CERN-TH.6836/93
DFTT-26/93
March 1993

Squeezed Thermal Vacuum and the Maximum Scale for Inflation

M. Gasperini *
Theory Division, CERN, Geneva, Switzerland
M. Giovannini
Dipartimento di Fisica Teorica, Via P.Giuria 1, 10125 Turin, Italy
and INFN, Sezione di Torino, Turin, Italy
and
G. Veneziano
Theory Division, CERN, Geneva, Switzerland

Abstract

We consider the stimulated emission of gravitons from an initial state of thermal equilibrium, under the action of the cosmic gravitational background field. We find that the low-energy graviton spectrum is enhanced if compared with spontaneous creation from the vacuum; as a consequence, the scale of inflation must be lowered, in order not to exceed the observed CMB quadrupole anisotropy. This effect is particularly important for models based on a symmetry-breaking transition which require, as initial condition, a state of thermal equilibrium at temperatures of the order of the inflation scale.

To appear in **Phys. Rev. D** *(Rapid Communications)*

* Permanent address: Dipartimento di Fisica Teorica, Via P.Giuria 1, 10125 Turin, Italy

It has recently been argued that the Universe cannot be in a state of "eternal" inflation: not only a primordial de Sitter exponential expansion, without a beginning in time, is impossible [1], but also there are quantum cosmological arguments [2] suggesting for the inflationary phase just the minimum duration required to bring in causal contact scales that are not much larger than our present Hubble radius H_0^{-1}.

If this is the case, the initial state, whose perturbations are amplified by the subsequent inflationary evolution, should be fixed by the dynamics of the pre-inflationary era and could differ considerably from the usually assumed vacuum. This difference influences the shape of the perturbation spectrum, as first pointed out in [3] for the case of tensor perturbations (graviton production), and recently discussed in [4] for the scalar perturbation case.

It is well known, in particular, that any inflationary model based on a temperature dependent phase transition necessarily requires a homogeneous thermal state as initial condition. The existence of initial thermal equilibrium constrains the parameters of such models, and a recent discussion [5] suggests that, in their context, a sufficient duration of inflation can be arranged only for low enough energy scales. The aim of this paper is to point out that also the conventional upper bounds on the scale arising from the gravity-wave contribution to the CMB quadrupole anisotropy are to be lowered, if the relic graviton background is produced from an initial thermal bath rather than from the zero-temperature vacuum.

Indeed, the inflationary amplification of the vacuum fluctuations can be properly represented, in second quantized formalism, as a process of pair production under the action of the external gravitational field [6], with the produced particles necessarily appearing in a final "squeezed vacuum" quantum state [6,7]. If the initial vacuum is changed into a state of thermal equilibrium, while the inflationary dynamics is left unchanged, the pair production process leads eventually to a thermal mixture of"squeezed number" states, instead of the pure squeezed vacuum. This modifies the spectral number density of the produced particles, in such a way that their energy distribution deviates in general from the flat Harrison-Zeldovich spectrum, even in the case of pure de Sitter inflationary dynamics. The isotropy properties of the CMB radiation then provide a bound on the inflation scale, which depends on the temperature of the initial thermal bath.

In order to discuss this effect let us recall [6–8], first of all, that the inflationary particle production can be described in terms of Bogoliubov transformations relating, for each mode k, the $|\mathrm{in}\rangle$ $(b_k, b_k^\dagger)$ to the $|\mathrm{out}\rangle$ $(a_k, a_k^\dagger)$ annihilation and creation operators :

$$a_k = c_+(k) b_k + c_-^*(k) b_k^\dagger \quad , \quad a_k^\dagger = c_-(k) b_k + c_+^*(k) b_k^\dagger. \tag{1}$$

