in the source term, the string stress tensor T_{AB} obtained by varying the action (2.1) with respect to the target metric G_{AB}

$$\sqrt{-G}T^{AB}(x) = \frac{1}{\pi\alpha'}\int d\sigma d\tau(\dot{X}^A\dot{X}^B - X'^AX'^B)\delta^D(X-x) \tag{3.2}$$

We are working in the conformal gauge for the world-sheet metric, as stressed before, and we are assuming for T_{AB} the spatial symmetries required by the source of a FRW geometry, so that, in a comoving frame, we can write

$$T_{00} = \rho, \quad T_{ij} = -pG_{ij} = pR^2\delta_{ij} \tag{3.3}$$

where ρ and p are functions of the cosmic time only.

According to the constraints (2.2), the following general identity holds

$$\pi\alpha'\sqrt{-G}(G_{00}T^{00} - G_{ij}T^{ij}) =$$

$$-2\int d\sigma d\tau\delta^D(X-x)[G_{00}(X'^0)^2 + G_{ij}\dot{X}^i\dot{X}^j] \tag{3.4}$$

For a configuration of unstable strings, the conditions (2.11) are also valid: as a consequence, the r.h.s. of (3.4) can be approximatively neglected with respect to each term appearing in the l.h.s., so that

$$\rho = T_0{}^0 \simeq G_{ij}T^{ij} = -p(D-1) \tag{3.5}$$

We thus obtain, in the case of unstable strings, an effective negative pressure, as opposed to stable strings or point-like matter for which, usually, $0 < p < \rho/3$.

This difference can be ascribed to the relative sign of the $\dot{X}^i$ and X'^i contributions to T^{ij} as they appear in eq.(3.2), and to the fact that the first contribution dominates for massless particles while the second one is the dominant term for unstable strings. For $D = 4$ we get $p = -\rho/3$, equation which has already appeared in the literature [6,9] in the context of string- driven cosmology. Our derivation provides, through use of the constraints, an explicit connection between this equation of state and the presence of instability ($|\dot{X}^i| << |X'^i|$).

Taking eq.(3.5) into account, and considering the case in which the dominant gravitational source is a homogeneous and isotropic configuration of unstable strings, the Einstein equations (3.1) imply

$$(D-1)(D-2)\frac{d^2R}{dt^2} = -8\pi G\rho(D-4) \tag{3.6}$$

Recalling that d^2R/dt^2 must be positive in order to allow instability, it turns out that, except for the case $D = 3$, the effective pressure (4.5) is not negative enough for allowing a possible unstable-string-sustained geometry. In four dimensions, in particular, the asymptotic contribution of unstable strings to the acceleration is exactly vanishing.

It should be stressed, however, that this result is valid in the case that all spatial dimensions expand isotropically, with no compactified dimensions. Moreover it is based on the fact that the string sources are represented by a stress tensor of the perfect fluid type, neglecting possible viscosity terms due to mutual and self- interactions. When such corrections are included, and an effective bulk viscosity is added to the components of the stress tensor, string-driven inflation (and deflation) can become possible, as discussed in ref.[9].

4. Discussion and conclusions

In this paper, by complementing the work of refs.[1,2], we have discussed the possible occurrence of a highly unstable regime for strings evolving in FRW

cosmological backgrounds. We recall from Sect.2 that all the results we have presented apply for the moment to the large R limit, i.e. to "late times". In this limit the universe, as well as our large strings, behave (semi) classically, justifying the neglect of quantum conformal anomalies which typically occur in non- Ricci-flat backgrounds.

The results of this paper, together with those of ref.[2], are summarized in **Table I** where, for each class of FRW metrics, we give the allowed asymptotic string evolutions. Let us describe the Table in some detail and discuss its physical meaning.

Starting from the trivial (flat) metric, in which obviously no instability can occur, we see that the situation is not changed as long as one considers standard cosmological evolution with $d^2R/dt^2 < 0$, including the limiting case of a linear expansion, $d^2R/dt^2 = 0$: no Jeans-like instability is possible in these geometries, in agreement with the absence of event horizons.

