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Pre-Big-Bang In String Cosmology

M.Gasperini
Dipartimento di Fisica Teorica dell'Università,
Via P.Giuria 1, 10125 Torino, Italy,
Istituto Nazionale di Fisica Nucleare, Sezione di Torino
and
G.Veneziano
Theory Division, Cern, Geneva, Switzerland

Abstract

The duality-type symmetries of string cosmology naturally lead us to expect a pre-big-bang phase of accelerated evolution as the dual counterpart of the decelerating expansion era of standard cosmology. Several properties of this scenario are discussed, including the possibility that it avoids the initial singularity and that it provides a large amount of inflation. We also discuss how possible tracks of the pre-big-bang era may be looked for directly in the spectral and "squeezing" properties of relic gravitons and, indirectly, in the distorsion they induce on the cosmic microwave background.

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1. Introduction

The standard cosmological model (SCM) provides the best account so far for the observed properties of our present Universe. Nevertheless, in spite of its success, the SCM cannot be extended backward in time to arbitrarily large curvatures or temperatures without running into problems. We are referring here not only to the well-known phenomenological problems (e.g. horizon, flatness, entropy), but also to what is probably the main conceptual difficulty of the SCM, the initial singularity. The aim of this paper is to present a possible alternative to the initial singularity, which arises naturally in a string cosmology context because of the "large scale-small scale" symmetry (target-space duality) typical of string theory (see Refs. [1–4] for previous pioneer work along this line).

What we shall refer to here as "*string cosmology*" is just the string-theory analogue of the usual Einstein-Friedman (EF) equations of the SCM. The string-cosmology equations are thus obtained by varying the low-energy string theory effective action -describing the massless background fields- supplemented with a phenomenological source term that accounts for possible additional bulk string matter at the classical level. As long as the equation of state of the classical sources [5] is compatible with the properties of string propagation in the massless backgrounds, the resulting system of coupled equations enjoys a much larger symmetry than that of standard cosmology [6].

This definition of string cosmology is probably insufficient for a truly "stringy" description of the Universe in the high curvature regime (near the Planck scale), where higher-order string corrections cannot be neglected, but it is certainly enough to suggest the alternative to the singularity of the SCM discussed here.

In the SCM, curvature, source energy density, and temperature all grow as we go backward in time: eventually, they blow up at the initial singularity, the so-called big bang. A possible way out of this conclusion is to postulate that the standard picture of space-time as a smooth manifold does not survive near and above the Planck scale, where some drastic modification of the classic geometry is to be introduced (see [7] for recent speculations on this subject). There are also, however, more conventional alternatives to the standard singular scenario. The two simplest possibilities are illustrated in **Fig. 1**. In the first case the curvature stops its growth after reaching a maximal value. In the second case, the curvature grows to a maximum value and then starts decreasing again as we go backward in time.

The first case corresponds to a de Sitter-like primordial inflationary phase with unlimited past extension. It is constrained by phenomenological bounds (based, as we shall see in Section 4, on the properties of the relic graviton spectrum), which imply for the final curvature scale a value at least four orders of magnitude below the Planck scale. This may seem unnatural if one believes that the curvature becomes stable just because of quantum gravity effects. Moreover, it has been recently argued [8] that eternal exponential expansion with no beginning is impossible in the context of the standard inflationary scenario. If this is so, the hypothesis of a primordial phase of constant curvature does not help to avoid the problem of the initial singularity.

The second possibility describes the alternative to the singularity suggested by the symmetries of string cosmology. In this case, as shown in Section 4, the quoted phenomenological constraints can be evaded and the curvature can reach the Planck scale. In this context the big-bang does no longer corresponds to a singularity, but to an instant of maximal curvature marking the transition from a "string-driven" growing-curvature regime (in the sense explained in Section 2) to the decelerated evolution of the standard scenario. We shall call, in general, *"pre-big-bang"* this new phase characterized by growing curvature and accelerated evolution (as one goes forward in time).

It is important to stress that the growth of curvature, during such a phase, may be due not only (and not necessarily) to the contraction of some internal space, but also to the superinflationary expansion [9] of the three physical spatial dimensions. Thus this scenario is something genuinely different from the old oscillating cosmological model, in which the present expansion is preceded by a contracting phase. Indeed, in this scenario, it is also possible for the Universe to be monotonically expanding (see Section 5). We also stress that this picture is by no means to be regarded as an alternative to the standard (even inflationary) cosmology. It is only a completion of the standard picture, which cannot be extended beyond the Planck era.

This paper is devoted to the discussion of various aspects of this possible alternative to the singularity of the SCM, and is organized as follows:

In Section 2 we shall discuss two string-theoretic motivations for the pre-big-bang scenario: one is based on the solution of the string equations of motion in a background with event horizons [5]; the second rests on a property of the low-energy string effective action, called scale-factor-duality [10–12], a particular case of a more general $O(d,d)$-covariance of the string cosmology equations [6,13,14].

In Section 3 a simple example of a string-driven pre-big-bang scenario is used to illustrate the behaviour of the density and of the temperature during the phase of growing curvature, and to discuss its possible relevance as a model of inflation.

If such a scenario is taken seriously, the important question to ask is whether observable tracks of the pre-big-bang era may survive and be available to present observations [cf. the observable tracks of the big bang, the electromagnetic cosmic microwave background (CMB)]. In Section 4 we show that these tracks may exist, and that they are to be looked for in the relic graviton background, either directly or through the distortions they induce on the CMB, perhaps not an impossible dream for the not-too-distant future.

In Section 5 we present solutions to the string cosmology equations which interpolate smoothly between the growing-curvature and the decreasing-curvature phases and in which both the curvature and the effective string coupling (the dilaton) are everywhere bounded. Although these solutions do not yet incorporate a completely self-consistent equation of state for the string sources, they represent good candidates for describing the background dynamics around the maximum curvature regime.

Our main conclusions are summarized in Section 6.

2. String theory motivations for the pre-big-bang scenario

2.1 String propagation in a background with shrinking event horizons

One argument supporting the pre-big-bang scenario is provided by the solution to the string equations of motion in a background manifold with event horizons [5]. Indeed, the curvature growth and the accelerated evolution, typical of a pre-big-bang epoch, are necessarily associated with event horizons whose proper size decreases in time. This has important consequences for objects of finite proper size, such as strings.

In order to present this argument, it is convenient to start by recalling some elementary, kinematical properties of Friedmann-Robertson-Walker spatially flat metrics:

$$g_{\mu\nu} = diag(1, -a^2(t)\delta_{ij}). \tag{2.1}$$

a is the scale factor, and $H = \dot{a}/a$ is the usual Hubble parameter, where a dot denotes differentiation with respect to the cosmic time t.

Consider the proper distance along a null geodesic

$$d(t) = a(t) \int_{t_1}^{t_2} dt' a^{-1}(t'). \tag{2.2}$$

In the limit $t_2 \to t_{Max}$, where t_{Max} is the maximal future extension of the cosmic time coordinate on the given background manifold, $d(t_1)$ defines the proper size of the *event horizon* at the time t_1 (i.e. the maximal size of the space-time region within which a causal connection can be established). In the limit $t_1 \to t_{min}$, where t_{min} is the maximal past extension of the cosmic time coordinate, $d(t_2)$ defines instead the *particle horizon* at the time t_2 (i.e. the maximal portion of space which can be included inside the past light cone of a given observer).

By means of these definitions, and by using $|H|$ as an indicator of the curvature scale, it is possible to provide a sort of "causal" classification of the isotropic and homogeneous backgrounds based on their asymptotic behaviour, which is reported in **Table I** for the expanding case, and in **Table II** for the contracting one. Note that, in both tables, the linear evolution of the horizon is referred to the cosmic time coordinate; note also the "dual" symmetry which exchanges particle with event horizon when passing from expanding to contracting backgrounds.

For the purpose of this paper, three points have to be stressed. The first point is that we are in presence of event horizons only if

$$sign\{\dot{a}\} = sign\{\ddot{a}\}, \tag{2.3}$$

that is only in the case of accelerated evolution. Inflation, on the other hand, is accelerated expansion, and inflation plus the simultaneous shrinking of some internal dimension may be self-sustained (in the absence of dominant vacuum contributions) only in the case of accelerated contraction [15]. Event horizons thus appear naturally during a phase of dimensional decoupling.

The second point to be stressed is that, for an accelerated evolution, the proper size of the regions that are initially in causal contact tends to evolve in time, asymptotically, like the scale factor $a(t)$. [This can be easily seen from eq. (2.2) by considering a causally connected region of proper initial size $d(t_1) = t_1$, in a background which is, for example, in accelerated expansion, i.e. $a \sim t^\alpha$, $\alpha > 1$ for $t \to \infty$. One then finds, for $t_2 >> t_1$, that $d(t_2) \simeq d(t_1)a(t_2)/a(t_1)$]. Comparing this behaviour with the time evolution of the event horizon, we may thus conclude that the regions which are initially in causal contact grow faster that the horizon in the case of accelerated expansion, and contract more slowly than the horizon in the case of accelerated contraction. In both cases, they will tend to cross the horizon. This is not a source of problems for point-like objects, of course, but extended sources may become, in such a situation, larger than the causal horizon itself.

The third important point is that, according to Tables I and II, the curvature is growing only if

$$sign\{\dot{a}\} = sign\{\dot{H}\}. \tag{2.4}$$

A typical pre-big-bang configuration with accelerated evolution and growing curvature is thus necessarily associated with the presence of an event horizon which shrinks linearly with respect to cosmic time: extended objects, in such backgrounds, are doomed to become causally disconnected, quite independently of their proper size.

