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Inflation, Deflation, and Frame-Independence in String Cosmology

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Abstract

The inflationary scenarios suggested by the duality properties of string cosmology in the Brans-Dicke (or String) frame are shown to correspond to accelerated contraction (deflation) when Weyl-transformed to the Einstein frame. We point out that the basic virtues of inflation (solving the flatness and horizon problems, amplifying vacuum fluctuations, etc.) have physically equivalent counterparts in the deflationary (Einstein-frame) picture. This could be the answer to some objections recently raised to superstring cosmology.

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1. Introduction

A potential source of difficulty for extended-inflation models [1] based on a Brans-Dicke theory of gravity [2] is the choice of the correct frame (metric) in which to describe the space-time geometry at a cosmological level. One may wonder, in particular, in which frame the metric should be of the inflationary type, and satisfy the conditions required to avoid the problems of the standard cosmological scenario.

While the choice of the Einstein (E) frame (in which the Einstein-Hilbert term takes the General-Relativity form) usually simplifies calculations and is quite popular, there are physical motivations for choosing instead the Brans-Dicke (BD) frame, in which matter couples to the metric-tensor in the standard way [3]. Arguments in favour of the BD choice can also be given in string theory [4], where the BD frame metric coincides with the σ-model metric to which test strings are directly coupled. Thus free string motions follow geodesic surfaces with respect to the BD (not the E) metric.

The physical observable properties of a given model should be independent, of course, from the field redefinition (Weyl rescaling) connecting BD and E frames. And indeed, in the case of extended inflation, the metric describing a phase of power-law inflation (with variable Newton constant) in the BD frame, is transformed into a metric, which is still describing power inflation (of the slow-roll type, with exponential potential) in the E frame, as discussed for instance in [5].

In a string theory context, the role of the BD scalar is played by the dilaton field. In such case, as pointed out in [6], there appear to be serious difficulties in arranging a successful phase of dilaton-driven, power-law, extended inflation, at least if theoretically motivated dilaton potentials are used. On the other hand, the cosmological equations obtained from the low-energy string effective action show that the dilaton can drive (even in the absence of a potential) a phase of accelerated expansion. This phase, supposedly describing the Universe before the big-bang (so-called "pre-big-bang" [7]), is characterized by being just the "dual" counterpart (in the sense of ref. [8]) of the "post-big-bang" standard cosmology.

The "pre-big-bang" phase corresponds, in the BD frame, to a superinflationary expansion. When transformed to the E frame, however, the same metric describes, as we shall see, a contracting Universe. Apparently, this represents a difficulty for the whole scenario, since the presence or absence of inflation (and of its bonuses) would seem to become frame-dependent.

In this paper we shall show that, on the contrary, even in the E frame the solutions of the string cosmology equations provide an adequate description of the inflationary phase, provided we generically mean, by "inflation", a phase of cosmological evolution that is able to avoid the problems (see for instance [9]) related to the decelerated kinematics of the standard cosmological model.

At the same time, and irrespectively of strings and/or BD theory, we shall argue that the solution of many of the standard-cosmology problems achieved by inflation is also possible through the introduction of an early phase of accelerated contraction, that we shall call deflation. This will be the content of the following section.

2. Inflation vs. deflation

It is well known that there are three possible classes of inflationary evolution [10], corresponding to a curvature scale that is constant (De Sitter inflation), decreasing (power inflation) or increasing (superinflation). Less known, however, seems to be the fact that in a phase of growing curvature the solution of the standard cosmological puzzles can be realized in two ways, namely by a metric describing *either* accelerated expansion, $\dot{a} > 0, \ddot{a} > 0$, *or* accelerated contraction, $\dot{a} < 0, \ddot{a} < 0$ (a is the scale factor of a homogeneous and isotropic model, and a dot denotes differentiation with respect to cosmic time).