The Bogoliubov coefficients $c_\pm(k)$ depend on the dynamics of the background geometry (in particular on the transition from the inflation to the standard decelerated phase), and satisfy $| c_+ |^2 - | c_- |^2 = 1$. By parametrizing $c_\pm$ as:

$$c_+(k) = \cosh r_k \qquad , \qquad c_-^* = e^{2i\vartheta_k} \sinh r_k, \tag{2}$$

the relations (1) can be rewritten as unitary transformations

$$a_k = \Sigma_k b_k \Sigma_k^\dagger \quad , \quad a_k^\dagger = \Sigma_k b_k^\dagger \Sigma_k^\dagger \tag{3}$$

generated by the squeezing operator *:

$$\Sigma_k = \exp[(z_k^* b_k^2 - z_k {b_k^\dagger}^2)/2] \quad , \quad z_k = r_k e^{2i\vartheta_k}. \tag{4}$$

* Note that, although the correct (momentum conserving) transformation should involve the two-mode squeezing operator [9], we are considering here the simpler one-mode formalism since it gives completely equivalent results for the quantities discussed in this paper.

The spectral properties of the relic radiation are usually derived by starting from the $|\mathrm{in}\rangle$ vacuum state $|0\rangle$, which satisfies $b_k|0\rangle = 0$. The pair production then process leads, for each mode, to the squeezed vacuum state $|z_k\rangle = \Sigma_k|0\rangle$, such that $a_k|z_k\rangle = 0$. The average particle number can be expressed in terms of the squeezing parameter r_k as

$$\overline{N}_k = \langle 0|a_k^\dagger a_k|0\rangle = |c_-(k)|^2 = \sinh^2 r_k. \tag{5}$$

If we start, however, with a number state $|n_k\rangle$ in which n particles are already present in the given mode, $b_k^\dagger b_k|n_k\rangle = n_k|n_k\rangle$, we obtain a "squeezed number" [10] state $|z_k, n_k\rangle = \Sigma_k|n_k\rangle$, with

$$\overline{N}_k = \langle n_k|a_k^\dagger a_k|n_k\rangle = |c_-|^2(1+n_k) + n_k(1+|c_-|^2), \tag{6}$$

in agreement with the rules of stimulated emission. If we start, more generally, with a statistical mixture,

$$\rho_k = \sum_n P_n(k)|n_k\rangle\langle n_k| \quad , \quad \sum_n P_n = 1, \tag{7}$$

then [11]

$$\overline{N}_k = \mathrm{Tr}(\rho_k a_k^\dagger a_k) = |c_-(k)|^2(1+\overline{n}_k) + \overline{n}_k(1+|c_-(k)|^2), \tag{8}$$

where $\overline{n}_k = \sum_n nP_n(k)$ is the initial averaged particle number. The spectral number distribution for the (bosonic) massless particles produced from an initial thermal bath, at a co-moving temperature $\overline{T} = \overline{\beta}^{-1}$, is thus obtained by inserting into eq.(8) the thermal average number

$$\overline{n}_k = (e^{\overline{\beta}k} - 1)^{-1}. \tag{9}$$

This result can be checked directly by explicit use of the "squeezed thermal" density operator [10], $\rho_{st} = \Sigma\rho_t\Sigma^\dagger$, where $\rho_t = \exp(-\overline{\beta}b^\dagger b)$ (from now on the mode index is to be understood, if not explicitly written; any correlation among modes is moreover neglected, as we are treating perturbations in the linear approximation). Indeed, in the convenient representation defined by the "superfluctuant" variable x, whose variance Δx is amplified by the squeezing process [9], ρ_{st} takes the form:

$$\rho_{st}(x,x') = \langle x|\rho_{st}|x'\rangle =$$
$$= [\frac{\pi(2\overline{n}+1)}{\sigma}]^{-1/2} \exp\{-\frac{2\overline{n}(\overline{n}+1)+1}{2(2\overline{n}+1)}(x^2+x'^2)\sigma + \frac{2\overline{n}(\overline{n}+1)}{(2\overline{n}+1)}xx'\sigma\}, \tag{10}$$

where $\sigma = \exp(-2r)$, and $\overline{n}$ is the thermal average number of eq. (9) (for $\sigma = 1$, eq. (10) reduces to the usual thermal density matrix in the configuration space representation). The computation of $\mathrm{Tr}(\rho_{\rm st} \mathrm{b}^\dagger \mathrm{b})$ then reproduces exactly eq. (8), with $|c_-|^2 = \sinh^2 r$.