This effect is peculiar to gravity, and can be easily understood in terms of tidal forces [16]. Consider indeed a free-falling object, of finite size λ, embedded in a cosmological background. Each point of the extended object will fall, in general, along different geodesics. In the free-falling frame of one end of the object, the other end, at a proper distance $|z| = \lambda$, will have a relative acceleration given by the equation of geodesic deviation,

$$|\frac{Du^\mu}{Ds}| = |R^\mu{}_{\nu\alpha\beta}u^\nu u^\alpha z^\beta| = \lambda|\frac{\ddot{a}}{a}|. \tag{2.5}$$

This acceleration defines a local Rindler horizon at a distance

$$d = \lambda^{-1}|\frac{\ddot{a}}{a}|^{-1}, \tag{2.6}$$

which depends only on the background curvature. If the curvature is growing, this distance becomes smaller and smaller, until the two ends become causally disconnected.

In the case of strings, the existence of a causally disconnected asymptotic regime implies that an approximate description of the string motion around the classical path of a point particle is no longer allowed [17]. Suppose indeed that the exact solution of the string equations of motion and constraints in a cosmological background is expanded around a comoving geodesic representing the point-particle motion of the centre of mass of the string [18]. Such an expansion is valid, provided [5,17]

$$|\dot{X}^i| \sim |\frac{\partial X^i}{\partial \sigma}| \tag{2.7}$$

(X^i is any spatial component of the target space coordinates, and σ is the usual world-sheet space variable) and provided the first-order fluctuations, whose Fourier components satisfy the linearized equation

$$\ddot{\chi}^i_n + (\frac{n^2}{\lambda^2} - \frac{\ddot{a}}{a})\chi^i_n = 0, \tag{2.8}$$

stay small [$\chi^i = a(t)X^i$, and $\lambda = m\alpha'$, where m is the string mass and α' the string tension].

If the background curvature is growing, this approximation breaks down asymptotically. Indeed, in the high curvature limit $|\ddot{a}/a| >> n^2\lambda^2$, the asymptotic solution to the fluctuation equation (2.8) can be written in general as

$$X^i = A + B\int\frac{dt}{a^2} \tag{2.9}$$

with A and B independent of time, and one obtains

$$|\dot{X}^i| << |\frac{\partial X^i}{\partial\sigma}| \tag{2.10}$$

in the case of expanding backgrounds, and

$$|\dot{X}^i| >> |\frac{\partial X^i}{\partial\sigma}| \tag{2.11}$$

in the contracting case.

In both cases the condition (2.7) is not satisfied, and the geodesic expansion is no longer valid, asymptotically. In both cases, however, the exact solution can be expanded around a new asymptotic configuration [5,19,20], which is "unstable", in the sense that it is non-oscillating with respect to the world-sheet time.

What is important, in our context, is that this kind of solution, when inserted into the string stress tensor, leads in general to a self-sustained scenario. In other words it leads to an effective equation of state characterizing a source which can drive, by itself, the phase of accelerated evolution and growing curvature. In the perfect fluid approximation and, in particular, for a background with d expanding and n contracting dimensions, such an equation of state takes the form [5]

$$\rho + dp - nq = 0 \tag{2.12}$$

(ρ is the energy density, p and q are respectively the pressure in the expanding space and in the internal shrinking dimensions).

According to this equation of state it is possible to obtain a "string-driven" pre-big-bang phase not only during an anisotropic situation of dimensional decoupling [5], but also in the case of isotropic expansion ($n = 0$) or isotropic contraction ($d = 0$), provided the correct string gravity equations, including the dilaton, are used instead of the Einstein equations [6]. This is to be contrasted with the case of point-like sources, where it is impossible to achieve, asymptotically, the causally disconnected regime characterized by non-oscillating configurations, and it is thus impossible to arrange a self-consistent equation of state able to sustain the pre-big-bang phase.

2.2 Scale factor duality

A second, probably stronger, motivation supporting the pre-big-bang picture follows from a property of the low-energy string effective action, which can be written in D dimensions

$$S = -\frac{1}{16\pi G_D}\int d^D x\sqrt{|g|}e^{-\phi}(R + \partial_\mu\phi\partial^\mu\phi - \frac{H^2}{12} + V) \qquad (2.13)$$

(here $H = dB$ is the antisymmetric tensor field strength and V is a constant which is vanishing in a critical number of space-time dimensions).

The cosmological equations obtained from this action for a homogeneous and spatially flat background are characterized by a symmetry called "scale factor duality" [10]. According to this symmetry, for any given set of solutions $\{a_i(t), \phi(t), 1 = 1, ..., D-1\}$ (a_i are the scale factors of a diagonal, not necessarily isotropic metric in the synchronous frame gauge $g_{00} = 1, g_{0i} = 0$), the configuration obtained through the transformation (for each i)

$$a_i \to \tilde{a}_i = a_i^{-1} \quad , \quad \phi \to \tilde{\phi} = \phi - 2\ln a_i \qquad (2.14)$$

is still a solution of the graviton-dilaton system of equations [10–12].

This transformation is just a particular case of a more general global $O(d,d)$ covariance of the theory [13,14] (d is the number of coordinates upon which the background fields are explicitly independent, in our case the $D-1$ spatial coordinates). In the case of anisotropic backgrounds with generally non-diagonal metrics, this covariance, besides transforming the dilaton field as

$$\phi \to \phi - \ln|\det g_{ij}| \qquad (2.15)$$

also mixes non-trivially the components of the metric and of the antisymmetric tensor as follows:

$$M \to \Omega^T M \Omega, \tag{2.16}$$

where Ω represents a global $O(d,d)$ transformation, and

$$M = \begin{pmatrix} G^{-1} & -G^{-1}B \\ BG^{-1} & G - BG^{-1}B \end{pmatrix} \tag{2.17}$$

($G \equiv g_{ij}$ and $B \equiv B_{ij} = -B_{ji}$ are matrix representations of the d by d spatial part of the metric and of the antisymmetric tensor, in the basis in which the $O(d,d)$ metric is in off-diagonal form [6,13]).

In the case of manifolds with spatial sections of finite volume (such as a torus), i.e. $(\int d^dx\sqrt{|G|})_{t=t_1} = const < +\infty$, this $O(d,d)$ covariance can be preserved even if the scalar poyential V is taken dilaton-dependent, provided ϕ appears in the potential only through the combination [13]

$$\overline{\phi} = \phi - \ln\sqrt{|G|} \tag{2.18}$$

(we have absorbed into ϕ the constant shift $-\ln\int d^dx$ required to secure the $GL(d)$ coordinate invariance of the corresponding non-local action).

Moreover, the $O(d,d)$ covariance holds even if the equations are supplemented by phenomenological source terms corresponding to bulk string matter [6]. Indeed, their effective equation of state can be expressed in terms of the $2d$-dimensional phase-space variables $Z^A = (P_i, \partial X^i/\partial\sigma)$, representing a solution of the string equations of motion: when the background is changed according to eq. (2.16), the new solution $\tilde{Z} = \Omega^{-1}Z$ generates a new effective equation of state which leaves the overall $O(d,d)$ symmetry unbroken [6].

The importance of scale factor duality, in our context, is that it allows the construction of a mapping relating any decreasing curvature background to a corresponding scenario in which the curvature is increasing.

It is true, indeed, that the Hubble parameter is odd under scale factor duality, $H \to -H$, so that a duality transformation $a \to a^{-1}$ maps an expanding solution of Table I into a contracting one of Table II (and vice versa). However, the cosmological solutions with monotonic evolution of the curvature (see for instance Section 3) are in general defined on a half-line in t, namely $-\infty \le t \le t_c$ and $t_c \le t \le \infty$, where t_c is some finite value of the cosmic time where a curvature singularity occurs [10]. We are thus allowed, in such a case, to change simultaneously the sign of $\dot{H}$ and $\ddot{a}$ inside a given table (i.e. keeping the sign of $\dot{a}$ constant), by performing a duality transformation and by changing, simultaneously, the domain of t from $[t_c, \infty]$ to $[-\infty, -t_c]$, i.e. by performing the inversion $t \to -t$.

This means that, to any given decelerated solution, with monotonically decreasing curvature, typical of the standard post-big-bang cosmology, it is always possible to associate a pre-big-bang solution, with accelerated evolution and increasing curvature, through the transformation

$$a(t) \to a^{-1}(-t). \tag{2.19}$$

It is important to stress that this mapping property (valid in the cases of both expanding and contracting backgrounds) cannot be achieved in the framework of Einstein's equations, where there is no dilaton. In that case, scale factor duality is broken; one is just left with the more conventional time reversal symmetry $a(t) \to a(-t)$.

Unlike straight scale-factor-duality, the transformation (2.19) has non-trivial fixed points $a(t) = a^{-1}(-t)$ describing models that are monotonically expanding (or contracting), since $H(t) = H(-t)$, and in which the duration of the primordial superinflationary evolution equals that of the dual decelerated phase, as $\dot{H}(t) = -\dot{H}(-t)$. We shall come back on this point in the following section.

3. An example of pre-big-bang dynamics

Consider a spatially flat background configuration, with vanishing antisymmetric tensor and dilaton potential. The dilaton field depends only upon time, and the metric, which is assumed to describe a phase of dimensional decoupling in which d spatial dimensions expand with scale factor $a(t)$, and n dimensions contract with scale factor $b(t)$, is given by

$$g_{\mu\nu} = diag(1, -a^2(t)\delta_{ij}, -b^2(t)\delta_{ab}) \tag{3.1}$$

(conventions: $\mu, \nu = 1, ..., D = d + n$; $i, j = 1, ..., d$; $a, b = 1, ..., n$).

We are working in a synchronous frame in which $g_{00} = 1$, $g_{0i} = 0 = g_{0a}$, so that the time parameter t coincides with the usual cosmic time coordinate. In this gauge, the stress tensor of a comoving source, in the perfect fluid approximation, becomes

$$T_\mu{}^\nu = diag(\rho(t), -p(t)\delta_i^j, -q(t)\delta_a^b), \tag{3.2}$$

where p and q are the pressures in the expanding and contracting space, respectively. It is also convenient to introduce the $O(d+n, d+n)$-invariant expressions for the dilaton and the matter energy density, that in this background take the form

$$\overline{\phi} = \phi - \ln\sqrt{|g|} = \phi - d\ln a - n\ln b \tag{3.3}$$

$$\overline{\rho} = \rho\sqrt{|g|} = \rho a^d b^n \tag{3.4}$$

(we also define, for the pressure, $\overline{p} = p\sqrt{|g|}$).

