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Quantum Squeezing and Cosmological Entropy Production†

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Abstract

The entropy growth in a cosmological process of pair production is completely determined by the associated squeezing parameter, and is insensitive to the number of particles in the initial state. The total produced entropy may represent a significant fraction of the entropy stored today in the cosmic black-body radiation, provided pair production originates from a change in the background metric at a curvature scale of the Planck order.

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One of the greatest challenges of modern cosmic physics is that of explaining the large level of entropy observed on a cosmological scale. While it seems clear that inflationary kinematics has to play a fundamental role in providing such an explanation, the nature of the (micro)physical mechanism that may have acted as a source of the cosmic entropy is, on the contrary, still unclear.

A natural candidate for such a mechanism, well known even before the advent of the inflationary models, is the production of particles by the changing background metric. The pair production from the vacuum (otherwise stated, in a wave-mechanics language, the parametric amplification of the background fluctuations) provides indeed a natural cosmological arrow [1], which is not inverted even if the expansion turns into a contraction, and which may thus be used to define also an appropriate arrow of time [2]. The problem is, however, that of quantifying in an unambiguous way, and in agreement with the usual notion of entropy, the information loss associated with pair production.

A possible solution to this problem comes from the observation that the production of particles by an external gravitational field can be conveniently represented in the squeezed state formalism [3]. Within this context, one can define indeed two canonically conjugate quantum variables (the so-called "quadrature operators"[4]): one operator has a variance that is "squeezed" with respect to the vacuum, while the variance of the other (the "superfluctuant" one) is correspondingly expanded.

Using this approach, we have recently proposed [5] a way to measure the loss of information associated to the cosmological particle production, in terms of the increased dispersion of the superfluctuant operator. According to the standard (information-theoretic) definition of non-equilibrium entropy, such a coarse graining approach then provides an expression in which the entropy growth ΔS_k is completely determined, for each mode k, by the squeezing parameter r_k [5] :

$$\Delta S_k = 2r_k. \tag{1}$$

Our definition of entropy is consistent with analogous results, recently obtained in the large squeezing limit, for the entropy of the cosmological perturbations [6]. In our case, however, eq. (1) is an exact result and can be applied also in the small squeezing regime.

By using eq. (1) we can estimate the entropy growth for a realistic case in which particles (e.g. gravitons) are produced by the transition between the inflationary era and the radiation-dominated era (occurring at a given scale H_1). We can assume, in particular, that their mode-number density has a power law behaviour, characterized as usual by the spectral index n (such that $k(d\rho/dk) \simeq k^4 \sinh^2 r_k \sim k^{n-1}$). In such a case the total produced entropy inside a co-moving horizon, obtained by summing the entropy density (1) over all squeezed modes, and measured in units of the usual black-body entropy S_γ associated to the electromagnetic CMB radiation, turns out to be [5]

$$\frac{\Delta S}{S_\gamma} \simeq \mid 5-n \mid \left(\frac{H_1}{M_P}\right)^{3/2}. \tag{2}$$

This result is particularly interesting for a recently developed string cosmology scenario [7], in which the spectral density of the cosmic graviton background increases with frequency, in such a way that the usual low-frequency bounds on the inflation scale are evaded, and the maximum allowed value of H_1 can be as large as the Planck mass M_P [8]. In such a context $H_1 \sim M_P$ is indeed the natural transition scale, so that the entropy stored in the relic graviton background could be comparable to the observed large scale entropy of the black-body radiation.

The expression (1) for the mode-entropy density, however, was obtained in Ref. [5] for the "minimum uncertainty" squeezed vacuum state, i.e. under the assumption that the initial state, whose fluctuation are amplified by the external gravitational field, is the vacuum, or a “displaced” vacuum (namely a coherent state, in general). In this paper we shall show that eq. (1) holds exactly, even if the initial state is any eigenstate of the number operator or, more generally, any statistical mixture of number states.

Consider indeed a pair production process described by a Bogoliubov transformation connecting, for each mode k, the particle $(b, b^\dagger)$ and antiparticle $(\tilde{b}, \tilde{b}^\dagger)$ annihilation and creation $|in\rangle$ operators to the $|out\rangle$ ones $(a,\ a^\dagger,\ \tilde{a},\ \tilde{a}^\dagger)$:

$$a_k = \Sigma_k b_k {\Sigma^\dagger}_k \qquad , \qquad \tilde{a}^\dagger_{-k} = \Sigma_k \tilde{b}^\dagger_{-k} \Sigma^\dagger_k. \tag{3}$$

Here Σ_k is the two-mode squeezing operator [4],

$$\Sigma_k = \exp(z_k^* b_k \tilde{b}_{-k} - z_k b_k^\dagger \tilde{b}^\dagger_{-k}) \qquad , \qquad z_k = r_k e^{2i\vartheta_k}, \tag{4}$$

parametrized by the complex number z_k, which depends on the dynamics of the external field leading to the process of pair creation. In terms of the variables x and $\tilde{x}$, whose variance in the final squeezed state is amplified with respect to their initial value, a and $\tilde{a}$ have the differential representation [5]

$$a_k = \frac{i}{2} e^{i\vartheta_k} \left[(\cosh r_k - \sinh r_k)(x - i\tilde{x}) + (\cosh r_k + \sinh r_k)(\partial_x - i\partial_{\tilde{x}})\right]$$

$$\tilde{a}_{-k} = \frac{i}{2} e^{i\vartheta_k} \left[(\cosh r_k - \sinh r_k)(x + i\tilde{x}) + (\cosh r_k + \sinh r_k)(\partial_x + i\partial_{\tilde{x}})\right] \tag{5}$$

(the relative phase has been chosen with respect to ϑ_k, in such a way as to identify the x and $\tilde{x}$ operators with the superfluctuant ones).