A possible equivalence of superinflation and accelerated contraction is clearly pointed out by an elementary analysis of the so-called flatness problem. If we want the contribution of the spatial curvature k to be suppressed with respect to the other terms of the cosmological equations, then the ratio

$$r_1 = \frac{k}{a^2 H^2} = \frac{k}{\dot{a}^2} \quad , \quad H \equiv \dot{a}/a \; , \tag{2.1}$$

must tend to zero during the inflationary era. Such a condition is clearly satisfied by a metric that behaves, for $t \to +\infty$, as

$$a \sim t^\alpha \quad , \quad t > 0 \quad , \quad \alpha > 1 \; , \tag{2.2}$$

but also by a metric, which, for $t \to 0_-$, behaves as

$$a \sim (-t)^\beta \quad , \quad t < 0 \quad , \quad \beta < 1 \; . \tag{2.3}$$

The case (2.2) corresponds to power inflation, and includes the standard De Sitter exponential inflation in the limit $\alpha \to \infty$. The second case, (2.3), corresponds, for $\beta < 0$, to the well-known case of pole inflation (superinflationary expansion, $\dot{a}, \ddot{a}, \dot{H}$ all positive). For $0 < \beta < 1$ it describes instead an accelerated contraction, or deflation ($\dot{a}, \ddot{a}, \dot{H}$ all negative). In both cases the curvature scale is growing, and $H, \dot{H}$ diverge as $t \to 0_-$.

A deflationary phase (2.3), with $0 < \beta < 1$, may also provide a solution to the so-called horizon problem. The presently observed large-scale homogeneity and isotropy requires the proper size of the particle horizon to become large enough during the inflationary era, and to go to infinity in the limiting case in which inflation extends for ever in the past. This means that the integral

$$d_p(t) = a(t) \int_{t_1}^{t} dt' a^{-1}(t') \tag{2.4}$$

must diverge, if a is the inflationary scale factor, when t_1 approaches the maximal past extension of the cosmic time coordinate for the given cosmological manifold.

For the metric (2.3) such a limiting time is $-\infty$, and $d_p \to \infty$ for $t_1 \to -\infty$, so that there are no particle horizons in a phase of accelerated contraction.

As a consequence of accelerated contraction, causally connected regions are pushed out of the event horizon, just as in the standard inflationary expansion. It is true that the proper size of a causally connected region tends to contract, asymptotically, like the scale factor. For a patch of initial size $d_1 \sim (-t_1)$ one finds in fact, from eqs. (2.3) and (2.4), that $d_p \to [a(t)/a(t_1)]d_1$ for $|t| << |t_1|$. However, the proper size of the event horizon, defined by

$$d_e(t) = a(t) \int_{t}^{t_2} dt' a^{-1}(t') \tag{2.5}$$

(t_2 is the maximal allowed future extension of the cosmic time coordinate), contracts always faster than d_p. Indeed, $t_2 = 0$ for the metric (2.3), and one finds that $d_e(t) \sim (-t)$ for $t \to 0$. The ratio of the two proper sizes at small t

$$r_2(t) = \frac{d_p(t)}{d_e(t)} \sim (-t)^{\beta-1} \tag{2.6}$$

shows that the causally connected regions will always cross the horizon, asymptotically, not only in the case of superinflationary expansion ($\beta < 0$), but even in the deflationary case ($0 < \beta < 1$).

We note, for later convenience, that the conditions for a successful resolution of the horizon and flatness problem, when expressed in terms of the conformal time coordinate η ($a = dt/d\eta$), are exactly the same for both superinflationary expansion and accelerated contraction. Moreover, if the contracting phase is long enough to solve the horizon problem, then also the flatness problem is automatically solved (and vice versa), as in standard inflation.