With these definitions, the equations obtained by varying the effective action (2.13) (with $V = 0$) and the action for the matter sources,

$$2(R_\mu{}^\nu + \nabla_\mu \nabla^\nu \phi) - \frac{1}{2}H_{\mu\alpha\beta}H^{\nu\alpha\beta} = 16\pi G_D e^\phi T_\mu{}^\nu \tag{3.5}$$

$$R - (\nabla_\mu\phi)^2 + 2\nabla_\mu \nabla^\mu\phi - \frac{1}{12}H_{\mu\nu\alpha}H^{\mu\nu\alpha} = 0 \tag{3.6}$$

$$\partial_\nu(\sqrt{|g|}e^{-\phi}H^{\nu\alpha\beta}) = 0 \tag{3.7}$$

(∇_μ is the Riemann covariant derivative), can be written in a particularly simple form (when $B = 0$). The dilaton equation (3.6) reduces to

$$\dot{\overline{\phi}}^2 - 2\ddot{\overline{\phi}} + dH^2 + nF^2 = 0, \tag{3.8}$$

where $H = \dot{a}/a$, $F = \dot{b}/b$. Moreover, by combining this equation with the time component of (3.5) we get

$$\dot{\overline{\phi}}^2 - dH^2 - nF^2 = 16\pi G_D\overline{\rho}e^{\overline{\phi}}. \tag{3.9}$$

From the space components of (3.5) we have finally the equations for the external and internal pressure

$$2(\dot{H} - H\dot{\overline{\phi}}) = 16\pi G_D\overline{p}e^{\overline{\phi}} \tag{3.10}$$

$$2(\dot{F} - F\dot{\overline{\phi}}) = 16\pi G_D\overline{q}e^{\overline{\phi}}. \tag{3.11}$$

[Note that the four equations (3.8) to (3.11) are only a particular case of the $O(d,d)$-covariant string cosmology equations of Ref. [6], for vanishing B,V and a perfect fluid description of the matter sources].

Suppose now that $a(t)$ describes accelerated expansion, $b(t)$ accelerated contraction, and that

$$p = \gamma_1\rho \quad , \quad q = \gamma_2\rho \quad , \quad b = a^{-\epsilon} \tag{3.12}$$

with $\gamma_1, \gamma_2, \epsilon$ constants, $\epsilon > 0$. "Stringy" sources, asymptotically consistent with such background [5], must satisfy the equation of state (2.12), which implies

$$d\gamma_1 - n\gamma_2 = -1. \tag{3.13}$$

As discussed in [5], the three possible asymptotic behaviours, $|q| << |p|$,$|q| >> |p|$ and $|q| \sim |p|$ correspond to $\epsilon < 1$, $\epsilon > 1$ and $\epsilon = 1$, respectively. In the first two cases, however, eqs. (3.8) to (3.11) can be consistently solved only for $\rho = p = q = 0$, so that one recovers the vacuum background [10] with a Kasner-like metric solution, and $\overline{\phi} \sim -\ln|t|$.

If we look for solutions with non-vanishing bulk string matter we have thus to consider the third possibility, $b = a^{-1}$. In such a case eqs. (3.10) and (3.11) imply $p = -q$, and the condition (3.13) is non-trivially satisfied by

$$\gamma_1 = -\gamma_2 = \frac{-1}{d+n}. \tag{3.14}$$

We can thus obtain, from eqs. (3.8) to (3.10), the particular exact solution

$$\begin{gathered} a(t) = b^{-1}(t) = (-\frac{t}{t_0})^{-2/(d+n+1)}, \\ \phi = \phi_0 + 2d\ \ln\ a \quad , \quad \rho = \rho_0 a^{n+1-d}, \end{gathered} \tag{3.15}$$

where t_0, ρ_0, ϕ_0 are integration constants, related by

$$\rho_0 e^{\phi_0} = 4\frac{(d+n)(d+n-1)}{(d+n+1)^2}. \tag{3.16}$$

This solution satisfies the properties

$$\begin{array}{ccccc} \dot{a} > 0 & , & \ddot{a} > 0 & , & \dot{H} > 0 \\ \dot{b} < 0 & , & \ddot{b} < 0 & , & \dot{F} < 0 \end{array} \tag{3.17}$$

Therefore, in the $t \to 0_-$ limit, it describes a background in which the evolution is accelerated, since $\{\dot{a}, \ddot{a}\}$ and $\{\dot{b}, \ddot{b}\}$ have the same sign, and the curvature is growing, since $\{\dot{a}, \dot{H}\}$ and $\{\dot{b}, \dot{F}\}$ have the same sign (recall Tables I and II). Moreover, the equation of state of the sources,

$$p = -q = -\frac{\rho}{d+n} \tag{3.18}$$

is compatible with the solution of the string equations of motion, so this background is a typical example of *string-driven pre-big-bang* cosmology.

It may be interesting to note that, in the four-dimensional isotropic limit $(n = 0, d = 3)$, the solution (3.15) simply reduces to

$$a = (\frac{-t}{t_0})^{-1/2} \quad , \quad \phi = \phi_0 - 3\ln(-\frac{t}{t_0}) \quad , \quad \rho = -3p = \rho_0(-\frac{t}{t_0}), \tag{3.19}$$

namely to the superinflationary background obtained, via the duality transformation (2.14), from the standard radiation-dominated cosmology (with constant dilaton),

$$a = (\frac{t}{t_0})^{1/2} \quad , \quad \phi = \phi_0 \quad , \quad \rho = 3p = \rho_0(\frac{t}{t_0})^{-2}, \tag{3.20}$$

as already pointed out in [6].

The simple example (3.15) can be used to illustrate two important properties of the pre-big-bang phase: not only the curvature, but also the total effective energy density and the temperature are growing together with the curvature.

For the behaviour of ρ we have indeed, from eq. (3.15):

$$\rho(t) = \rho_0(-\frac{t}{t_0})^{2(d-n-1)/(d+n+1)}. \tag{3.21}$$

Since we are working in a Brans-Dicke frame, we have to include also the time variation of the gravitational constant, $G \sim e^{\phi}$, in the term that plays the role of the total gravitational source. We thus obtain, for the effective energy density,

$$G\rho \sim \rho_0 a^{d+n+1} = \rho_0(-\frac{t}{t_0})^{-2}, \tag{3.22}$$

which is always growing for $t \to 0_-$.

As far as the temperature behaviour is concerned, it must be recalled that, according to the string cosmology equations (3.5) to (3.7), the dilaton gives no contribution to the covariant conservation of the energy of the sources (in spite of its coupling to the tensor $H_{\mu\nu\alpha}$ [6]). Moreover, when $H_{\mu\nu\alpha} = 0$, such a conservation equation defines a regime of adiabatic evolution for the perfect fluid. By using the standard thermodynamical arguments (see for instance [21]) one can thus obtain, for any given equation of state $p = \gamma\rho$, a relation between temperature and scale factor which, in the case of d isotropic spatial dimensions, takes the form

$$a(T) \sim T^{-1/d\gamma}. \tag{3.23}$$

The conservation equation, on the other hand, provides also a relation between scale factor and energy density:

$$\rho(a) \sim a^{-d(1+\gamma)}. \tag{3.24}$$

For a standard radiation-dominated background ($\gamma = 1/d$), such as that of eq. (3.2), one then finds the usual adiabatic decreasing of the temperature:

$$\rho \sim \frac{1}{a^{d+1}} \sim T^{d+1} \quad , \quad T \sim \frac{1}{a}. \tag{3.25}$$

For the dual background (3.19), on the contrary, $\gamma = -1/d$ and the temperature increases together with the scale factor,

$$\rho \sim \frac{1}{a^{d-1}} \sim \frac{1}{T^{d-1}} \quad , \quad T \sim a. \tag{3.26}$$

The same result holds in the more general anisotropic pre-big-bang background of eq. (3.15), with equation of state (3.18). Indeed in such a case the conservation equation reads

$$\dot{\overline{\rho}} = H\overline{\rho} \tag{3.27}$$

and implies

$$\rho(a) \sim a^{n+1-d}. \tag{3.28}$$

In terms of the total proper volume $V = V_d V_n = a^{d-n}$, eq. (3.27) can be rewritten as an adiabaticity condition

$$dS \equiv \frac{1}{T}d(\rho V) + \frac{1}{T}(p\frac{dV_d}{V_d} + q\frac{dV_n}{V_n})V = \frac{1}{T}d(\rho V) + \frac{p'}{T}dV = 0 \tag{3.29}$$

corresponding to the effective pressure

$$p' = p(\frac{d+n}{d-n}) = \frac{\rho}{n-d}. \tag{3.30}$$

The integrability condition $\partial^2 S/\partial V\partial T = \partial^2 S/\partial T\partial V$ then gives

$$\rho(t) \sim T^{n+1-d}, \tag{3.31}$$

which, using eq.(3.28), leads to

$$T \sim a \sim (-t)^{-2/(n+1+d)}. \tag{3.32}$$

Thus we find that, during the pre-big-bang phase, the temperature, also in the anisotropic case, grows proportionally to the expanding scale factor.

Two comments are in order:

The first is that, according to this example, the temperature seems to remain unchanged ($T = \tilde{T}$) when passing from a background a to the dual one $\tilde{a} = a^{-1}$. Indeed, from eqs. (3.25) and (3.26),

$$\tilde{T} \sim \tilde{a} = a^{-1} \sim T. \tag{3.33}$$

This behaviour is to be contrasted with recent results, obtained in a black-hole background, which suggest that the temperature should be inverted under a duality transformation [22].

We must note, however, that the pre-big-bang solution considered here is related to the decelerated post-big-bang phase not only through a duality transformation, but also through a change of the cosmic time range from $[-\infty, 0]$ to $[0, \infty]$. Moreover, what we have considered is the temperature of the sources, and not the horizon temperature as in the black hole case.