Note, incidentally, that, unlike $\overline{N}$, the entropy growth ΔS associated with pair production is not affected by finite-temperature corrections to the initial state; for a squeezed thermal mixture, ΔS turns out to be just the same as that obtained starting from the vacuum [12,13], namely $\Delta S = -\ln\sigma = 2r$. This can be easily verified by computing $-\mathrm{Tr}\rho_{\rm st}\ln\rho_{\rm st}$ for the two-mode generalization of the matrix (10), and by subtracting the initial thermal contribution.

It is also worth noting that, in the real-time formalism of the thermo-field dynamics [14], the thermal vacuum is related to the $T = 0$ vacuum by a kind of Bogoliubov transformation, with squeezing parameter $r_t = \sinh^{-1}[(e^{\overline{\beta}k} - 1)^{-1/2}]$. Such a transformation acts on a "doubled" Hilbert space, obtained by introducing fictitious operators associated with each physical operator. In this context, the average number of eq. (8) can be recovered, formally, by considering the state obtained from the vacuum by the product of two $SU(1,1)$ Bogoliubov matrices, with parameters $r_1 = r_t$, and $r_2 = \sinh^{-1}|c_-|$, provided the relative phase is chosen to be $\vartheta_1 - \vartheta_2 = (2m+1)\pi/4$, with m integer.

We shall now concentrate, in particular, on the stimulated emission of gravitons from an initial thermal bath, under the action of a changing background geometry which describes the transition from an inflationary phase to the subsequent radiation-dominated and matter-dominated eras.

The spectral energy density $\rho(\omega)$, which is the variable usually adopted to characterize today's distribution of the produced gravitons [8,15,16] is given by

$$\rho(\omega) = \omega(\frac{d\rho_g}{d\omega}) \simeq \omega^4 \overline{N}(\omega), \tag{11}$$

where $\overline{N}$ is defined in eq. (8), with the corresponding $\overline{n}$ of eq. (9) (we have neglected numerical factors of order unity, and ω is the proper frequency, related to the comoving one, k, by $\omega = k/a(t)$, where a is the scale factor of the background isotropic metric).

The Bogoliubov coefficients $c_{\pm}(\omega)$, connecting the $|\text{in}\rangle$ and $|\text{out}\rangle$ graviton modes for the inflation $\rightarrow$ radiation $\rightarrow$ matter transition, have been computed by many authors [8,16,17], in the sudden approximation. In such an approximation, one ignores the details of the transition among the three different cosmic phases, and the particle production is neglected for modes which never "hit" the effective potential barrier appearing in the graviton wave equation. As a consequence, the Bogoliubov coefficients are not modified, in this approximation, if an initial phase dominated by a thermal radiation bath is inserted before the de Sitter era, since the radiation dominated evolution gives no contribution to that potential barrier.

We then insert the known expression of $c_-(\omega)$ in eq. (11) and measure $\rho(\omega)$, as usual, in units of critical energy density ρ_c, defining $\Omega(\omega) = \rho(\omega)/\rho_c$. By exploiting the fact that $|c_-| \geq 1$ for all the modes undergoing the parametric amplification, we finally get (we follow in particular the notations of [17]):

$$\Omega(\omega, t_0) \simeq G{H_1}^2 \Omega_\gamma(t_0) (\frac{\omega}{\omega_1})^{2-2\alpha} \coth(\frac{\beta_0 \omega}{2}) \quad , \qquad \omega_2 < \omega < \omega_1$$

$$\Omega(\omega, t_0) \simeq G{H_1}^2 \Omega_\gamma(t_0) (\frac{\omega}{\omega_1})^{2-2\alpha} (\frac{\omega}{\omega_2})^{-2} \coth(\frac{\beta_0 \omega}{2}) \quad , \qquad \omega_0 < \omega < \omega_2. \tag{12}$$

