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$O(d,d)$-COVARIANT STRING COSMOLOGY

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Abstract

The recently discovered $O(d,d)$ symmetry of the space of (slowly-varying) cosmological string vacua in $d+1$ dimensions is shown to be preserved in the presence of bulk string matter. The existence of $O(d,d)$ conserved currents allows all the equations of string cosmology to be reduced to first-order differential equations. The perfect-fluid approximation is not invariant under $O(d,d)$ transformations, implying that stringy fluids possess in general a non-vanishing viscosity.

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1. Introduction

As a candidate theory for the unification of all interactions, including gravity, string theory must possess all kinds of conventional field-theoretic symmetries, in particular gauge and general coordinate transformation invariance, as well as their supersymmetric generalizations.

It has long been suspected that the above are only a tiny subset of the full symmetries of string theory, and much research has gone in recent years into trying to unravel the "higher" stringy symmetries which are not shared by conventional quantum field theories.

Although this program is still in its infancy, some interesting stringy symmetries have indeed emerged, in particular those which go under the generic name of target-space duality (or modular invariance) [1]. These symmetries might play a crucial role in connection with compactification of the extra dimensions [2] and in determining the mechanism of supersymmetry breaking [3] in string theory.

It is very natural to ask whether some stringy symmetries can also be found in the string analogue of the Einstein–Friedmann equations, which govern the time evolution of a spatially homogeneous Universe, i.e. in what we shall refer to as "string cosmology". Indeed, it has been suspected for some time [4] that string cosmology should also exhibit a "duality" between large and small scale factors and/or large and small temperatures. Finding the precise form of this symmetry has proven to be a non-trivial task, however.

In the case of closed strings moving in a compact (target) space, many authors [5,6] have recently discussed the generalization of target-space duality to the non-static case. This leads to the physical identification of apparently unrelated cosmologies, in the sense that the contraction of a compact dimension below the self-dual point is shown to be actually equivalent to its expansion above it.

It can be shown however that, even for open strings and/or a non-compact target space, duality-like discrete symmetries (scale-factor duality) survive [7] as symmetries of the field equations that determine the possible consistent vacua of the theory. In this case, more than of a symmetry, one should talk of a group acting on the space of solutions by transforming non-equivalent vacua into each other. Furthermore, the discrete symmetry of the field equations persists [7] even in the presence of classical string sources.

An enlargement of the symmetry group of cosmological string vacua in $d+1$ dimensions was discovered in Ref. [8]. Accordingly, discrete scale-factor duality is embedded in a continuous $O(d,d)$ "Narain" group [9]. Obviously, such a large group can only be interpreted as a symmetry of the equations of motion and not of the full theory. In more modern terminology, we would say that the generators of this group are "moduli" in the space of classical solutions. In this formulation the basic, $O(d,d)$-covariant objects are a shifted dilaton Φ, which absorbs the volume factor $\sqrt{|G|}$, and a symmetric $O(d,d)$ matrix M, which mixes non-trivially the metric and the antisymmetric tensor fields (thus implementing, in a perhaps unexpected way, Einstein's old dream of a non-symmetric unified theory [10]).

Arguments for the validity of the $O(d,d)$ symmetry to all orders have been presented, both from the string field theory [11] and from the σ model [12] point of view. Extensions to the case of more general backgrounds that are just independent of a subset of coordinates have also been given [13,12], and some amusing applications of the symmetry, both to cosmological solutions [14] and to $2D$ black holes [15] have been found.

In this paper we add classical string sources to the manifestly $O(d,d)$ invariant (low-energy) cosmological vacuum equations of Ref. [12]. As in the case of scale-factor duality, we find that manifest $O(d,d)$ invariance is maintained in the presence of sources. This leads to the conclusion that string cosmology in $d+1$ dimensions is $O(d,d)$-covariant. A welcome consequence of this symmetry is the existence of conserved $O(d,d)$ currents. By constructing them explicitly we are able to reduce all our constraints to first-order differential equations. We shall also derive a general continuity equation for the string sources, showing that the antisymmetric field, unlike the dilaton, contributes explicitly to the covariant conservation of the total source energy.

