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Electromagnetic Origin of the CMB Anisotropy in String Cosmology

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Abstract

In the inflationary scenarios suggested by string theory, the vacuum fluctuations of the electromagnetic field can be amplified by the time-evolution of the dilaton background, and can grow large enough to explain both the origin of the cosmic magnetic fields and of the observed CMB anisotropy. The normalization of the perturbation spectrum is fixed, and implies a relation between the perturbation amplitude at the COBE scale and the spectral index n. Working within a generic two-parameter family of backgrounds, a large scale anisotropy $\Delta T/T \simeq 10^{-5}$ is found to correspond to a spectral index in the range $n \simeq 1.11 - 1.17$.

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In the standard inflationary scenario, the anisotropy of the Cosmic Microwave Background (CMB) recently detected by COBE [1, 2] is usually attributed to the cosmological amplification of the quantum fluctuations of the metric. These consist of both tensor (gravitational waves) and scalar perturbations, the latter being coupled to the energy density fluctuations. The observed inhomogeneities of the CMB radiation could also emerge from the vacuum fluctuations of the electromagnetic radiation itself, through their contribution to $\delta\rho/\rho$. However, the minimal coupling of photons to the metric is conformally invariant in $d = 3$ spatial dimensions, and it is difficult, in general, to obtain a significant amplification of the electromagnetic fluctuations in the context of the standard inflationary scenario [3].

In the inflationary models based on the low energy limit of critical superstring theory [4, 5, 6], the electromagnetic field $F_{\mu\nu}$ is coupled not only to the metric ($g_{\mu\nu}$) but also to the dilaton (ϕ) background, according to the dimensionally reduced, effective action [7]

$$S = -\int d^4x |det(g_{\mu\nu})|^{1/2} e^{-\phi}(R + \partial_\mu\phi\partial^\mu\phi + \frac{1}{4}F_{\mu\nu}F^{\mu\nu}) \tag{1}$$

In this context ϕ, which controls the tree-level four-dimensional gauge coupling $g^2 = e^\phi$, is rapidly changing in time and can amplify directly the electromagnetic vacuum fluctuations [8, 9]. On the other hand, tensor metric perturbations, as well as the scalar perturbations induced by dilatonic fluctuations, are characterized in this context by "blue" spectra strongly tilted towards large frequencies [6, 10, 11], with an amplitude on large angular scales which is far too low to match COBE's observations. It becomes thus crucial, for the purpose of testing string theory through its astrophysical consequences [12], to decide whether or not the CMB anisotropy could originate from the vacuum quantum fluctuations of the electromagnetic field itself, after they have been amplified by the time-evolution of the dilaton background.

The purpose of this paper is to show that, in an appropriate range for two arbitrary parameters characterizing a generic string cosmology scenario, such an electromagnetic origin of the anisotropy is possible, is consistent with the linearization of perturbations around a nearly homogeneous background, and is also consistent with various phenomenological constraints (following from pulsar timing data and nucleosynthesis). In that range of parameters, moreover, the same mechanism that amplifies the electromagnetic vacuum fluctuations can also be responsible for the production of the observed galactic (and extragalactic) magnetic fields [8], and can thus explain why the average energy density of the cosmic magnetic fields and of the CMB radiation are of the same order.

Let us consider the evolution of the quantum fluctuations of the electromagnetic field, according to the action (1). In a four-dimensional, conformally flat cosmological background, the Fourier modes A_k^μ of the (canonically normalized) electromagnetic variable satisfy the equation

$$A_k'' + [k^2 - V(\eta)]A_k = 0 \quad , \qquad V(\eta) = g(g^{-1})'' \quad , \qquad g(\eta) \equiv e^{\phi/2} \tag{2}$$

This equation is valid for each polarization component, and is obtained from the action (1) by imposing the standard radiation gauge which can be written, in our time-dependent background, $\partial_\nu[e^{-\phi}\partial^\mu(e^{\frac{\phi}{2}}A^\nu)] = 0$ (a prime denotes differentiation with respect to the conformal time η). Note the analogy with the tensor part of the metric perturbation equations [13], which has the same form as (2) with the inverse of the coupling, g^{-1}, replaced simply by the Einstein-frame scale factor $a_E = g^{-1}a$.

In our context $V(\eta)$ represents an effective potential barrier, approaching zero as $\eta \to \pm\infty$. A mode of comoving frequency k, "hitting" the barrier at the time $\eta = \eta_{ex}(k)$, is thus parametrically amplified just like in the case of tensor perturbations. The modulus of the Bogoliubov coefficient $|c(k)|$ describing this amplification turns out to be given, to leading order, by the ratio of the gauge coupling at reentry and at exit [8]

$$|c(k)| \simeq \frac{g_{re}}{g_{ex}} \equiv \exp\{-\frac{1}{2}[\phi(\eta_{ex}) - \phi(\eta_{re})]\} \tag{3}$$

where $\eta_{ex}(k)$ and $\eta_{re}(k)$ are defined by $k^2 = |V(\eta_{ex})| = |V(\eta_{re})|$. This equation is only valid for k smaller than the maximal eight of the barrier, $k^2 < |V_{max}|$, while $|c(k)| \simeq 0$, in our approximation, if $k^2 > |V_{max}|$ (see also [14] for a detailed computation of $c(k)$). The Bogoliubov coefficient $c(\omega)$ defines the energy distribution $\rho(\omega)$ of the amplified fluctuation spectrum, through the relation $d\rho/d\ln\omega \simeq (\omega^4/16\pi^2)|c(\omega)|^2$, where $\omega(t) = k/a(t)$ is the red-shifted, present value of the amplified proper frequency. We are interested, in particular, in the ratio

$$r(\omega) = \frac{\omega}{\rho_{cmb}}\frac{d\rho}{d\omega} \simeq \frac{\omega^4}{16\pi^2\rho_{cmb}}\left[\frac{g_{re}(\omega)}{g_{ex}(\omega)}\right]^2 \tag{4}$$

measuring the fraction of electromagnetic energy stored in the mode ω, relative to the CMB energy density ρ_{cmb}.