Here ${\beta_0}^{-1} \equiv \beta^{-1}(t_0)$ is the proper temperature of the initial thermal state, adiabatically rescaled down to the present observation time t_0 [β_0 is defined in terms of the comoving temperature $\overline{\beta}$ as $\beta(t_0) = \overline{\beta}a(t_0)]$; $\Omega_\gamma(t_0) \simeq 10^{-4}$ is the fraction of the critical energy density present today in the form of radiation; $\alpha \geq 1$ is a coefficient parametrizing (in conformal time) the power-law behaviour of the scale factor; $H_1 \equiv H(t_1)$ is the curvature scale at the time t_1 marking the end of inflation and the beginning of the radiation-dominated era; $\omega_0 \simeq 10^{-18}$ Hz is the minimum amplified frequency crossing today the Hubble radius H_0^{-1}; $\omega_2 \simeq 10^2\omega_0$ is the frequency corresponding to the matter radiation transition; ω_1, finally, is the maximum amplified frequency, related to the inflation scale by $\omega_1 \simeq 10^{11}(H_1/M_P)^{1/2}$ Hz (M_P is the Planck mass).

Equation (12) provides the present energy distribution of a gravity-wave background of inflationary origin, obtained from a primordial state of thermal equilibrium at a proper temperature β^{-1}. In the limit $\beta \to \infty$, and for $\alpha = 1$, we recover the well-known flat de Sitter spectrum obtained from the vacuum, with the usual frequency dependence ($\sim \omega^{-2}$) at low energy, due to the radiation-matter transition [15–17].

The primary effect of the initial finite temperature is to enhance the low-frequency graviton production, with respect to the high-frequency sector of the spectrum. In this respect, the initial thermal vacuum mimics the effect of putting "more power on larger scales" [18], typical of power-law inflation; with the difference, however, that the thermal effects are rapidly damped at high ω, for realistic values of the rescaled initial temperature β_0. In Fig.1 the spectrum has been plotted for the de Sitter case ($\alpha = 1$), in such a way as to represent, for various values of β_0, the maximum allowed fraction of critical energy density compatible with the CMB isotropy.

The relic graviton spectrum is mainly constrained, indeed, by three kinds of direct observations [15]: CMB isotropy, pulsar timing data, and critical density. However, as discussed in [15] and [17], the most significant constraint for flat or decreasing spectra, such as those of eq. (12), turns out to be the isotropy bound imposed at the minimum frequency ω_0, where it presently implies $\Omega(\omega_0, t_0) \lesssim 10^{-10}$ (we are making here a conservative use of the COBE data [19], as an upper limit on the graviton contributions to the quadrupole anisotropy). Such a condition, imposed on eq. (12), provides a bound on the inflation scale H_1, which can be conveniently expressed, in terms of the usual spectral index $n = 3 - 2\alpha$,

$$\log_{10}(\frac{\mathrm{H_1}}{\mathrm{M_P}}) \lesssim \frac{2}{3+\mathrm{n}}\left(29\mathrm{n} - 39 + \log_{10}(\tanh\frac{\beta_0\omega_0}{2})\right). \tag{13}$$

In the limit $\beta_0 \to \infty$ this generalizes, to any value of n, the usual isotropy constraint on the curvature scale of de Sitter ($n = 1$) inflation [20], namely $H_1 \lesssim 10^{-5} M_P$ (with an improvement of one order of magnitude with respect to [15–17,20], due to the use of the more constraining COBE data). The new effect, however, is that for finite initial temperature the maximum allowed scale is in general depressed with respect to vacuum production, as illustrated in Fig.2 (for $\beta_0\omega_0 \ll 1$, in particular, H_1 scales like $\beta_0^{2/(3+n)}$). The inclusion of the thermal correction is thus expected to modify the existing relations (see e.g. [21]) between the power index and the scale of inflation, obained by fitting the observed anisotropy on a 10 degree angular scale.

In particular, for the inflationary models based on a thermal symmetry breaking mechanism, the initial temperature β^{-1} is not independent from the scale of inflation itself. Suppose, indeed, that the inflationary phase transition occurs at an energy scale M, which is the scale at which the time-independent vacuum energy becomes dominant (M is related to the curvature scale H_1 by $M/M_P = (H_1/M_P)^{1/2}$, according to the Einstein equations). At earlier times, such that $\beta^{-1}(t) > M$, the symmetry is restored, and the Universe becomes radiation-dominated. The temperature of the initial thermal ensemble, rescaled at the beginning ($t = t_i$) of inflation, must therefore satisfy $\beta(t_i)M \leq 1$.