We stress that, in order not to spoil $O(d,d)$ invariance, string sources must evolve in time in a way consistent with the equations [16] describing the motion of each string in the (self-consistent) background generated by the sources themselves. Thus, under $O(d,d)$, not only the backgrounds but also the sources and their equations of state must change. Perhaps surprisingly, we shall find that a perfect-gas equation of state in not invariant under $O(d,d)$, so that, generically, our cosmological backgrounds are sustained by a string fluid with some specific kind of viscosity.

2. Background field equations with string sources

At low energy, the tree-level effective action for closed string theory, in D dimensions, can be written as [17]

$$I = \frac{1}{2\kappa}\int d^D x \sqrt{|G|} e^{-\phi}[R + (\nabla\phi)^2 - V + \frac{H^2}{12}]\,, \tag{2.1}$$

where $H = dB$ is the antisymmetric tensor field strength, κ is a dimensionful parameter related to the string tension (see [7]), and V is the cosmological constant (proportional to $D - D^{crit}$). We shall consider in particular homogeneous cosmological backgrounds which are independent of all space-like coordinates, and for which a synchronous frame exists where $G_{00} = -1$, $G_{0i} = 0 = B_{0i}$ $(i, j = 1, 2, ..., D - 1 \equiv d)$.

Defining [8]

$$\Phi = \phi - ln\sqrt{|detG|} \tag{2.2}$$

$$M = \begin{pmatrix} G^{-1} & -G^{-1}B \\ BG^{-1} & G - BG^{-1}B \end{pmatrix} \tag{2.3}$$

where G and B are $d \times d$ matrices representing respectively $G_{ij}(t)$ and $B_{ij}(t)$, the action (2.1) can be written in a manifestly $O(d, d)$-invariant form [8,12]:

$$I = -\frac{1}{2\kappa} \int dte^{-\Phi}[\dot{\Phi}^2 + \frac{1}{8}Tr(\dot{M}\eta\dot{M}\eta) + V] \ , \tag{2.4}$$

where η is the $O(d, d)$ metric in off-diagonal form

$$\eta = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} \ ,$$

a dot denotes differentiation with respect to the cosmic time t, and we have allowed for a more general scalar self-interaction $V(\Phi)$ than just a constant.

We also stress that $O(d, d)$-invariance takes a simple form only if one works directly with the (unrescaled) σ-model metric G. The fact that G is also the physical metric of string theory was shown in ref. [18] (see also [19]).

Let us now add to the action (2.4) the contribution of classical string sources. Their effective Lagrangian (in the conformally flat gauge for the world-sheet metric) can be written as

$$L(t) = \frac{1}{2\pi\alpha'} \int d\sigma d\tau \delta(t - X^0(\sigma, \tau))(P_0\partial_\tau X^0 + P_i\partial_\tau X^i - H) \ , \tag{2.5}$$

where

$$H = \frac{1}{2}[Z^T MZ - P_0^2 - (X^{0\prime})^2] \ , \tag{2.6}$$

$P_0 = -\partial_\tau X^0$ according to the string equations of motion, and

$$Z^A(\sigma, \tau) = (P_i, X'^i) \tag{2.7}$$

are the $2d$-dimensional phase-space coordinates (σ and τ are as usual the world-sheet variables, and a prime denotes $\partial/\partial\sigma$). Note that we write explicitly partial differentiation with respect to τ, as we have reserved a dot for cosmic time derivatives. Moreover, a sum over different strings in eqs. (2.5) and (2.6) is understood.

Since Φ is not directly coupled to the sources, the variation of the total action with respect to Φ leads to the same dilaton equation as already obtained in [8], namely

$$\dot{\Phi}^2 - 2\ddot{\Phi} - \frac{1}{8}Tr[\dot{M}\eta\dot{M}\eta] + \frac{\partial V}{\partial \Phi} - V = 0 \ . \tag{2.8}$$

The G_{00} variation (see [8]) provides the (zero-energy) equation

$$\dot{\Phi}^2 + \frac{1}{8}Tr[\dot{M}\eta\dot{M}\eta] - V = 2\kappa\overline{\rho}e^{\Phi} \ , \tag{2.9}$$

where

$$\overline{\rho}(t) \equiv \sqrt{|G|}\rho = -\frac{\delta L}{\delta G_{00}} = \frac{1}{4\pi\alpha'}\int d\sigma \frac{d\tau}{dX^0}[P_0^2 - (X'^0)^2] \tag{2.10}$$

represents the effective energy density of the string sources.