Indeed, the appearance of a dynamical event horizon during the pre-big-bang phase is not sufficient to introduce a thermal bath of geometric origin, and to define an intrinsic "background temperature". The reason is that, in contrast with the Rindler or de Sitter case, the Green functions for fields embedded in a manifold with shrinking horizons are not, in general, periodic with respect to the Euclidean time coordinate. In other words, a De Witt detector [23] in the frame of a comoving pre-big-bang observer will measure a particle background, but the response function of the detector is not of the thermal type and cannot define an intrinsic temperature.

Moreover, even introducing (naïvely) a temperature associated with the pre-big-bang event horizon in terms of its local surface gravity, $T_H \sim |t|^{-1}$, it would be impossible in this cosmological context to discuss the transformation of the temperature under duality by using the horizon thermodynamics, because there is no event horizon in the dual, decelerated phase.

It is possible, however, to obtain a thermal spectrum from the geometry, in a cosmological context, by considering the high frequency sector of the particle spectrum produced by a background that evolves smoothly between two asymptotically flat states [23–25]. In this way one can associate a temperature with the geometry, independently from the presence of event horizons, and one can easily show that for a "self-dual" metric [i.e. a metric that satisfies $a^{-1}(t) = a(-t)$, see for instance Section. 5], this "geometric" temperature is inverted under duality, just as in the black hole case.

The demonstration is based on the properties of the Bogoliubov coefficients, $c_+(k)$ and $c_-(k)$, parametrizing for each mode k the transformation between the $|in>$ and $|out>$ vacuum, and normalized in such a way that $|c_-(k)|^2 =< n_k >$ measures the spectral number of produced particles, and $|c_+|^2 = 1 + |c_-|^2$.

Consider indeed the transformation $t \to -t$, connecting an expanding to a contracting background. The $|in>$ and $|out>$ states get interchanged, but the constant equilibrium temperature T_0 associated with the thermal distribution of the produced particles is preserved, as it depends on the norm of the Bogoliubov coefficients, which is left unchanged

$$|c_+| = | < in, +|out, + > | \quad , \quad |c_-| = | < in, +|out, - > |. \tag{3.34}$$

The local temperature $T(t) = T_0/a(t)$, characteristic of the given geometry, changes however from a regime of adiabatic red-shift to an adiabatic blue-shift, $T(t) \to T(-t) = T_0/a(-t)$. If the metric is self-dual we obtain, in units of T_0,

$$T(a(-t)) = T(a^{-1}(t)) \equiv \tilde{T}(t) = a(t) = \frac{1}{T(a(t))} \tag{3.35}$$

so that the temperature and its dual $\tilde{T}$ satisfy

$$T\tilde{T} = T_0^2 = const, \tag{3.36}$$

in agreement with earlier arguments of string thermodynamics in a cosmological context [2,3].

The second comment regards the growth of the temperature of the source during the pre-big-bang phase [see eqs. (3.25) and (3.26)]. In the regime of radiation-dominated expansion, aT stays constant, and the usual way to get the very large present value of the entropy, starting from reasonable initial conditions, is through the occurrence of some non-adiabatic "reheating" era. In the dual pre-big-bang regime, on the contrary, the temperature of the sources grows with the scale factor just because of the adiabatic evolution. It is thus possible (at least in principle) for the Universe to emerge from the big-bang with hot enough sources and large enough scale factor to solve the entropy and the horizon problems.

Consider, for example, a cosmological evolution which is symmetric around the phase of maximal curvature, in the sense that the post-big-bang decelerated expansion of the three-dimensional space [eq. (3.20)] is preceeded by a corresponding phase of accelerated expansion [eq. (3.19)], which extends in time as long as the dual decelerated phase.

The pre-big-bang superinflation ends at the maximum curvature scale, which is expected to be of the order of the Planck mass, $H_f \sim M_P$. On the other hand, our present curvature scale H_0 is about 60 orders of magnitude below the Planck scale. In the case of a symmetric evolution evolution around the maximum (see e.g. the example presented in Section 5), the pre-big-bang superinflation era must start at a scale $H_i \lesssim H_0 \sim 10^{-60} M_P$, thus providing a total amount of inflation

$$\frac{a_f}{a_i} \sim (\frac{H_f}{H_i})^{1/2} \gtrsim 10^{30}, \tag{3.37}$$

which is just the one required for the solution of the standard cosmological puzzles [26].

Apart from this amusing numerical coincidence, what is important to stress is that in the case of a symmetric temporal extension the pre-big-bang scenario seems capable of providing automatically the required amount of inflation, *for any given value of the final observation time* t_0, thus avoiding the anthropic problem that is in general included in the more conventional, post-big-bang, inflationary models (see [27] for a lucid discussion of the anthropic principle in an inflationary context).

It should be clear, however, that even in this context the large present value of the total entropy would be the consequence of a non-adiabatic conversion of the hot string gas into hot radiation, occurring during the transition to the standard scenario. A possible example of such process is the decay of highly excited string states, created by the background evolution through the mechanism described (for the case of gravitons) in the following section. One should also mention, in this context, the existence of an additional source of radiation entropy due to the presence of non-trivial antisymmetric tensor backgrounds. Indeed, even if the evolution is globally adiabatic, entropy exchanges may occur between B and the fluid part of the source, because of the covariant conservation of the total energy density. As a consequence, the damping of B in time (which is expected to occur if the standard isotropic scenario is to be recovered) should be accompanied by an entropy increased in the radiation fluid, as already stressed in [6].

4. Observable tracks of a pre-big-bang phase

As discussed in the previous sections, there are various reasons for which the occurrence of a period of accelerated evolution and growing curvature, in the past history of our Universe, may seem both plausible (from a string theory point of view) and attractive (for its phenomenological aspects). The important question to ask, therefore, is whether the occurrence of such a phase may be tested experimentally in some way.

An affirmative answer to this question is provided by the study of the cosmic graviton background [28–30]: indeed, gravitons decoupled from matter earlier than any other field (nearly at the Planck scale), so that the shape of the spectrum of the gravitons produced by the transition from the pre-big-bang to the post-big-bang scenario should survive, nearly unchanged, up to the present time. Given a model of cosmological evolution, in particular, one can compute the expected graviton spectrum, and then constrain the model by using the present observational bounds.

Consider, for example, a generic model of pre-big-bang evolution in which the dilaton is growing, d dimensions expand with scale factor $a(\eta)$ and n dimensions contract with scale factor $b(\eta)$ in such a way that, for $\eta < -\eta_1 < 0$,

$$a \sim (-\eta)^{-\alpha} \quad , \quad b \sim (-\eta)^{\beta} \quad , \quad \phi \sim \gamma \ln a, \tag{4.1}$$

where η is the conformal time coordinate, related to the cosmic time t by $dt/d\eta = a$ (we shall denote, in this section, derivatives with respect to η with a prime). Note that in eq. (4.1) η ranges over negative values, so that α, β and γ are all positive parameters. We shall assume that this phase is followed (at $\eta = -\eta_1$) by the standard radiation-dominated, and matter-dominated (at $\eta = \eta_2$), expansion of three spatial dimensions, with frozen dilaton and radius of the internal space, namely

$$\begin{aligned} &a \sim \eta \quad , \quad b = 1 \quad , \quad \phi = \phi_0 \quad , \quad -\eta_1 < \eta < \eta_2, \\ &a \sim \eta^2 \quad , \quad b = 1 \quad , \quad \phi = \phi_0 \quad , \quad \eta > \eta_2 > 0. \end{aligned} \tag{4.2}$$

The spectrum of the gravitons produced from the vacuum, because of the background variation, is to be computed from the free propagation equation of a metric fluctuation, $h_{\mu\nu} = \delta g_{\mu\nu}$, obtained by perturbing the background equations at fixed sources [31–33] (in our case dilaton and antisymmetric tensor included), $\delta T_\mu^\nu = \delta\phi = \delta H_{\mu\nu\alpha} = 0$. The contribution to the graviton production from a possible variation of the effective gravitational coupling can be accounted for, in general, by perturbing a Brans-Dicke background [34]. In our case we shall perturb the string gravity equations (3.5) to (3.7) around the cosmological configuration $\{H_{\mu\nu\alpha} = 0, \phi = \phi(t), g_{\mu\nu} = diag(1, g_{ij}, g_{ab})\}$, where $g_{ij} = -a^2\delta_{ij}$ and $g_{ab} = -b^2\delta_{ab}$ are the d- and n-dimensional conformally flat metrics of the expanding and contracting manifolds, respectively.

By imposing the gauge

$$h_{0\mu} = 0 \quad , \quad \nabla_\nu h_\mu{}^\nu = 0 \quad , \quad g^{\mu\nu} h_{\mu\nu} = 0, \tag{4.3}$$

and by considering a perturbation $h_i{}^j(\vec{x}, t)$ propagating in the expanding "external" space, one then gets the linearized equation [34]

$$g^{\mu\nu} \nabla_\mu \nabla_\nu h_i{}^j - \dot{\phi} \dot{h}_i{}^j = 0, \tag{4.4}$$

where ∇_μ denotes the covariant derivative with respect to the background metric g. By performing a Fourier expansion, and introducing the variable

$$\psi_i{}^j = h_i{}^j b^{n/2} a^{(d-1)/2} e^{-\phi/2}, \tag{4.5}$$

one can conveniently rewrite eq. (4.4), for each mode $\psi_i{}^j(k)$ in terms of the conformal time variable, in the form:

$$\psi''(k) + [k^2 - V(\eta)]\psi(k) = 0, \tag{4.6}$$

where

$$\begin{aligned} V(\eta) = {} & \frac{d-2}{2}\frac{a''}{a} + \frac{n}{2}\frac{b''}{b} - \frac{\phi''}{2} + \frac{1}{4}(d-1)(d-3)(\frac{a'}{a})^2 + \frac{n}{4}(n-2)(\frac{b'}{b})^2 \\ & + \frac{1}{4}\phi'^2 + \frac{n}{2}(d-1)\frac{a'}{a}\frac{b'}{b} - \frac{1}{2}(d-1)\frac{a'}{a}\phi' - \frac{n}{2}\frac{b'}{b}\phi' \end{aligned} \tag{4.7}$$

For the phenomenological background (4.1), (4.2) we thus have the effective potential barrier

$$\begin{aligned} V(\eta) &= \frac{1}{4\eta^2}\{[\alpha(d-1-\gamma) - n\beta + 1]^2 - 1\} \quad , \quad \eta < -\eta_1 \\ V(\eta) &= 0 \quad , \quad -\eta_1 < \eta < \eta_2 \\ V(\eta) &= \frac{2}{\eta^2} \quad , \quad \eta > \eta_2, \end{aligned} \tag{4.8}$$

which clearly displays the usual contribution of the accelerated expansion ($\alpha \neq 0$) [28,29], of the dimensional reduction process ($n\beta \neq 0$) [35,36] and, in addition, of the dilaton variation ($\gamma \neq 0$) [34].