In the $(x, \tilde{x})$-space representation, the normalized squeezed vacuum wave function $\psi_{o,z_k}(x, \tilde{x})$ (such that $a\psi_o = 0 = \tilde{a}\psi_o$) is thus given by [5]

$$\psi_{o,z_k}(x, \tilde{x}) = \langle x\tilde{x}|\Sigma_k|00\rangle = \left(\frac{\sigma}{\pi}\right)^{\frac{1}{2}} e^{-\frac{\sigma_k}{2}(x^2+\tilde{x}^2)} \qquad , \qquad \sigma_k = e^{-2r_k}, \tag{6}$$

where $|0\rangle$ is the $|in\rangle$ vacuum state, $b|0\rangle = 0 = \tilde{b}|0\rangle$. When n pairs are already present in the mode k ($b_k^\dagger b_k |n_k n_{-k}\rangle = n|n_k n_{-k}\rangle$), however, the pair production leads to a two-mode squeezed number state, whose wave function in this space has the explicit representation

$$\psi_{n,z_k}(x,\tilde{x}) = \langle x\tilde{x}|\Sigma_k|n_k n_{-k}\rangle = \langle x\tilde{x}|\frac{\left(a_k^\dagger \tilde{a}_{-k}^\dagger\right)^n}{n!}\Sigma_k|0\rangle$$

$$= e^{-2in\vartheta_k}\left(\frac{\sigma_k}{\pi}\right)^{\frac{1}{2}} L_n\left(\sigma_k(x^2+\tilde{x}^2)\right) e^{-\frac{\sigma_k}{2}(x^2+\tilde{x}^2)}$$

$$= e^{in(\pi-2\vartheta_k)}\left(\frac{\sigma_k}{\pi}\right)^{\frac{1}{2}} e^{-\frac{\sigma_k}{2}(x^2+\tilde{x}^2)} \sum_{m=0}^{n} \frac{1}{m!(n-m)!} H_{2m}(\sqrt{\sigma}_k x) H_{2n-2m}(\sqrt{\sigma}_k \tilde{x}) \tag{7}$$

where L_n and H_n are the Laguerre and Hermite polynomials, respectively. It should be noted that, because of pair correlations, this wave function cannot be simply factorized in terms of two decoupled squeezed oscillators in an excited state, which are known to provide the usual representation for the one-mode squeezed number wave function [9]. One can easily verify, however, that x and $\tilde{x}$ are superfluctuant operators for the state (7), i.e.

$$\frac{(\Delta x)^2_{n,z_k}}{(\Delta x)^2_n} = \frac{(\Delta \tilde{x})^2_{n,z_k}}{(\Delta \tilde{x})^2_n} = e^{2r_k} > 1, \tag{8}$$

where

$$(\Delta x)^2_{n,z_k} = \langle n_k n_{-k}|\Sigma_k^\dagger x^2 \Sigma_k|n_k n_{-k}\rangle$$

$$(\Delta x)^2_n = \lim_{z_k\to 0}(\Delta x)^2_{n,z_k} \tag{9}$$

and the same for $\tilde{x}$.

Suppose now that the initial state is a generic mixture of number states,

$$\overline{\rho}_k = \sum_n P_n(k)|n_k n_{-k}\rangle\langle n_k n_{-k}| \quad , \quad \sum_n P_n = 1. \tag{10}$$

The final state will correspond to the squeezed mixture:

$$\overline{\rho}_{sk} = \sum_n P_n(k)\Sigma_k|n_k n_{-k}\rangle\langle n_k n_{-k}|\Sigma_k^\dagger. \tag{11}$$

Apart from the possible initial entropy, the pair production process is accompanied by a loss of information corresponding to a larger dispersion of x and $\tilde{x}$ around their mean values. According to [5], the reduced density operator associated to such an increased uncertainty, for the initial mixture (10), can be written

$$\rho_{sk} = \sum_n P_n(k) \int dx d\tilde{x} \mid \psi_{n,z_k}(x,\tilde{x}) \mid^2 |x\tilde{x}\rangle\langle x\tilde{x}|. \tag{12}$$

By using the explicit form of the wave function (7), the net entropy growth due to the pair production is thus

$$\Delta S_k = -Tr(\rho_{sk} \ln \rho_{sk}) + [Tr(\rho_{sk} \ln \rho_{sk})]_{\sigma_k=1} = -\sum_n P_n(k) \ln \sigma_k = 2r_k \tag{13}$$

i.e. exactly the same as that obtained starting from the vacuum (eq. (1)). This supports the internal consistency of our definition of entropy [5], and the whole approach to the cosmological entropy based on the squeezed-state formalism [5,6].

We conclude by noting that the result (13) has an important application to the case of an initial thermal mixture.

The inflationary models based on a temperature-dependent phase transition require indeed, as initial condition, a thermal bath at a finite temperature higher than the inflation scale. Such a non-zero temperature affects the spectral energy density of the amplified perturbations, which may deviate from the flat Harrison-Zeldovich form even in the case of pure exponential inflation [10]. The result (13) shows instead that the produced entropy is not affected by finite temperature corrections to the initial state. The entropy depends only on the squeezing parameter (i.e. on the coefficients of the Bogoliubov transformation, determined by the changing geometry), and the expression (2) for the total entropy of a given cosmic background of relic particles is always valid, irrespective of the initial field configuration.

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