Indeed the ratio r_2 scales in conformal time like η^{-1}, while the ratio r_1 scales like η^2. The horizon problem is solved if $r_2(\eta_f)$, evaluated at the end of the accelerated evolution ($\eta = \eta_f$), is larger than the present value $r_2(\eta_0) \simeq 1$, rescaled down at η_f. This implies

$$\frac{|\eta_i|}{|\eta_f|} \gtrsim \frac{|\eta_0|}{|\eta_f|} \simeq 10^2 (\frac{T_{rh}}{eV}) \; . \tag{2.7}$$

Here η_i denotes the beginning of the contracting (or expanding) accelerated evolution, T_{rh} the final reheating temperature at $\eta = \eta_f$, and the last equality holds in the hypothesis of standard, adiabatic, radiation-dominated and matter-dominated expansion from η_f down to the present time η_0.

The solution of the flatness problem, on the other hand, is obtained if the ratio r_1 at the end of the accelerated phase is tuned to a value that is small enough, so that the subsequent decelerated evolution leads to a present value of r_1 satisfying the condition $r_1(\eta_0) \lesssim 1$. This means

$$(\frac{\eta_f}{\eta_i})^2 \lesssim (\frac{\eta_f}{\eta_0})^2 \; , \tag{2.8}$$

which is clearly equivalent to eq. (2.7), and which implies a resolution of the flatness and horizon problems (as well as of their rephrasing in terms of the entropy [9]) for both expanding and contracting metrics of the type (2.3).

Besides solving the kinematical problems, a phase of successful inflation is also expected to efficiently amplify the vacuum fluctuations of the metric background. We shall conclude this section by noting that such an amplification can also be provided by a long period of deflation.

Consider, for instance, the amplification of tensor perturbations h_μ^ν (similar arguments hold for the scalar case also). In a four-dimensional conformally flat background, the wave equation for each Fourier component of h can be written in terms of the rescaled variable $\psi = ah$ as [11]

$$\psi'' + (k^2 - \frac{a''}{a})\psi = 0 \tag{2.9}$$

(a prime denotes differentiation with respect to conformal time). In a realistic case, the phase of accelerated evolution is followed by the standard radiation-dominated expansion, with $a \sim \eta$, and the amplification of the fluctuations can be described as a process of graviton production from the vacuum (such an approach will be used in Section 3). Equivalently, in a Schrödinger-like language, the process corresponds to a parametric amplification of the perturbation wave function [11], which is oscillating at $\eta \to \pm\infty$, and evolves with a power-law behaviour in the regions where the co-moving frequency k is negligible with respect to the effective potential a''/a of eq. (2.9).

By inserting into (2.9) a generic parametrization (in conformal time) of the accelerated metric, $a(\eta) = (-\eta)^{-\delta}$, one finds indeed that the solution behaves like

$$h \sim A_{\pm} \frac{e^{\pm ik\eta}}{a} \quad , \quad k\eta >> 1 \tag{2.10}$$

$$h \sim A + B\frac{(-\eta)}{a^2} = A + B(-\eta)^{1+2\delta} \quad , \quad k\eta << 1 \tag{2.11}$$

($A_{\pm}, A, B$ are integration constants). In the case of accelerated expansion ($\delta > 0, a \to \infty$ for $\eta \to 0_-$), the perturbations are amplified because their amplitude tends to stay constant in the $\eta \to 0$ limit, instead of decreasing adiabatically as in the oscillating regime (2.10).

In the case of deflation ($\delta < 0, a \to 0$ for $\eta \to 0_-$), the amplification process is even more efficient than in the previous case, as the amplitude of h grows (with respect to the adiabatic red-shift of the subsequent radiation-dominated expansion) even in the oscillating regime. Moreover, as shown by eq. (2.11), h may even grow asymptotically (instead of being constant) provided $\delta < -1/2$. As we shall see in Section 3, this condition is satisfied in particular, in the E frame, by a 3-dimensional phase driven by stretched strings.

Note that the amplification coefficient corresponding to a phase of accelerated contraction is different, in general, from the one corresponding to a phase of accelerated expansion. It is just because of this difference that the perturbation spectrum may remain unchanged, when an inflationary background is transformed into a deflationary one through a conformal rescaling, as we shall see in the following Section.