The general solution of eq. (4.6) can be written in terms of the first- and second-kind Hankel functions $H_\nu^{(1)}(k\eta), H_\nu^{(2)}(k\eta)$, of index

$$\nu = \frac{1}{2}[\alpha(d-1-\gamma) - n\beta + 1] \tag{4.9}$$

during the pre-big-bang phase, and of index $|\nu| = 1/2$ and $|\nu| = 3/2$ during the radiation- and matter-dominated phase, respectively. Starting with the initial solution

$$\psi(k) = N\eta^{1/2} H_\nu^{(2)}(k\eta) \quad , \quad \eta < -\eta_1 \tag{4.10}$$

representing positive frequency modes in the $\eta \to -\infty$ limit (and corresponding to the Bunch-Davies "conformal" vacuum [23]), one has in general a linear combination of positive ($H^{(2)}$) and negative ($H^{(1)}$) frequency solutions for $\eta \to \infty$.

The Bogoliubov coefficients $c_{\pm}(k)$, describing such mixing, can be obtained by matching the solutions at $\eta = \eta_1$ and $\eta = \eta_2$ ($\eta_2 >> \eta_1$).

For $k\eta_1 < 1$, using the small argument limit of the Hankel functions, and considering only the first background transition, this "sudden" approximation gives

$$|c_-(k)| \simeq (k\eta_1)^{-|\nu+1/2|} \tag{4.11}$$

(we neglect numerical factors of order unity). For $k\eta_1 \gtrsim 1$, i.e. for modes with a comoving frequency k higher than the potential barrier $[V(\eta_1)]^{1/2} \simeq \eta_1^{-1}$, this approximation is no longer valid; then the mixing coefficient c_- is to be computed by replacing the potential step with a smooth transition from $V(\eta_1)$ to 0 (in order to avoid ultraviolet divergences). In this way one finds, however, that for $k\eta_1 > 1$ the mixing is exponentially suppressed [23,25,35], so that it may be neglected for our purposes.

The modes with sufficiently small frequency $k < 1/\eta_2$ are significantly affected also by the second background transition, from the radiation-dominated to the matter-dominated regime [28,37-39]. In the same approximation one finds that, for this frequency sector,

$$|c_-(k)| \simeq (k\eta_1)^{-|\nu+1/2|}(k\eta_2)^{\mp 1}, \tag{4.12}$$

where the sign of the second exponent is -1 ($+1$) if $\nu + 1/2 > 0$ (< 0) [34]. For all the models of dynamical dimensional decoupling proposed up to now (see for instance [5,15,40]), including the pre-big-bang example of the previous section, one has however $\nu + 1/2 > 0$. We shall thus disregard here, for simplicity, the alternative possibility (but we note that for $\nu + 1/2 < 0$ the phenomenological constraints on the graviton spectrum that we shall present below turn out to be somewhat relaxed [34]).

The spectral energy density $\rho(\omega) = \omega(d\rho_g/d\omega)$, which is the variable usually adopted to characterize the energy distribution of the produced gravitons [28,29,39], is simply related to the Bogoliubov coefficient $c_-(k)$ that gives, for each mode, the final number of produced particles:

$$\rho(\omega) \simeq \omega^4 |c_-|^2, \tag{4.13}$$

where $\omega = k/a(t)$ is the proper frequency (again we neglect numerical factors of order unity). On the other hand, $|\eta|^{-1} \simeq |a(\eta)H(\eta)|$; moreover, η_1 corresponds to the beginning of the radiation era, so we can conveniently express the maximum curvature scale $H_1 \equiv H(\eta_1)$, reached at the end of the pre-big-bang phase, as $H_1^2 \simeq G\rho_\gamma(\eta_1)$, where G is the usual Newton constant, and ρ_γ is the radiation energy density [29]. It follows that the spectral energy density (4.13), at the present observation time t_0, and in units of critical energy density ρ_c, can be finally expressed as

$$\begin{aligned}
\Omega(\omega,t_0) &\equiv \frac{\rho(\omega)}{\rho_c} \simeq GH_1^2\Omega_\gamma(t_0)(\frac{\omega}{\omega_1})^{4-2|\nu+1/2|} \quad , \qquad \omega_2<\omega<\omega_1 \\
\Omega(\omega,t_0) &\equiv \frac{\rho(\omega)}{\rho_c} \simeq GH_1^2\Omega_\gamma(t_0)(\frac{\omega}{\omega_1})^{4-2|\nu+1/2|}(\frac{\omega}{\omega_2})^{-2} \quad , \quad \omega_0<\omega<\omega_2
\end{aligned} \tag{4.14}$$

Here $\Omega_\gamma(t_0) \sim 10^{-4}$ is the fraction of critical energy density present today in radiation form; $\omega_0 \sim H_0 \sim 10^{-18}$ Hz is the minimal frequency inside the present Hubble radius;

$$\omega_2 = \frac{H_2 a_2}{H_0 a_0}\omega_0 \sim 10^2\omega_0 \tag{4.15}$$

is the frequency corresponding to the radiation $\rightarrow$ matter transition and, finally,

$$\omega_1 = \frac{H_1 a_1}{H_0 a_0}\omega_0 \sim 10^{29}(\frac{H_1}{M_P})^{1/2}\omega_0 \tag{4.16}$$

is the maximum cut-off frequency depending on the height of the potential barrier $V(\eta_1)$, i.e. on the big-bang curvature scale H_1.

The two important parameters of the spectrum (4.14) are the maximum scale H_1 and the power ν which depends on the pre-big-bang kinematics and fixes the frequency behaviour of the energy density of the relic gravitons. In four dimensions ($d=3, n=0$), and in the absence of dilaton contributions ($\gamma=0$), the high-frequency behaviour of the spectrum mimics exactly the behaviour of the curvature scale for $\eta < -\eta_1$, as was stressed in [29,39] (we recall indeed that in such a case $4-2|\nu+1/2| = 2-2\alpha$, so that a de Sitter phase at constant curvature, $\alpha=1$, corresponds to a flat spectrum). If, however, the other contributions to the graviton production are included, then the total spectrum may be flat or decreasing even if the curvature is growing.

What is important to stress is that for a flat or decreasing spectrum the most significant bound is provided by the isotropy of the electromagnetic background radiation [28], $\Omega < \Omega_i$, while if the spectrum is growing fast enough it is only constrained by the critical density bound [34,39], $\Omega < \Omega_c$. The first bound turns out to be most constraining if imposed at the minimum frequency ω_0, the second bound at the maximum frequency ω_1 (there is also a bound obtained from the pulsar-timing data [28], to be imposed at a frequency $\omega \sim 10^{-8}$ Hz, but it is not significant in our context [34]).

The two constraints

$$\Omega(\omega_0) < \Omega_i \qquad , \qquad \Omega(\omega_1) < \Omega_c \tag{4.17}$$

define an allowed region in the (H_1, ν) plane, which is bounded by the curves [obtained from eq.(4.14)]

$$\begin{aligned} &Log(\frac{H_1}{M_P}) < 2 + \frac{1}{2} Log\Omega_c, \\ &Log(\frac{H_1}{M_P}) < \frac{1}{|\nu + 1/2|}(116 + Log\Omega_i) - 58. \end{aligned} \tag{4.18}$$

The region allowed by the present experimental data ($\Omega_c \sim 1, \Omega_i \sim 10^{-8}$) is shown in **Fig. 2**.

Because of the many approximations made, and of the uncertainties in the experimental data, this figure is expected to give, of course, only a qualitative picture of the possible phenomenological scenario. We can see, nevertheless, that there is a maximum allowed scale of curvature, $H_1 \sim 10^2 M_p$, which can be reached only if the spectrum is growing fast enough, $|\nu + 1/2| \lesssim 1.8$. The general bound on the pre-big-bang kinematics to allow a given final scale $H_1 \leq 10^2 M_P$, in particular, is given by eq. (4.18) as

$$|\nu + \frac{1}{2}| \leq \frac{108}{58 + Log(H_1/M_P)}. \tag{4.19}$$

The Planck scale, in particular, can be reached if $|\nu + 1/2| \lesssim 1.88$. Flat or decreasing spectra, $|\nu + 1/2| \geq 2$, can only be extended to a maximum scale $H_1 \lesssim 10^{-4} M_P$. One thus recovers the well-known bound [41] (already quoted in Section 1) that applies to a four-dimensional primordial phase of the de Sitter type (which has $\nu + 1/2 = 2$, and thus corresponds to a flat spectrum).

This bound is avoided by the pre-big-bang example given in Section .2. In that case

$$\alpha = \beta = \frac{2}{3 + d + n} \qquad , \qquad \gamma = 2d, \tag{4.20}$$

and one gets an increasing-with-energy graviton spectrum with

$$\nu + \frac{1}{2} = \frac{2}{3 + d + n}. \tag{4.21}$$

As a result, the curvature can reach the maximum allowed scale for any number of dimensions.

The situation that we have discussed refers to the case of a "sudden" transition from the growing curvature regime to the decreasing one. The transition, however, could pass through an intermediate de Sitter phase of maximal constant curvature. In such a case, the high-frequency part of the graviton spectrum is flattened, while the lower-frequency part continues to grow. It is still possible to evade the bound $H_1 \lesssim 10^{-4} M_P$, and to consider a de Sitter inflation occurring at the Planck scale, but its duration turns out to be constrained by the present pulsar-timing data, in such a way that $10^{-1} M_P$ is practically the maximum allowed value of H_1 in such a Planck-scale inflation [39].