We have now to vary the action with respect to M. Proceeding as in Ref. [12], we perform an infinitesimal transformation

$$\delta M = \Omega^T M \Omega - M = \epsilon^T M + M\epsilon \ , \tag{2.11}$$

where $\Omega = I + \epsilon$ belongs to $O(d,d)$. One easily finds (using $\eta^2 = I$, $\eta\epsilon = -\epsilon^T\eta$) that the source contribution is given by

$$\frac{\delta L}{\delta(\eta\epsilon)} = -\frac{1}{4\pi\alpha'}\int d\sigma \frac{d\tau}{dX^0}(ZZ^T M\eta - \eta M Z Z^T) \equiv -\frac{1}{2}(SM\eta - \eta MS) \ , \tag{2.12}$$

where we have defined the symmetric matrix

$$S(t) = \frac{1}{2\pi\alpha'}\int d\sigma \frac{d\tau}{dX^0} ZZ^T(\sigma, \tau(\sigma,t)) \ . \tag{2.13}$$

By adding the massless field contributions (see [12]), the variation of the full action finally provides the equation of motion

$$\frac{d}{dt}(e^{-\Phi}M\eta\dot{M}) = 2k\overline{T} \ , \tag{2.14}$$

where

$$\overline{T} = \frac{1}{2}(MS\eta - \eta SM) \tag{2.15}$$

represents a generalized "stress matrix" for the string sources.

The three equations (2.8), (2.9) and (2.14) are manifestly invariant under the global transformation group defined by

$$\Phi \to \Phi \quad , \quad X'^0 \to X'^0 \quad , \quad P_0 \to P_0 \ ,$$

$$Z \to \tilde{Z} = \Omega^{-1} Z \quad , \quad M \to \tilde{M} = \Omega^T M \Omega \, , \tag{2.16}$$

where Ω is an $O(d,d)$ constant matrix satisfying

$$\Omega^T \eta \Omega = \eta \, . \tag{2.17}$$

It is important to stress that if a given set $\xi(\sigma,\tau) = \{Z^A, P_0, X'^0\}$ is a solution of the string equations in a background M, than the transformed set $\tilde{\xi}(\sigma,\tau) = \{\tilde{Z}^A = \Omega^{-1} Z, P_0, X'^0\}$ is a solution for the transformed background $\tilde{M}$, as one can easily verify from the string equations of motion following from the Hamiltonian (2.6):

$$\partial_\tau P_0 = -\partial_\tau^2 X^0 = -X^{0\prime\prime} - \frac{1}{2} Z^T \dot{M} Z \tag{2.18}$$

$$\partial_\tau Z = (\eta M Z)' \tag{2.19}$$

and from the constraints

$$H = 0 \tag{2.20}$$

$$Z^T \eta Z + 2 P_0 X'^0 = 0 \, . \tag{2.21}$$

The coupled string-background system of equations (2.8), (2.9), (2.14), (2.18)–(2.21) defines for us string cosmology. When they are all fulfilled, the stringy analogue of the stress tensor transforms covariantly under $O(d,d)$, namely

$$\overline{\rho} \to \overline{\rho} \quad , \quad \overline{T} \to \Omega^T \overline{T} \Omega \, . \tag{2.22}$$

In view of future applications, it may be useful to express $\overline{T}$ in terms of the components of the usual energy–momentum tensor, θ^{ij}, and of the antisymmetric current J^{ij} coupled to the torsion field B_{ij}, which are defined by

$$\overline{\theta}^{ij} = \sqrt{|G|} \theta^{ij} = \frac{\delta L}{\delta G_{ij}} \quad , \quad \overline{J}^{ij} = \sqrt{|G|} J^{ij} = \frac{\delta L}{\delta B_{ij}} \, . \tag{2.23}$$

According to the Hamiltonian equations $\partial_\tau X^0 = \partial H / \partial P_0$, $\partial_\tau X^i = \partial H / \partial P_i$, one has

$$P_0 = -\partial_\tau X^0 \quad , \quad P_i = \partial_\tau X^j G_{ji} - X'^j B_{ji} \tag{2.24}$$

and by writing explicitly $L(t)$ in terms of G and B, one readily obtains

$$\overline{\theta}^{ij}(t) = \frac{1}{4\pi\alpha'} \int d\sigma \frac{d\tau}{dX^0} (\partial_\tau X^i \partial_\tau X^j - X'^i X'^j) \tag{2.25}$$

$$\overline{J}^{ij}(t) = \frac{1}{4\pi\alpha'} \int d\sigma \frac{d\tau}{dX^0} (\partial_\tau X^i X'^j - X'^i \partial_\tau X^j) \, . \tag{2.26}$$