3. Pre-big-bang cosmology in the Brans-Dicke and Einstein frames

In a string cosmology context [7,12], a global (at least semi-quantitative) description of the evolution and symmetries of the early Universe is expected to be provided by the low-energy string effective action, possibly supplemented by the action S_m for macroscopic matter sources:

$$S = -\frac{1}{16\pi G}\int d^{d+1}x\sqrt{|g|}e^{-\phi}[R + (\partial_\mu\phi)^2 - \frac{1}{12}H^2_{\mu\nu\alpha} + V] + S_m \tag{3.1}$$

Here $H_{\mu\nu\alpha}$ is the antisymmetric tensor field strength, and V a (possibly non-zero) dilaton potential.

In this paper we will consider a $(d+1)$-dimensional, anisotropic metric background of the Bianchi I type, with time-dependent dilaton,

$$g_{00} = 1 \quad , \quad g_{ij} = -a_i^2 \delta_{ij} \quad , \quad \phi = \phi(t) \quad , \quad i, j = 1, 2, ..., d \tag{3.2}$$

and with vanishing $H_{\mu\nu\alpha}$ and $V(\phi)$. The additional matter sources, which are decoupled from the dilaton in this frame, will be represented by a perfect fluid with anisotropic pressure:

$$T_0^0 = \rho \quad , \quad T_i^j = -p_i \delta_i^j = -\gamma_i \rho \delta_i^j \ . \tag{3.3}$$

By defining as usual [8,7,12]

$$\overline{\phi} = \phi - \ln \sqrt{|g|} \quad , \quad \overline{\rho} = \rho \sqrt{|g|} \quad , \quad \overline{p} = p \sqrt{|g|} \tag{3.4}$$

the field equations following from the variation of the action (3.1) can be written in the form [8]

$$\dot{\overline{\phi}}^2 - 2\ddot{\overline{\phi}} + \sum_i H_i^2 = 0 \tag{3.5}$$

$$\dot{\overline{\phi}}^2 - \sum_i H_i^2 = \overline{\rho} e^{\overline{\phi}} \tag{3.6}$$

$$2(\dot{H}_i - H_i \dot{\overline{\phi}}) = \overline{p}_i e^{\overline{\phi}} \tag{3.7}$$

where $H_i = \dot{a}_i / a_i$, and we use units in which $8\pi G = 1$. Their combination gives the usual conservation equation

$$\dot{\overline{\rho}} + \sum_i H_i \overline{p}_i = 0 \ . \tag{3.8}$$

By applying the general procedure illustrated in [7], the background field variables can be separated, and the equations can be integrated exactly, by introducing a suitable time-like coordinate x such that

$$\overline{\rho} = \frac{1}{L} \frac{dx}{dt} \tag{3.9}$$

(L is a constant with dimensions of length, in such a way that x is dimensionless). For constant γ_i we obtain the following general exact solution of eqs.(3.5–3.7) (a similar problem was first solved in a different context in [13]):

$$a_i = a_{0i} |(x - x_+)(x - x_-)|^{\gamma_i/\alpha} \left| \frac{x - x_+}{x - x_-} \right|^{\alpha_i} \tag{3.10}$$

$$e^{\overline{\phi}} = e^{\phi_0}|(x-x_+)(x-x_-)|^{-1/\alpha}|\frac{x-x_+}{x-x_-}|^{-\sigma} \tag{3.11}$$

$$\overline{\rho} = \frac{\alpha}{4L^2}e^{\phi_0}|(x-x_+)(x-x_-)|^{(\alpha-1)/\alpha}|\frac{x-x_+}{x-x_-}|^{-\sigma} \tag{3.12}$$

where

$$\alpha = 1-\sum_i \gamma_i^2 \quad , \quad \sigma = \sum_i \alpha_i\gamma_i \quad , \quad \alpha_i = \frac{\alpha x_i + \gamma_i(\sum_i \gamma_i x_i - x_0)}{\alpha[(\sum_i \gamma_i x_i - x_0)^2 + \alpha(\sum_i x_i^2 - x_0^2)]^{1/2}}$$

$$x_\pm = \frac{1}{\alpha}\{\sum_i \gamma_i x_i - x_0 \pm [(\sum_i \gamma_i x_i - x_0)^2 + \alpha(\sum_i x_i^2 - x_0^2)]^{1/2}\} \tag{3.13}$$

and a_0, ϕ_0, x_0, x_i are integration constants.