Note that, in this paper, we are using the data on the isotropy of the electromagnetic CMB only as a bound on the possible presence of a relic graviton background, since we are mainly interested in the limiting value of the curvature for a phase of pre-big-bang evolution. It is also possible, however, to interpret the CMB anisotropy recently detected by COBE [42] as entirely (or at least partially, but significantly) due to a stochastic background of cosmic gravitons [43,34]. By making such an assumption, and by using the graviton spectrum corresponding to our pre-big-bang model (4.21), the COBE result can be read as providing an interesting relation between the maximum scale H_1 and the total number of spatial dimensions $d + n$.

We should mention, finally, that an additional phenomenological signature of a pre-big-bang phase is contained in the squeezing parameter $r(\omega)$ which characterizes the quantum state of the relic cosmic gravitons [44], produced from the vacuum by the pre-big-bang $\rightarrow$ post-big-bang transition.

Such a parameter can be approximately expressed in terms of the Bogoliubov coefficients as [34,44]

$$r(\omega) \simeq \ln |c_-|. \tag{4.22}$$

For the high-frequency part of the spectrum ($\omega > \omega_2$) we thus find, from eq. (4.11),

$$r(\omega) \simeq |\nu + \frac{1}{2}|[25 - \ln(\frac{\omega}{Hz}) + \frac{1}{2}\ln(\frac{H_1}{M_P})]. \tag{4.23}$$

The first term on the right-hand side is expected to be dominant (at least in the presently allowed frequency range for a possible graviton detection); the second term determines the variation with frequency of the squeezing, and the last term provides a correction if the final curvature scale differs from the Planck one. In the case of a future *direct* observation of the cosmic graviton background, a measure of the parameter $r(\omega)$ would provide information both on the maximum curvature scale H_1 and on the kinematics of a possible pre-big-bang phase.

5. The high curvature regime

In the examples of pre-big-bang scenarios so far considered the growth of the curvature, as well as that of the effective coupling e^{ϕ}, were unbounded [see for instance eq. (3.15)]. According to the phenomenological constraints discussed in the previous section, however, this growth must stop, at the latest around the Planck scale. A realistic model for this scenario should thus describe also a smooth transition from the growing to the decreasing curvature phase, avoiding the singularity both in the curvature and in the dilaton field.

Of course, in the high-curvature regime in which one approaches the Hagedorn and Planck scale, string corrections (of order α' and higher) are expected to modify the naïve low-energy effective action (2.13). Moreover, in such regime a more realistic cosmology is probably obtained by including the contributions of a dilaton potential $V(\phi)$, and also of the antisymmetric tensor field. Indeed, there are regular examples (without sources) in which both a constant dilaton potential [12] and an $O(d,d)$-generated torsion background [45] contribute to violate the strong energy condition and to avoid the singularity.

Let us resort, therefore, to the full $O(d,d)$-covariant equations of string cosmology, obtained from the action (2.13) (supplemented by a phenomenological source term describing bulk string matter). Let us assume, furthermore, that the d-dimensional space has finite volume (even if flat, for simplicity) such as a torus. It is thus possible to make the $O(d,d)$ covariance of the background fields compatible with the $GL(d)$ coordinate invariance, even in the presence of a non-trivial dilaton potential, provided $V = V(\overline{\phi})$, where $\overline{\phi}$ is as defined in eq. (2.18). In such case the field equations, obtained from the total (generally non-local) action, can be written in explicit $O(d,d)$-covariant form as [6]

$$\dot{\overline{\phi}}^2 - 2\ddot{\overline{\phi}} - \frac{1}{8}Tr(\dot{M}\eta)^2 + \frac{\partial V}{\partial\overline{\phi}} - V = 0 \tag{5.1}$$

$$\dot{\overline{\phi}}^2 + \frac{1}{8}Tr(\dot{M}\eta)^2 - V = \overline{\rho}e^{\overline{\phi}} \tag{5.2}$$

$$(e^{-\overline{\phi}}M\eta\dot{M})^{\cdot} = \overline{T} \tag{5.3}$$

where M is the matrix defined in eq. (2.17), $\overline{T}$ is a $2d \times 2d$ matrix representing the spatial part of the string stress tensor (including the possible antisymmetric current density, source of torsion [6]), and η is the $O(d,d)$ metric in off-diagonal form

$$\eta = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} \tag{5.4}$$

(Note that, throughout this section, both $\overline{T}$ and $\overline{\rho}$ are expressed in units of $16\pi G_D$, so that they have dimensions L^{-2}.)

These three equations generalize, respectively, the dilaton equation (3.8), and the time and space part of the graviton equations, (3.9) to (3.11), to which they reduce exactly for $V=0$, vanishing torsion, and diagonal metric background. Their combination provides the useful covariant conservation equation of the source energy density, which can be written in compact form as [6]

$$\dot{\overline{\rho}} + \frac{1}{4} Tr(\overline{T}\eta M \eta \dot{M} \eta) = 0. \tag{5.5}$$

As in ordinary cosmology, this system of equations must be supplemented by an equation of state, which in this context reads as a relation of the form $\overline{T} = \overline{T}(\overline{\rho}, M)$. In order to provide an explicit example of regular pre-big-bang scenarios, we note that the variables of eqs. (5.1) to (5.3) can be separated, and the equations can be solved, for any given $\overline{T}/\overline{\rho}$, provided

$$\frac{\partial V}{\partial \overline{\phi}} = 2V. \tag{5.6}$$

Indeed, using this condition, the combination of eqs. (5.1) and (5.2) gives

$$(e^{-\overline{\phi}})^{\cdot\cdot} = \frac{1}{2}\overline{\rho}. \tag{5.7}$$

Defining a time coordinate ξ such that

$$d\xi = \frac{\overline{\rho}}{\rho_0} dt \tag{5.8}$$

(ρ_0 is a constant, and we shall denote with a prime differentiation with respect to ξ), eqs. (5.7) and (5.3) can then be easily integrated a first time to give

$$\overline{\rho}(e^{-\overline{\phi}})' = \frac{1}{2}\rho_0^2(\xi + \xi_0) \tag{5.9}$$

$$\overline{\rho} M \eta M' = \rho_0^2 \Gamma e^{\overline{\phi}}. \tag{5.10}$$

where

$$\Gamma = \int \frac{\overline{T}}{\overline{\rho}} d\xi \tag{5.11}$$

(ξ_0 is an integration constant).

On the other hand, by using (5.10) to eliminate $M\eta\dot{M}$, and by noting that $\overline{T}/\overline{\rho} = \Gamma'$, we can rewrite the conservation equation as

$$\overline{\rho}' e^{-\overline{\phi}} = -\frac{1}{8}\rho_0^2 Tr(\Gamma\eta\Gamma\eta)'. \tag{5.12}$$

By adding eqs. (5.9) and (5.12), integrating a second time and defining (for later convenience)

$$D(\xi) = 4\beta + (\xi + \xi_0)^2 - \frac{1}{2}Tr(\Gamma\eta)^2 \tag{5.13}$$

(β is an integration constant), we obtain finally

$$\overline{\rho} e^{-\overline{\phi}} = \frac{1}{4} D\rho_0^2, \tag{5.14}$$

which, inserted into (5.9) and (5.10), gives

$$\overline{\phi}' = -\frac{2}{D}(\xi + \xi_0) \tag{5.15}$$

$$M\eta M' = \frac{4\Gamma}{D}. \tag{5.16}$$

We have thus satisfied eq. (5.3), and a combination of eqs. (5.1) and (5.2). We must still impose eq. (5.2) which, using eqs. (5.15) and (5.16) and the identity [6]

$$(M\eta M'\eta)^2 = -(M'\eta)^2 \tag{5.17}$$

reduces to the condition

$$(\xi + \xi_0)^2 - \frac{1}{2}Tr(\Gamma\eta)^2 = D + \frac{VD^2}{4}(\frac{\rho_0}{\overline{\rho}})^2. \tag{5.18}$$

This condition, together with eq. (5.6), can be satisfied in two ways. The first way is trivial, $V = 0$ and $\beta = 0$. The equations (5.15) and (5.16) can then be integrated for any given $\Gamma(\xi)$, but the singularity in this case cannot be avoided, because of the zeros of $D(\xi)$.

The second possibility corresponds to a non-trivial two-loop potential $V(\overline{\phi}) = -V_0 e^{2\overline{\phi}} < 0$, with $\beta = V_0$. In this case too eqs. (5.6) and (5.18) are both satisfied, and we can obtain examples of backgrounds that are exact solutions of the string cosmology equations (5.14)–(5.16) (with $\beta = V_0$), and which describe a smooth evolution from asymptotically growing to asymptotically decreasing curvature, without singularities. We shall give some examples below.