From the definition (2.15) we are thus led to the explicit expression

$$\overline{T} = \begin{pmatrix} -\overline{J}, & -\overline{\theta}G + \overline{J}B \\ G\overline{\theta} - B\overline{J}, & G\overline{J}G + B\overline{J}B - G\overline{\theta}B - B\overline{\theta}G \end{pmatrix} \tag{2.27}$$

where $\overline{\theta}$ and $\overline{J}$ represent the $d \times d$ matrices of eqs. (2.25) and (2.26).

We note, finally, that the conservation of the charge associated with the global $O(d,d)$ invariance allows a first integration of eq. (2.14). Let us define, indeed,

$$\overline{\Theta}(t) = -\frac{1}{2}\int d\sigma d\sigma' \epsilon(\sigma - \sigma') F(\sigma,\tau) Z(\sigma,\tau) Z^T(\sigma',\tau') F(\sigma',\tau') \tag{2.28}$$

$$F = \eta - (X'^0/\partial_\tau X^0) M \tag{2.29}$$

where $\epsilon(x) = \frac{1}{2} sign(x)$ and $\tau(\sigma,t)$ $(\tau'(\sigma',t))$ is solution of $t = X^0(\sigma,\tau)$ $(t = X^0(\sigma',\tau'))$. One finds:

$$\begin{aligned}
\dot{\overline{\Theta}} = & -\frac{1}{2}\int d\sigma d\sigma' \epsilon(\sigma - \sigma') \{[\partial_\tau X^0(\sigma)]^{-1} [\partial_\tau F(\sigma) Z(\sigma)] Z^T(\sigma') F(\sigma') \\
& + F(\sigma) Z(\sigma) [\partial_\tau Z^T(\sigma') F(\sigma')] [\partial_\tau X^0(\sigma')]^{-1}\} \\
= & -\frac{1}{2}\int d\sigma d\sigma' \epsilon(\sigma - \sigma') \{[\partial_\sigma (\partial_\tau X^0)^{-1} M Z(\sigma)]_t Z^T(\sigma') F(\sigma') \\
& + F(\sigma) Z(\sigma) [\partial_{\sigma'} Z^T M (\partial_\tau X^0)^{-1}(\sigma')]_t\}
\end{aligned} \tag{2.30}$$

where we used the equations of motion (2.19), and the following relation between derivatives with respect to σ performed at constant t and τ:

$$[\partial_\sigma f]_t = [\partial_\sigma f]_\tau - (X'^0/\partial_\tau X^0)[\partial_\tau f]_\sigma \equiv f' - (X'^0/\partial_\tau X^0)\partial_\tau f \; . \tag{2.31}$$

Integrating finally by parts in eq. (2.30) we get

$$\dot{\overline{\Theta}} = \overline{T} \tag{2.32}$$

so that eq.(2.14) simply gives

$$e^{-\Phi} M \eta \dot{M} = C(t) \equiv 2k\overline{\Theta} + A \; . \tag{2.33}$$

Here A is a constant antisymmetric matrix, and the matrix C, because of the $O(d,d)$ properties of M (i.e. $M\eta M = \eta$), satisfies the property

$$M\eta C = -C\eta M \; . \tag{3.34}$$

In the absence of sources ($Z = 0, C = const$) one then finds the general solution presented in [12].

In the next section we shall derive a first-order energy conservation equation which, together with eqs. (2.9) and (2.33), implies also the second order dilaton equation (2.8). We thus conclude that, thanks to the $O(d,d)$ symmetry, the equations of string cosmology can always be reduced to first-order differential equations.

3. Covariant conservation of the source energy

In general relativity, the energy–momentum tensor of the gravitational sources is covariantly conserved as a consequence of the contracted Bianchi identity. This identity could be applied to obtain a conservation equation also in our case, of course, by re-writing the field equations so as to include all the dilaton, torsion, and string contributions to the "right-hand side" of a generalized Einstein equation. However, such a generalized conservation law can be obtained already in $O(d,d)$-invariant form, by using directly the $O(d,d)$-covariant equations derived in the previous section.