This solution has various interesting properties, which we shall discuss elsewhere [14]. Here we only note that there are two curvature singularities at $x = x_\pm$, and that the region between the singularities is unphysical, in the sense that the critical density parameter

$$\Omega(x) \equiv \frac{\rho e^{\phi}}{(d-1)\sum_i H_i^2} = \frac{(x+x_0)^2 - \sum_i(\gamma_i x + x_i)^2}{(d-1)\sum_i(\gamma_i x + x_i)^2} \tag{3.14}$$

becomes negative. This parameter tends to zero at the singularities, and in this limit the metric (3.10) goes over to the vacuum solutions of string cosmology [15,8]. For $x \to x_\pm$ one finds indeed

$$a_i(t) \sim |t-t_\pm|^{\beta_i^\pm} \ , \tag{3.15}$$

where

$$\beta_i^\pm = \frac{x_i \pm \gamma_i x_\pm}{x_0 + x_\pm} \qquad , \qquad \sum_i(\beta_i^\pm)^2 = 1 \ . \tag{3.16}$$

However, because of the neglect in the original action (3.1) of truly "stringy" contributions (such as α' and loop corrections), this solution is not expected to provide a reliable description of the very high curvature regime. The appropriate range of validity of the solution is instead the large $|x|$ limit, and in particular $x \to -\infty$, where it provides a typical example of pre-big-bang evolution, characterized by acceleration and growing curvature scale [7].

If we consider, in particular, the isotropic case with negative pressure ($a_i = a, \gamma_i = \gamma < 0$ for all d spatial directions), then at large negative x we have $|x| \sim |t|^{\alpha/(2-\alpha)}$, and the solution (3.10-3.12) becomes, in this limit,

$$a(t) \sim (-t)^{2\gamma/(1+d\gamma^2)} \qquad , \qquad \overline{\phi}(t) \sim -\frac{1}{\gamma}\ln a$$

$$\phi = \overline{\phi} + d \ln a \sim \frac{d\gamma - 1}{\gamma} \ln a \quad , \quad \overline{\rho} \sim a^{-d\gamma} \, . \tag{3.17}$$

For $\gamma = -1/d$, which is the typical equation of state for a perfect gas of stretched (or unstable) strings [16], one thus recovers the particular solution already considered in [7,12] ("string-driven" pre-big-bang). More generally, however, the background (3.17) describes a phase of superinflationary expansion, $H > 0, \ddot{a}/a > 0$, and growing curvature scale, $\dot{H} > 0$, for all $\gamma < 0$.

This is the picture in the BD frame, which may be regarded as the natural one in a string theory context [3]. The passage to the E frame, defined as the frame in which the graviton and dilaton kinetic terms are diagonalized and the action takes the standard form,

$$S_E = \frac{1}{16\pi G} \int d^{d+1}x \sqrt{|\tilde{g}|}[-\tilde{R}(\tilde{g}) + \frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu \tilde{\phi} \partial_\nu \tilde{\phi}] + S_m \, , \tag{3.18}$$

is obtained through the conformal rescaling

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} e^{-2\phi/(d-1)} \quad , \quad \tilde{\phi} = \sqrt{\frac{2}{d-1}} \phi. \tag{3.19}$$

The E-transformed scale factor, $\tilde{a}$, and cosmic time coordinate, $\tilde{t}$, are thus related to the original BD ones by

$$\tilde{a} = a e^{-\phi/(d-1)} \quad , \quad d\tilde{t} = dt e^{-\phi/(d-1)}. \tag{3.20}$$