Let us consider, first of all, a d-dimensional isotropic background, with scale factor a, vanishing torsion ($B = 0$), and sources with a diagonal stress-tensor, so that [6]

$$M\eta M' = 2\frac{a'}{a}\begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix} \quad , \quad \overline{T} = \overline{p}\begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}, \tag{5.19}$$

where I is the d-dimensional identity matrix. By defining $\overline{p}/\overline{\rho} = \gamma$ we thus have

$$D = 4V_0 + (\xi + \xi_0)^2 - d(\int \gamma d\xi)^2. \tag{5.20}$$

The integration of the cosmological equations is thus immediate in the case of a perfect fluid source, with $\gamma = const.$ Consider for example the interesting case of the radiation-like solution,

$$\gamma = \frac{1}{d} \quad , \quad \int \gamma d\xi = \frac{1}{d}(\xi + \xi_1) \quad , \quad \xi_1 = const. \, , \tag{5.21}$$

and define, for convenience, the constant parameters

$$\alpha = \frac{d-1}{d} \quad , \quad b = 2(\frac{\xi_0}{\xi_1} - \frac{1}{d}) \quad , \quad c = \frac{4V_0 + \xi_0^2}{\xi_1^2} - \frac{1}{d}. \tag{5.22}$$

By choosing the arbitrary constants ξ_0 and ξ_1 in such a way that

$$\Delta^2 \equiv 4\alpha c - b^2 > 0 \tag{5.23}$$

the integration of eqs. (5.15) and (5.16) then provides the non-singular solution

$$\overline{\phi} = \phi_0 + \ln(\alpha\frac{\xi^2}{\xi_1^2} + b\frac{\xi}{\xi_1} + c)^{-\frac{d}{d-1}} - \frac{4(\xi_1 - \xi_0)}{(d-1)\Delta\xi_1}\arctan(\frac{2\alpha\xi + b\xi_1}{\Delta\xi_1}) \tag{5.24}$$

$$a = a_0(\alpha\frac{\xi^2}{\xi_1^2} + b\frac{\xi}{\xi_1} + c)^{\frac{1}{d-1}}\exp\{\frac{4(\xi_1 - \xi_0)}{(d-1)\Delta\xi_1}\arctan(\frac{2\alpha\xi + b\xi_1}{\Delta\xi_1})\} \tag{5.25}$$

(a_0 and ϕ_0 are dimensionless integration constants). Their combination gives

$$e^{\phi} = a_0^d e^{\phi_0}\exp\{\frac{4(\xi_1 - \xi_0)}{(d-1)\Delta\xi_1}\arctan(\frac{2\alpha\xi + b\xi_1}{\Delta\xi_1})\}, \tag{5.26}$$

which is growing for $\xi_1 > \xi_0$.

Note that the dilaton potential $V(\overline{\phi})$ provides its strongest contribution just in correspondence of the phase of maximum curvature, while it rapidly decays to zero away from this regime. We thushave an example of "running cosmological constant" $\Lambda = V(\overline{\phi})$, which may suggest a new possible approach to the cosmological constant problem based on the damping of Λ obtained from a non-local effective action (see also [46]).

It is also interesting to point out that, in the case of a radiation-like source, the time coordinate ξ coincides with the conformal time η. From eq. (5.14) we indeed obtain, for $\overline{\rho}$

$$\overline{\rho} = \frac{\xi_1^2 \rho_0^2 e^{\phi_0}}{4(\alpha\frac{\xi^2}{\xi_1^2} + b\frac{\xi}{\xi_1} + c)^{\frac{1}{d-1}}} \exp\{-\frac{4(\xi_1 - \xi_0)}{(d-1)\Delta\xi_1} \arctan(\frac{2\alpha\xi + b\xi_1}{\Delta\xi_1})\}, \tag{5.27}$$

so that, from the definition of ξ,

$$\frac{d\xi}{dt} = \frac{\overline{\rho}}{\rho_0} = \frac{1}{4}\xi_1^2 \rho_0 e^{\phi_0}(\frac{a_0}{a}). \tag{5.28}$$

Equations (5.25) and (5.26) describe a radiation-dominated solution, which interpolates without singularities between an initial asymptotic phase of *accelerated contraction*, $a \sim (-t)^{2/(d+1)}$, $\phi = \phi_1 = const.$, *growing curvature*, and a final phase of *decelerated expansion*, $a \sim t^{2/(d+1)}$, $\phi = \phi_2 = const.$, *decreasing curvature*. There are no horizons, and both the curvature and the dilaton field are regular.

In this example the final expanding state is reached starting from an initial contraction. There is, however, also the dual solution $\tilde{\phi}, \tilde{a}$ corresponding to the equation of state

$$\overline{p} = -\frac{\overline{\rho}}{d} \quad , \quad \int \gamma d\tilde{\xi} = -\frac{1}{d}(\tilde{\xi} + \xi_1), \tag{5.29}$$

where $\tilde{\xi}$ is the "dual" time coordinate defined by eq. (5.8) for this new solution. From eqs.(5.14)–(5.16) one obtains, in terms of $\tilde{\xi}$, that the new solution $\{\tilde{\phi}, \tilde{a}\}$ is related to the old one $\{\phi, a\}$ by

$$\begin{aligned} \tilde{\overline{\phi}} &= \overline{\phi}(\tilde{\xi}) \\ (\frac{\tilde{a}}{a_0}) &= a_0 a^{-1}(\tilde{\xi}) \\ \tilde{\overline{\rho}} &= \frac{1}{4}\xi_1^2\rho_0^2 e^{\phi_0}\frac{a_0}{a(\tilde{\xi})} = \frac{1}{4}\rho_0^2 e^{\phi_0}(\frac{\tilde{a}}{a_0}) \end{aligned} \quad , \tag{5.30}$$

where $d\tilde{\xi} \sim \tilde{a}dt$. In this solution, therefore, the background evolves smoothly from an initial accelerated phase of *superinflationary expansion*, $a \sim (-t)^{-2/(d+1)}$, with a logarithmically *increasing dilaton*, towards a final asymptotic phase of *decelerated contraction*, $a \sim t^{-2/(d+1)}$, a logarithmically *decreasing dilaton*, and *decreasing curvature*. Again the maximum curvature regime is crossed over without singularities.

These two kinds of solution smoothly connect the pre-big-bang phase to the post-big-bang one by changing the sign of H (from expansion to contraction, and vice-versa). Their existence may suggests a possible scenario in which the transition to the standard cosmology occurs after a period of background oscillations [11,47], namely a series of anisotropic contractions and expansions in which "external" and "internal" coordinates exchange their roles periodically. Such a scenario may also recall the Kasner-like oscillating behaviour, typical in general relativity of a cosmological metric approaching the initial singularity [48], with the only difference that in our case the singularity is smoothed out by a phase of maximal curvature.

This possibility is suggested by solutions obtained for a fixed equation of state. We might expect, however, that the transition from a given kinematical class of background evolution to the dual one might be associated with a corresponding transition between duality-related regimes also in the matter sources. In such a case it becomes possible to go across the phase of maximum curvature, without singularities, even for monotonically expanding (or contracting) "self-dual" solutions.

Consider indeed (always in an isotropic context, with diagonal pressure) a model of sources performing a transition from an equation of state that is typical of string-driven pre-big-bang, $p = -\rho/d$ (see Section 3), to the dual, radiation-dominated regime with $p = \rho/d$. We shall model this transition by

$$\frac{\overline{p}}{\overline{\rho}} = \gamma(\xi) = \frac{\xi}{d\sqrt{\xi^2 + \xi_1^2}} \quad , \quad \int \gamma d\xi = \frac{1}{d}\sqrt{\xi^2 + \xi_1^2}, \tag{5.31}$$

where $|\xi_1|$ is a phenomenological parameter (typically of order M_P^{-1}) that characterizes the time scale of the transition regime. Asymptotically, i.e. for $|\xi| >> |\xi_1|$, one recovers radiation for $\xi > 0$ and the dual state for $\xi < 0$.

By inserting this matter behaviour into the equations (5.14), (5.15) and (5.20), we get a regular solution provided ξ_0 and ξ_1 are chosen such that

$$\alpha c > \xi_0^2, \tag{5.32}$$

where α and c are the constants defined in eq. (5.22). This condition can be satisfied, in particular, by the choice

$$\xi_0 = 0 \quad , \quad 4V_0 = \xi_1^2, \tag{5.24}$$

which nicely simplifies the final expression for the background fields (a more general choice does not change the qualitative behaviour of the solution, which is presented here for illustrative purposes only).

With this choice, the integration of eqs. (5.15) and (5.16) gives

$$e^{\overline{\phi}} = e^{\phi_0}(1+\frac{\xi^2}{\xi_1^2})^{\frac{-d}{d-1}} \tag{5.25}$$

$$a = a_0(\frac{\xi}{\xi_1}+\sqrt{1+\frac{\xi^2}{\xi_1^2}})^{\frac{2}{d-1}} \tag{5.26}$$

where a_0 and ϕ_0 are integration constants. From their combination we have

$$e^{\phi} = a_0^d e^{\phi_0}(1+\frac{\xi}{\sqrt{\xi^2+\xi_1^2}})^{\frac{2d}{d-1}}. \tag{5.27}$$

Moreover, according to eq. (5.14),

$$\frac{\overline{\rho}}{\rho_0} = \frac{d-1}{4d}\xi_1^2\rho_0 e^{\phi_0}(1+\frac{\xi^2}{\xi_1^2})^{-\frac{1}{d-1}} = \frac{d\xi}{dt} \tag{5.28}$$

so that

$$e^{\phi}\frac{\rho}{\rho_0} = \frac{d-1}{4d}\xi_1^2\rho_0 e^{\phi_0}(1+\frac{\xi^2}{\xi_1^2})^{-\frac{d+1}{d-1}} \tag{5.29}$$

and

$$e^{\phi}\frac{p}{\rho_0} = \frac{\xi}{\xi_1}\frac{d-1}{4d}\xi_1^2\rho_0 e^{\phi_0}(1+\frac{\xi^2}{\xi_1^2})^{-\frac{3d+1}{2(d-1)}} \tag{5.30}$$

We note that, for $\xi \to \infty$, eq. (5.26) gives $a(\xi) \sim \xi^{2/(d-1)}$, while for $\xi \to -\infty$, $a(\xi) \sim (-\xi)^{-2/(d-1)}$. From the definition (5.8), it thus follows that, in the asymptotic future $d\xi/dt \sim a^{-1}$, namely ξ tends to coincide with the conformal time, while in the asymptotic past ξ tends to coincide with the "dual" time coordinate, as $d\xi/dt \sim a$.

This solution describes a model that is expanding ($H > 0$), for all t; the Universe, starting from a flat space ($H \to 0$) and weak coupling ($e^{\phi} \to 0$) regime, evolves through a superinflationary phase [$a \sim (-t)^{-2/(d+1)}$] dominated by string-like unstable matter ($p = -\rho/d$), towards a final decelerated [$a \sim t^{2/(d+1)}$], radiation-dominated ($p = \rho/d$) phase with frozen gravitational coupling ($e^{\phi} = const.$). In $d = 3$ spatial dimensions it provides an explicit realization of the model discussed at the end of Section 3.