The situation starts to change as we move on to inflationary geometries ($d^2R/dt^2 > 0$), where we have to distinguish three cases: for power-law inflation ($dH/dt < 0$) both stable and unstable string evolutions are possible, for super-inflation ($dH/dt > 0$) only the highly unstable regime is allowed. Finally, for the physically interesting case of a de Sitter geometry ($H = const$) instabilities are always allowed while the stable, point-like behaviour is only possible [2] for sufficiently small H (i.e. $\alpha' MH < 1$).

We recall that the two possible behaviours are characterized by different identifications of the world-sheet time τ: in the stable regime τ turns out to be proportional to cosmic time t, while, in the unstable regime, the same is true for conformal time η.

It is perhaps surprising to find that some FRW geometries (e.g. power- law inflation) allow for both kinds of behaviours, and one may ask what determines

the late time evolution of a given string in such geometries. We believe the answer to this question to lie in the complete history of the string and, in particular, on its initial state.

Consider, for instance, the case of de Sitter geometry with two initial strings, one with $\alpha' MH < 1$, the second with $\alpha' MH > 1$. The first string will evolve like a massive point particle, with the center of mass following the corresponding geodesic and keeping a constant mass. The second string, as shown in [2], will start developing an instability in its first mode ($n = 1$) which will produce an increase of the mass with time. The mass stored in the first mode will increase until, at $\alpha' M(t)H > 2$, the second mode will become unstable and the mass will grow even faster. The process continues and, one after the other, more and more modes develop the Jeans-like instability: the string approaches then the highly unstable regime considered in this paper, in which the approximate description in terms of the point-particle expansion is no longer appropriate.

Analogously, in the case of power-law inflation, the instability will develop [2] if, at some time,

$$\frac{(\alpha' M)^2}{R}\frac{d^2R}{dt^2} = (\alpha' M)^2\left(\frac{dH}{dt} + H^2\right) > 1 \tag{4.1}$$

Since, in this case, $R^{-1}(d^2R/dt^2)$ is a decreasing function of cosmic time, the instability will continue only if the mass increase is fast enough to keep the above inequality satisfied. In other words the string size has to grow faster than the "horizon size" H^{-1}, which is expanding. This is to be contrasted with the case of super-inflation, where H^{-1} is decreasing: the horizon will always be crossed by the string at late times, and the highly unstable regime will unavoidably set in.

Concerning the possibility of string-driven inflation, our results do not appear to support it, at least in the perfect fluid approximation for the string energy-momentum tensor. It may be possible, however, that the inclusion of other effects, such as mutual string interactions or quantum string production from the vacuum,

could sustain an inflationary expansion, and phenomenological models for these effects have indeed been considered [6,9].

We finally stress the importance of finding a smooth way to join together stable and unstable regimes: this would allow to deal, for instance, with physical situations in which an inflationary epoch with unstable strings is followed by an era of standard decelerating expansion in which only the stable behaviour is allowed. We plan to come back to this case in the near future.

One of us (MG) wishes to thank the theory division at CERN, and in particular S. Fubini, for the kind hospitality received during the early stages of this work.

References

1. H.J.De Vega and N.Sánchez, Phys.Lett.B197 (1987) 320
2. N.Sánchez and G.Veneziano, Nucl.Phys.B333 (1990) 253
3. I.Antoniadis, C.Bachas, J.Ellis and D.V.Nanopoulos, Nucl.Phys.B238(1989) 117
4. see e.g. W.C.Saslaw, in "Gravitational physics of stellar and galactic systems" (Cambridge University Press, Cambridge, 1985), Chapt.21
5. N.Turok and P.Bhattacharjee, Phys.Rev.D29(1984)1557
6. N.Turok, Phys.Rev.Lett.60 (1988) 549
7. Y.Aharonov, F.Englert and J.Orloff, Phys.Lett.B199(1987)366
8. F.Lucchin and S.Matarrese, Phys.Lett.B164 (1985) 282
9. J.D.Barrow, Nucl.Phys.B310 (1988) 743

Table caption

Table I

A classification of spatially flat FRW backgrounds, according to their asymptotic compatibility with stable and unstable string configurations. Here "unstable" denotes the regime, discussed in this paper, in which the string proper size grows in time like the scale factor, while "stable" refers to a regime in which the solution to the string equations can be consistently expanded around a geodesic, characterized by the mass parameter M (see refs.[1,2]).