Indeed, by differentiating eq. (2.9) with respect to cosmic time, combining the result with eqs. (2.8) and (2.14), and by using the identity

$$(M\eta\dot{M}\eta)^2 = -(\dot{M}\eta)^2 \ , \tag{3.1}$$

we get the conservation equation in the form

$$\dot{\overline{\rho}} = \frac{1}{4} Tr[\overline{T}\eta M\eta\dot{M}\eta] \tag{3.2}$$

or, equivalently

$$\dot{\overline{\rho}} = \frac{1}{4} Tr[S\dot{M}] \ . \tag{3.3}$$

It is important to note that, according to this equation, there is no direct contribution of the dilaton field to the covariant evolution of the string-matter energy density (no dilaton-induced violation of the weak equivalence principle). This is a usual result in many scalar-tensor gravitational theories (like, for example, in Brans-Dicke gravity), but a somewhat unexpected property in our context, where the dilaton is directly coupled to the torsion part of the total Lagrangian, and the general scalar-tensor theorems (see for instance [20]) are no longer applicable.

We also note that eq. (3.3) is actually implied by the string equations of motion. This is explicitly checked by writing, upon use of eq. (2.18)

$$\frac{1}{4} Tr[S\dot{M}] = \frac{1}{8\pi\alpha'} \int d\sigma \frac{d\tau}{dX^0} Z^T \dot{M} Z = \frac{1}{4\pi\alpha'} \int d\sigma \frac{d\tau}{dX^0} (\partial_\tau^2 X^0 - X^{0\prime\prime}). \tag{3.4}$$

On the other hand, the explicit differentiation of eq. (2.10) gives

$$\dot{\overline{\rho}}(t) = \frac{1}{4\pi\alpha'} \int d\sigma \frac{d\tau}{dX^0} [\partial_\tau^2 X^0 - \partial_\tau (X'^0)^2 (\partial_\tau X^0)^{-1}] \ . \tag{3.5}$$

By using (2.31) one can easily see that the two integrands in eqs.(3.4) and (3.5) differ by a total derivative in σ (at fixed t), thus yielding eq. (3.3).

By working out explicitly the components of $\overline{T}$, the torsion contribution to the conservation equation can be separated out as follows

$$\dot{\overline{\rho}} - \frac{1}{2} Tr[(\overline{\theta} G)(G^{-1} \dot{G})] + \frac{1}{2} Tr[\overline{J} \dot{B}] = 0 \ . \tag{3.6}$$

For an isotropic, D-dimensional Friedmann–Robertson–Walker (FRW) metric, $G = a^2(t) I$, and, in the perfect fluid approximation ($\overline{\theta} G = -\overline{p} I$, where $\overline{p} = \sqrt{|G|} p$ is the isotropic pressure), eq. (3.6) takes the more familiar form

$$\dot{\rho} + (D-1) H (\rho + p) + \frac{1}{2} Tr[J \dot{B}] = 0 \ , H = \dot{a}/a, \tag{3.7}$$

which admits an interesting thermodynamical interpretation.

Let us write indeed $\rho = E/V$ and $J = \omega/V$, where E and ω are, respectively, the energy and the "torsional charge" of the source inside a proper spatial volume $V = (a\ell)^d$ ($\ell = const$). Eq. (3.7) then becomes, in differential form,

$$dE + p dV = -\frac{1}{2} \omega^{ij} dB_{ij} \ . \tag{3.8}$$

This equation suggests that, even if the source evolution is globally adiabatic, entropy exchanges occur between the perfect-fluid part and the "torsional" part of the source. In particular, a possible damping of torsion in time, $\dot{B} < 0$, should be accompanied by an entropy increase in the fluid part.

We note, finally, that the thermodynamical role played by ω_{ij} in eq. (3.8) is (formally) identical to that expected for the intrinsic vorticity tensor, in the context of a spinning-fluid model of the cosmological sources [21].