The curvature is everywhere bounded, growing from $-\infty$ to 0, and decreasing from 0 to ∞. The behaviour of H and $\dot{H}$ is given by

$$H = \frac{a'\overline{\rho}}{a\rho_0} = \frac{\xi_1\rho_0 e^{\phi_0}}{2d}(1+\frac{\xi^2}{\xi_1^2})^{-\frac{d+1}{2(d-1)}} \tag{5.31}$$

$$\dot{H} = H' \frac{\overline{\rho}}{\rho_0} = -\frac{(d+1)\xi\xi_1\rho_0^2 e^{2\phi_0}}{8d^2}(1+\frac{\xi^2}{\xi_1^2})^{-\frac{3d+1}{2(d-1)}}. \quad (5.32)$$

The absence of singularity can be traced back to the fact that this solution is self-dual, in the sense that

$$[\frac{a(-t)}{a_0}]^{-1} = \frac{a(t)}{a_0}. \quad (5.33)$$

In general relativity, such a solution is not allowed, as the field equations are time-symmetric but not invariant under the inversion of the scale factor. The behaviour of $a, e^{\phi}, e^{\phi}\rho, e^{\phi}p, H$ and $\dot{H}$ is plotted in **Fig. 3** for $d = 3$, $\phi_0 = 0$, $a_0 = \rho_0 = \xi_1 = 1$.

An analogous solution obviously exists in which H is always negative (the universe is always isotropically contracting); it is easily obtained in correspondence with the dual equation of state $\gamma(\xi) = -\xi/d\sqrt{\xi^2+\xi_1^2}$. More interesting, in our context, is however the Bianchi I type anisotropic background in which, during the pre-big-bang phase, d dimensions expand with scale factor a, while n dimensions shrink with scale factor $b = a^{-1}$, with an equation of state $p = -q = -\rho/(d+n)$ (the example discussed in Section 3).

In this case, by setting $\gamma = p/\rho$, we have

$$\frac{1}{2}Tr(\Gamma\eta)^2 = (d+n)(\int \gamma d\xi)^2, \quad (5.34)$$

and the equations (5.15) and (5.16) for a and $\overline{\phi}$ reduce to

$$\begin{aligned} \overline{\phi}' &= -\frac{2}{D}(\xi+\xi_0) \\ \frac{a'}{a} &= \frac{2}{D}\int \gamma d\xi = -\frac{b'}{b}, \end{aligned} \quad (5.35)$$

where

$$D = 4V_0 + (\xi+\xi_0)^2 - (d+n)(\int \gamma d\xi)^2. \quad (5.36)$$

In analogy with the previous example, we represent the evolution in time of the sources between the dual asymptotic regimes by

$$\begin{aligned} \frac{p}{\rho} &= \gamma(\xi) = \frac{1}{d+n}\frac{\xi}{\sqrt{\xi^2+\xi_1^2}} = -\frac{q}{\rho} \\ \int \gamma d\xi &= \frac{\sqrt{\xi^2+\xi_1^2}}{d+n} \end{aligned} \quad (5.37)$$

and we choose conveniently the arbitrary parameters in such a way that $\xi_0 = 0$ and $\xi_1^2 = 4V_0$.

The integration of eqs. (5.35) then provides

$$e^{\overline{\phi}} = e^{\phi_0}(1+\frac{\xi^2}{\xi_1^2})^{-\frac{d+n}{d+n-1}} \tag{5.38}$$

$$a = b^{-1} = a_0(\frac{\xi}{\xi_1}+\sqrt{1+\frac{\xi^2}{\xi_1^2}})^{\frac{2}{d+n-1}} \tag{5.39}$$

with a_0 and ϕ_0 arbitrary integration constants. It follows that

$$e^{\phi} = a_0^{d-n}e^{\phi_0}(\frac{\xi}{\xi_1}+\sqrt{1+\frac{\xi^2}{\xi_1^2}})^{\frac{2(d-n)}{d+n+1}}(1+\frac{\xi^2}{\xi_1^2})^{-\frac{d+n}{d+n-1}} \tag{5.40}$$

and that

$$\frac{\overline{\rho}}{\rho_0} = \frac{d\xi}{dt} = \frac{d+n-1}{4(d+n)}\xi_1^2\rho_0 e^{\phi_0}(1+\frac{\xi^2}{\xi_1^2})^{-\frac{1}{d+n-1}}. \tag{5.41}$$

By comparing this last equation with eq. (5.39) we can easily check that this solution, in the $\xi \to -\infty$ limit, exactly describes the regime of pre-big-bang and dynamical dimensional reduction discussed in Section 3, with $a(t) \sim (-t)^{-2/(d+n+1)} = 1/b$. This background, according to the solution (5.39), (5.40), evolves in such a way as to reach a phase of maximal, finite curvature, after which it approaches the dual, decelerated regime in which the internal dimensions are not frozen, but keep contracting like $b(t) = 1/a \sim t^{-2/(d+n+1)}$ for $t \to +\infty$.

It is important to stress that the dilaton coupling, in this case, does not settle down to a finite constant value after the big-bang, but, according to eq.(5.40), tends to decrease during the phase of decreasing curvature. Such a decrease of ϕ is driven by the decelerated shrinking of the internal dimensions which are not frozen, unlike in the previous case.

We have thus provided two examples of self-dual solutions which can model the transition from the growing to the decreasing curvature regime. These examples are not intended to apply too far away from the maximum curvature scale (i.e. in too low a curvature regime where, for example, there are constraints which rule out too fast a variation of the gravitational coupling [49] and of the radius of the internal dimensions [50], such as the ones predicted by our last example). In the low curvature regime $O(d,d)$ symmetry and scale factor duality may indeed be expected to be broken, for instance by a dilaton potential $V(\phi)$. They suggest, however, the interesting possibility of a transition, which may occur even for monotonically evolving scale factors, and which approximates a de Sitter phase in the neighbourhood of the big-bang region ($\dot{H} = 0$).

We note, finally, that in the model of sources that we have used the ratio p/ρ is not a constant, but the pressure is still diagonal. The effective, time-dependent equations of state (5.31) and (5.37) may thus be re-expressed in terms of an effective bulk viscosity, which depends on ρ and which becomes negligibly small in the "in" and "out" asymptotic regimes where the perfect fluid behaviour is recovered. It is thus significative, in this context, that the evolution of a phase of exponential de Sitter expansion down to the decelerated regime, in a model of string-driven inflation [51], can also be interpreted in terms of an effective ρ-dependent, bulk viscosity [52].

We have already seen, on the other hand, that in the pre-big-bang phase the entropy of the sources is constant, as the matter evolution is adiabatic. Near the maximum ($\dot{H} \simeq 0$), however, one can define a horizon entropy [53] which shrinks like the pre-big-bang horizon area. Viscosity, and other dissipative processes, may thus naturally be expected to occur, in that regime, possibly yielding a large entropy increase.

6. Conclusions

In this paper we have described how string theory suggests and supports a picture of the Universe in which the present decelerated expansion is preceeded by a dual phase in which the evolution is accelerated and the curvature is growing (what we have called **"pre-big-bang"**). We stressed that such a scenario is not to be regarded as an alternative to the usual post-Planckian cosmology, whether inflationary or not, but that it represents **a complement** of the standard scenario, which cannot be extended beyond the Planck scale.

In the pre-big-bang phase the growth of curvature may be accompanied (but not necessarily sustained) by the shrinking of some extra "internal" spatial dimensions down to a final compactification scale. This scale, as well as the maximal (big-bang) curvature scale, are both expected to be given by the fundamental length parameter of string theory, i.e. roughly by the Planck scale itself. In the subsequent decelerated phase the internal radius may be frozen, or may keep shrinking, as shown by the examples presented in Section 5.

As we pointed out there, the possibility of avoiding the big-bang singularity appears to be deeply rooted in a typical stringy symmetry, the scale factor duality, which acts on homogeneous cosmological backgrounds. Indeed, the product of scale-factor-duality and time reversal admits, as non-trivial fixed points, backgrounds which connect smoothly, at $t = 0$, two duality-related regimes, e.g. a superinflationary era and a standard decelerating expansion.

Moreover, the generalization of scale-factor-duality to a full continuous $O(d,d)$ symmetry provides a framework in which entire classes of non-singular cosmological models can be obtained via $O(d,d)$ "boosts" of trivial, or even singular [45] initial conformal backgrounds.

Obviously, the picture presented here is in many respects qualitative and preliminary. Many problems remain to be solved, such as that of formulating a simple, yet consistent equation of state for string sources in curved backgrounds. At a more fundamental level, we might expect that, unless higher-order quantum string effects are consistently taken into account, it will be impossible to avoid a singularity once all physical constraints are imposed.

Nevertheless, we wish to stress that the global picture emerging from our simplified approach is already well defined and allows, in principle, for experimental verifications. We have shown, indeed, that the string cosmology equations and the assumption of a pre-big-bang era provide definite predictions for the spectrum of relic gravitons that should be filling up space not less than the electromagnetic CMB does. Thus such a scenario can be confirmed, disproved, or at least significantly constrained, by some (hopefully near-future) direct or indirect observations and measurements of a gravitational-wave background of cosmological origin. It is perhaps encouraging to recall at this point that the-hot big-bang hypothesis [54] was definitively confirmed through the observation of the relic electromagnetic background radiation [55].

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Figure captions

Fig. 1
Two possible alternatives to the curvature singularity of the standard cosmological model.

Fig. 2
Present constraints on the maximum allowed curvature scale H_1 (in units of Planck mass), versus the index ν which parametrizes the background kinematics. The allowed region extends down to $|\nu + 1/2| = 0$.

Fig. 3
Time evolution with respect to ξ of the self-dual solution given by eqs. (5.26) and (5.27) around the phase of maximal curvature ($\xi = 0$). a) The scale factor and the dilaton coupling; b) the effective density and pressure; c) the Hubble parameter and its time derivative.