The pre-big-bang configuration (3.17) becomes, in the E frame,

$$\begin{aligned} &\tilde{a}(\tilde{t}) \sim (-\tilde{t})^\beta \quad , \quad \tilde{\phi} \sim \sqrt{\frac{2}{d-1}} \frac{(d-1)(1-d\gamma)}{(\gamma-1)} \ln \tilde{a} \\ &\tilde{\rho} \sim \tilde{a}^{-2/\beta} \quad , \quad \beta = \frac{2(1-\gamma)}{(d-1)(1+d\gamma^2) - 2(d\gamma - 1)} \end{aligned} \tag{3.21}$$

where $\tilde{\rho}$ is conformally related to the original density ρ as

$$\tilde{\rho} = \rho \frac{\sqrt{|g|}}{\sqrt{|\tilde{g}|}} = \rho e^{\phi(d+1)/(d-1)} \tag{3.22}$$

(see for instance [17]). For all $d > 1$ and $\gamma < 0$, the transformed metric (3.21) satisfies

$$\frac{\ddot{\tilde{a}}}{\tilde{a}} < 0 \quad , \quad \tilde{H} < 0 \quad , \quad \dot{\tilde{H}} < 0 \, , \tag{3.23}$$

where $\tilde{H} = \dot{\tilde{a}}/\tilde{a}$, and the dot denotes here differentiation with respect to $\tilde{t}$. The BD superinflation thus becomes an accelerated contraction of the type (2.3).

This result is a consequence of the non-trivial evolution of the dilaton background that determines the transformation between the two frames, and it is of crucial importance. It implies that, if inflation is long enough in the BD frame to solve the kinematical problems of the standard model, then such problems are also solved in the E frame. Indeed, according to eq. (3.20), the two frames have the same conformal time

$$d\tilde{\eta} = \frac{d\tilde{t}}{\tilde{a}(\tilde{t})} = \frac{dt}{a(t)} = d\eta \tag{3.24}$$

and we have shown in Section 2 that the conditions to be satisfied for solving the kinematical problems, when expressed in conformal time, are the same for both superinflationary expansion and accelerated contraction.

Moreover, the spectrum of the metric perturbations amplified in the course of the background evolution is also the same in both frames. This can be easily shown by considering , for instance, the case of tensor perturbations, and assuming a generic model of background evolution characterized by the transition (at $\eta = \eta_1$) from the accelerated phase to the standard radiation-dominated one. In conformal time, such evolution can be parametrized as

$$\begin{aligned} &a \sim (-\eta)^{-\delta} \quad , \quad \phi \sim \epsilon \ln a \quad , \quad \eta << -\eta_1 \\ &a \sim \eta \quad , \quad \phi \sim const \quad , \quad \eta >> -\eta_1 \, . \end{aligned} \tag{3.25}$$

In order to verify the equality of the spectral behaviour, it is crucial to take into account the fact that not only the background solutions, but also the perturbation equations are different, when the frame is changed. In the BD frame, the tensor perturbation equation contains explicitly the contribution of the dilaton background, and for each component of h_μ^ν the equation can be written [7,17]

$$\psi'' + (k^2 - V)\psi = 0 \, , \tag{3.26}$$

where [7,17]

$$\begin{aligned} &\psi = h a^{(d-1)/2} e^{-\phi/2} \\ &V = \frac{(d-1)a''}{2a} - \frac{\phi''}{2} + \frac{(d-1)(d-3)a'^2}{4a^2} + \frac{\phi'^2}{4} - \frac{(d-1)a'\phi'}{2a} \, . \end{aligned} \tag{3.27}$$

By matching the solutions of (3.26) corresponding to the two phases of background evolution, one can compute the Bogoliubov coefficients relating $|in\rangle$ and

$|out\rangle$ vacua, and describing the associated graviton production. For co-moving frequencies k that are small enough with respect to the height of the effective potential barrier ($k\eta_1 << 1$), the modulus of the Bogoliubov coefficient is [7,18]

$$|c_-(k)| \simeq (k\eta_1)^{-|\nu|-1/2} \tag{3.28}$$

where

$$\nu = \frac{\delta}{2}(d-1-\epsilon) + \frac{1}{2} \tag{3.29}$$

and the corresponding spectral distribution of gravitons is determined as $\rho(k) = k^4|c_-|^2$. In the case of four-dimensional exponential inflation ($\delta = 1, d = 3, \epsilon = 0$) one thus finds, in particular, the flat Harrison-Zeldovich spectrum.