4. $O(d,d)$ transformations of the equation of state

For the microscopic model of matter sources that we are considering, based on classical strings, the source equation of state compatible with a given background is determined by the solution of the string equations of motion. It follows that, in the case of torsionless, isotropic FRW backgrounds, sources with equation of state of the perfect-fluid type are allowed, at least asymptotically, as discussed in previous papers [16]. It should be stressed, however, that the presence of shear and viscosity is in general required for a phenomenological fluid description of the sources, even when the antisymmetric tensor is vanishing. This point may be conveniently elucidated by recalling that, in the context of our model, the matter sources transform in an $O(d,d)$-covariant way, according to eq. (2.22).

Consider for example a perfect fluid, with given equation of state ($J=0$)

$$\overline{\theta}^i{}_j = -\overline{p}\delta^i_j \quad , \quad p = \gamma\rho \tag{4.1}$$

which is the source of a torsionless FRW background

$$B = 0 \quad , \quad G(t) = a^2(t) I \tag{4.2}$$

(we shall work, for simplicity, in $D = 2+1$ dimensions, so that I is the 2×2 unit matrix). We apply to $\overline{T}$ the one-parameter $O(d,d)$ transformation with

$$\Omega(\alpha) = \frac{1}{2}\begin{pmatrix} 1+c & s & c-1 & -s \\ -s & 1-c & -s & 1+c \\ c-1 & s & 1+c & -s \\ s & 1+c & s & 1-c \end{pmatrix} \tag{4.3}$$

where $c=$coshα, $s=$sinhα (previously called "boost" [14], but somewhat improperly since for $\alpha \to 0$ it reduces not to the identity, but to a matrix representing the discrete inversion of one scale factor). For the transformed sources (denoted by a tilde) we then have

$$\tilde{\overline{\rho}} = \overline{\rho} \quad , \quad \tilde{\overline{J}} = 0 \quad , \quad \tilde{\overline{\theta}}^i{}_j = -\overline{p}\tau^i{}_j \ , \tag{4.4}$$

where

$$\tau = \begin{pmatrix} c & -s \\ -s & -c \end{pmatrix} \tag{4.5}$$

A perfect-fluid interpretation of this stress tensor is not possible, as $\tilde{\theta}$ is not diagonal. For a co-moving viscous fluid, on the other hand, the stress tensor can be written in general as [22]

$$\theta^i{}_j = -(p - \xi\vartheta)\delta^i{}_j + 2\eta\sigma^i{}_j \ , \tag{4.6}$$

where ξ and η are the bulk and shear viscosity coefficients, $\vartheta = \nabla_\mu u^\mu$ (u^μ is the co-moving, geodesic velocity field), and

$$\sigma^i{}_j = \nabla^i u_j - \frac{\vartheta}{D-1}\delta^i{}_j \tag{4.7}$$

is the traceless shear tensor. For the metric obtained by applying to M the transformation (4.3):

$$\tilde{G} = \frac{1}{2ca^2}\begin{pmatrix} c(a^4+1)+a^4-1 & -s(a^4+1) \\ -s(a^4+1) & c(a^4+1)-a^4+1 \end{pmatrix}, \tag{4.8}$$

one finds that $\vartheta = 0$, and

$$\sigma^i{}_j = \Gamma_0{}^i{}_j = H\tau^i{}_j \ . \tag{4.9}$$

A comparison with eq. (4.4) shows that the transformed sources can be consistently described as a pressureless fluid with shear viscosity, characterized by the equation of state

$$\tilde{p} = 0 \quad , \quad \tilde{\eta} = -\frac{\gamma\tilde{\rho}}{2H} \ . \tag{4.10}$$

Therefore, the perfect-fluid equation of state is not an $O(d,d)$-invariant property, and viscosity is needed, in general, for a phenomenological characterization of the sources. Consistent equations of state (with viscosity) can be obtained by applying directly $O(d,d)$ transformations to a known consistent solution of the coupled string-background equations. An example worth investigating could be, for $V=0$, the background

$$\begin{aligned} &B = 0 \ , \quad \Phi = -\frac{2}{D}(D-1)ln(\pm t/t_0) \\ &G = a^2 I \quad , \quad a = (\pm t/t_0)^{\pm 2/D} \end{aligned} \tag{4.11}$$

whose source is a perfect fluid with equation of state

$$p = \pm\frac{1}{(D-1)}\rho \ . \tag{4.12}$$

Such an equation of state was shown [16] to be consistent with the string equations of motion and constraints in the backgrounds (4.11) at sufficiently small t.

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