This condition, rescaled down adiabatically at the present time t_0, provides a bound on the spectral parameter β_0, which depends on the duration Z of the inflationary phase ($Z = a(t_1)/a(t_i)$, where t_1 marks the end of inflation):

$$\beta_0\omega_0 = M\beta_i \left(\frac{a_0}{a_i}\right)\left(\frac{\omega_0}{M}\right) \leq Z\left(\frac{H_0}{M_P}\right)^{\frac{1}{2}}. \tag{14}$$

For $Z \to \infty$ this bound is washed out by the inflationary supercooling of the original thermal ensemble. However, for models whose parameters are adjusted to give just the minimal amount of inflation required to solve the standard problems [22],

$$Z \simeq Z_{min} = e^{53}\left(\frac{M}{10^{14}\mathrm{GeV}}\right)^{\frac{2}{3}}\left(\frac{T_{rh}}{10^{10}\mathrm{GeV}}\right)^{\frac{1}{3}} \tag{15}$$

(T_{rh} is the reheating temperature), the bound (14) can be re-expressed in terms of the reheating efficiency, $Q = T_{rh}/M$, as

$$2\,\log_{10}(\beta_0\omega_0) \lesssim \log_{10}\left(\frac{\mathrm{H_1}}{\mathrm{M_P}}\right) + \frac{2}{3}\log_{10}\mathrm{Q}. \tag{16}$$

For each given value of n and Q one has then a minimum allowed temperature β_0^{-1} and a maximum allowed scale H_1, which are fixed by the combination of the constraints (13) and (16), as illustrated in Fig.3 for three different spectral indices. This effect represents a truly "remnant" of the pre-inflationary Universe, in the sense of [22]. For $Z > Z_{min}$, the maximum allowed value of H_1 scales up like $Z^{2/(3+n)}$ and becomes Z-independent for $\beta\omega_0 \gtrsim 1$.

In conclusion, we want to stress that the numerical results presented here should be regarded, in many respects, as semi-quantitative results, because of the approximations made and of the uncertainty of the experimental data, which has not been completely taken into account in our discussion. Nevertheless, we believe that already at this qualitative level two important indications emerge rather clearly.

The first is that *any fit* of the CMB anisotropy in terms of the gravity wave background *should include a thermal dependence in the spectrum* [according to eq.

(12)], in order to take into account the possible finite temperature of the initial state.

The second indication is that, even if a thermal phase transition at the GUT scale is certainly not ruled out as a possible inflationary mechanism, the effects discussed here seem to provide some motivation for investigating the same mechanism also at lower scales, such as the electroweak one [23]. The constraints on the scale imposed by our results seem to challenge, in particular, thermal models of inflation obtained in the context of string theory, where the natural scale is very near the Planck one. No difficulty seems to arise, on the contrary, in the context of a "duality-symmetric" string cosmology [24] where the inflationary phase is the dual counterpart of the present decelerating expansion, and starts naturally from a flat and cold low-energy vacuum.

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Figure Captions

Fig. 1 Maximum allowed spectral energy density [according to eq. (12)] in a relic graviton background, produced after a phase of de Sitter inflation, from an initial thermal bath at finite temperature $T = \beta^{-1}$. The rescaled temperature β_0^{-1} is measured here in units of $\omega_0 = 10^{-18}$ Hz.

Fig. 2 Maximum allowed inflation scale versus the spectral index n, according to eq. (13), for three different values of the initial temperature β_0^{-1} (in units of ω_0).

Fig. 3 Maximum scale and minimum initial temperature for models with the inflation rate of eq. (15), and with reheating efficiency $Q = 1$ and $Q = 10^{-3}$. The allowed region lies <u>below</u> the curves labelled by the spectral index n, and <u>to the left</u> of the curves labelled by Q.