In the more general case of the background (3.17), one finds that, in conformal time, the kinematics is parametrized according to eq. (3.25) by

$$\delta = -\frac{2\gamma}{1-2\gamma+d\gamma^2} \quad , \quad \epsilon = \frac{d\gamma-1}{\gamma} \ . \tag{3.30}$$

The coefficient $|\nu|$ determining the pre-big-bang graviton spectrum in the BD frame is thus

$$|\nu| = \frac{1}{2}|\frac{d\gamma^2-1}{1-2\gamma+d\gamma^2}| \ . \tag{3.31}$$

In the E frame, there is no explicit dilaton contribution to the perturbation equation for h, which is exactly the same equation as that satisfied by a minimally coupled scalar field [11] (the dilaton contribution, however, is implicitly contained in the rescaled metric background). Such an equation can still be written in the form (3.26), (3.27), but with $\phi = const$. As a consequence, the spectral coefficient $|\nu|$ of eq. (3.28) is determined by the metric background only, and becomes

$$\nu = \frac{\tilde{\delta}}{2}(d-1) + \frac{1}{2} \ , \tag{3.32}$$

where $\tilde{\delta}$ is the exponent parametrizing, in conformal time, the evolution of the contracting E metric (3.21):

$$\tilde{\delta} = \frac{2(\gamma-1)}{(d-1)(1-2\gamma+d\gamma^2)} \ . \tag{3.33}$$

This value, when inserted into eq. (3.32), provides exactly the same expression for $|\nu|$ as in eq. (3.31), and thus the same graviton spectrum as in the BD frame.

We want to stress, finally, that the same results hold in the case of conformal vacuum backgrounds, namely for solutions of eqs. (3.5-3.7) with $\rho = p = 0$ [8,15]

(the general vacuum solution for the action (3.1) with non-zero $H_{\mu\nu\alpha}$ is given in [19]).

In the vacuum case the analogous of the isotropic, d-dimensional solution (3.17) is, in the BD frame,

$$\begin{aligned} & a_\mp(t) \sim |t|^{\mp 1/\sqrt{d}} \\ & \phi_\mp(t) \sim -(1 \pm \sqrt{d}) \ln |t| = \pm(\sqrt{d} \pm d) \ln a_\mp \end{aligned} \tag{3.34}$$

The two signs correspond to the two duality-related solutions [8], and the upper sign describes a "dilaton-driven", pre-big-bang, superinflationary expansion for t ranging from $-\infty$ to 0.

In the E frame the solution (3.34) becomes (in conformal time)

$$\tilde{a}(\tilde{\eta}) = |\tilde{\eta}|^{1/(d-1)} \quad , \quad \tilde{\phi}(\tilde{\eta}) = \mp\sqrt{2d(d-1)} \ln \tilde{a} \tag{3.35}$$

and it always describes an accelerated contraction of the type (2.3), independently of the choice of sign in eq. (3.34). It is interesting to note that the duality transformation, which is represented in the BD frame as an inversion of the scale factor and a related dilaton shift,

$$a_+ \to a_- = a_+^{-1} \quad , \quad \phi_+ \to \phi_- = \phi_+ - 2d \ln a_+ \tag{3.36}$$

becomes, in the E frame, a transformation between what we may call a strong-coupling and a weak-coupling regime, $\tilde{\phi} \to -\tilde{\phi}$, without changing the metric background described by $\tilde{a}$.

4. Conclusions

The main goal of this paper has been to show that, for what concerns the solution of the kinematical problems (horizon, flatness) of the standard model, and the amplification of the vacuum fluctuations, an accelerated contraction of the metric is equally good as an accelerated expansion.

This observation was motivated by the fact (also discussed in this paper) that accelerated contraction is the behaviour of the metric in a general pre-big-bang cosmological string scenario, when seen in the Einstein frame. Indeed, as already stressed in [7], there are only two ways of implementing a phase of cosmic acceleration and simultaneous growth of the curvature scale: accelerated contraction and superinflationary (or pole-like) expansion. The latter corresponds to the pre-big-bang picture in the conformally related Brans-Dicke frame.

Obviously, a contracting phase cannot dilute the relic abundance of some unwanted remnants, such as the monopoles of the GUT phase transition. However, the same is true for the pre-big-bang scenario in the BD frame, as well as for all models in which the phase of inflationary expansion occurs at some higher fundamental (near Planckian) scale, which is indeed what is expected in a string cosmology context. In this respect, we recall [7] that a pre-big-bang phase should be regarded not necessarily as an alternative, but possibly as a complement to the more conventional inflationary models, which cannot be extended (at least semiclassically) beyond the Planck era.

Moreover, it is clear that deflationary contraction is adiabatic for what concerns radiation, just like the usual inflationary expansion. Therefore, as recently stressed also in [20], a kinematical modification of the standard model can explain the large present value of the cosmic black-body entropy, only if the accelerated evolution is matched to the standard one through a phase dominated by some non-adiabatic process (the so-called "reheating" era).

In the BD picture of the pre-big-bang scenario (see eq. (3.17)), the radiation is supercooled and diluted with respect to the sources that drive inflation. The conservation equation (3.8) leads in fact to an effective source temperature $T_s \sim a^{-d\gamma}$, which grows together with the scale factor for $\gamma < 0$, and satisfies

$$\frac{T_s}{T_r} = \frac{\rho_\gamma}{\rho_r} \sim a^{1-d\gamma} \tag{4.1}$$

(r corresponds here to the radiation-like equation of state, $\gamma = 1/d$). The reheating process is thus expected to represent, in this frame, a sort of non-adiabatic conversion of the hot sources into radiation, such as a possible isothermal decay of the highly excited states of a gas of stretched strings [7].

In the E frame (see eq. (3.21)) the fluid sources satisfy a modified conservation equation,

$$\dot{\tilde{\rho}} + d\tilde{H}(\tilde{\rho} + \tilde{p}) - \frac{\dot{\tilde{\phi}}}{\sqrt{2(d-1)}}(\tilde{\rho} - d\tilde{p}) = 0 \; . \tag{4.2}$$

Radiation still evolves adiabatically, now with a blue-shifted temperature because of the contraction, $\tilde{T}_r \sim \tilde{a}^{-1}$. The effective temperature of the pre-big-bang sources is also blue-shifted, however, since, in the perfect fluid approximation, eq. (4.2) leads to

$$\tilde{T}_s \sim \tilde{a}^{(d^2\gamma^2+1-d\gamma-d\gamma^2)/(\gamma-1)} \tag{4.3}$$

and thus

$$\frac{\tilde{T}_s}{\tilde{T}_r} = \frac{\tilde{\rho}_s}{\tilde{\rho}_r} \sim \tilde{a}^{\gamma(d-1)(d\gamma-1)/(\gamma-1)} \; . \tag{4.4}$$

For $\tilde{a} \to 0$ the temperature of the sources that drive the acceleration ($\gamma < 0, d > 1$) is always growing, even with respect to the radiation temperature. The physical picture of reheating as a non-adiabatic decay of the hot sources is still valid, therefore, also in the Einstein frame.

We would like to stress, finally, that the absence of problems related to some "preferred frame" description of a string cosmology inflation is to be ascribed, to a large extent, to the crucial role played by the dilaton field, which transforms conformally a superinflationary expansion into a deflationary contraction. This is to be traced back to the duality properties of the string effective action [8,12,19,21], and thus gives support to the consistency of an approach to string cosmology based on the effective action (3.1).

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