In order to compute this ratio, we must use the explicit time evolution of the dilaton background, as predicted by the inflationary models based on the string effective action [5, 6]. In such models the dilaton undergoes an accelerated evolution from the string perturbative vacuum ($g = 0$, $\phi = -\infty$) towards the strong coupling regime, where it is expected to remain frozen at its present value ($g = g_1 = e^{\phi_1/2}$ = const). The initial phase of growing curvature and dilaton coupling (also called "pre-big-bang" scenario [5, 6]) is driven by the kinetic energy of the dilaton field (with negligible contributions from the dilaton potential), and can be described in terms of the lowest order string effective action only up to the time $\eta = \eta_s$ at which the curvature reaches the string scale $H_s = \lambda_s^{-1} \equiv (\alpha')^{-1/2}$ [determined by the string tension $(\alpha')^{-1}$]. The value ϕ_s of the dilaton at $\eta = \eta_s$ is the first important parameter of our scenario. Provided such value is sufficiently negative it is also arbitrary, since we are still in the perturbative regime at $\eta = \eta_s$, and there is no perturbative potential to break invariance under shifts of ϕ.

For $\eta > \eta_s$, however, higher orders in α' become important in the string effective action, and the background enters a genuinely "stringy" phase of unknown duration, assumed to end at $\eta = \eta_1$ with a smooth transition to the standard radiation-dominated regime (where $\phi = \phi_1$ =const). As shown in [15], it is impossible to have a graceful exit to standard cosmology without such an intermediate stringy phase, after which the dilaton, feeling a non-trivial potential, is attracted to its present constant value. The total red-shift $z_s = a_1/a_s$ encountered during the stringy epoch, will be the second crucial parameter (besides ϕ_s) entering our discussion. For our purpose, two parameters are enough to specify completely our model of background, if we accept that during the string phase the curvature stays controlled by the string scale, that is $H \simeq \lambda_s^{-1}$ for $\eta_s < \eta < \eta_1$.

We will work in the so-called String frame, in which the string scale λ_s is constant, while the Planck scale $\lambda_p = \lambda_s e^{\phi/2}$ grows from zero (at the initial vacuum) to its present value $\lambda_p \simeq 10^{-19} (GeV)^{-1}$ reached at the end of the string phase. In the low energy phase driven by the dilaton, the dilaton evolution is exactly known [4, 5, 6] and is given, in the String frame, by

$$\phi = (3+\sqrt{3})\ln a + const = -\sqrt{3}\ln|\eta| + const, \qquad \eta < \eta_s \tag{5}$$

Internal dimensions affects slightly the above numerical constants without affecting the results of this paper. During the string phase the curvature stays constant ($H \simeq \lambda_s^{-1}$) so that, in this frame, the string epoch is characterized by a de Sitter-like evolution of the metric background, with $\eta_s/\eta_1 = a_1/a_s \equiv z_s$. In addition, the rate of growth of the coupling during the string phase should also be bounded by the string scale (like the space-time curvature). Defining $\dot{\phi} \simeq 2\beta H \simeq 2\beta\lambda_s^{-1}$, where β is some constant of order of unity [for instance, eq.(5) gives $\beta \simeq 2.37$ in the dilaton driven epoch], the "average" time behaviour of the dilaton between η_s and η_1 can be parameterized as

$$\phi = -2\beta\ln|\eta| + const, \qquad \beta = -\frac{(\phi_s - \phi_1)}{2\ln z_s}, \qquad \eta_s < \eta < \eta_1 \tag{6}$$

For this model of background, the effective potential $V(\eta)$ of eq.(2) grows like η^{-2} for $\eta \to 0_-$ in the dilaton-driven phase, keeps growing during the string phase , where it reaches a maximal value $\sim \eta_1^{-2}$ around the final time η_1, and then goes rapidly to zero at the beginning of the radiation dominated era, where $g(\eta) = g_1$ =const. A mode hitting the barrier (or "crossing the horizon") during the dilaton phase thus remains under the barrier during the whole string phase. As a consequence, $\eta_{re} > \eta_1$ and $\phi_{re}(\omega) = \phi_1$ =const for all ω.

The spectral distribution $r(\omega)$ is now completely fixed in terms of our two parameters ϕ_s and z_s. Using $\rho_{cmb}(\eta_1) \simeq M_p^2 H_1^2 \simeq g_1^{-2} H_1^4$, ($M_p$ being the present value of the Plank Mass), eq.(4) leads simply to

$$r(\omega) \simeq \frac{g_1^2}{16\pi^2}\left(\frac{\omega}{\omega_1}\right)^4 e^{-\Delta\phi_{ex}(\omega)} \tag{7}$$

where $\Delta\phi_{ex}(\omega) = \phi_{ex}(\omega) - \phi_1$, and $\omega_1 = H_1 a_1/a \simeq (g_1/4\pi)^{1/2} 10^{11}$Hz is the maximal amplified frequency (the amplitude of modes $\omega > \omega_1$ is exponentially suppressed, and will be neglected throughout this paper). For modes $\omega > \omega_s \equiv \omega_1/z_s$, crossing the horizon during the string phase, we thus obtain the spectrum

$$r(\omega) \simeq \frac{g_1^2}{16\pi^2}\left(\frac{\omega}{\omega_1}\right)^{4-2\beta} \qquad , \quad \omega_s < \omega < \omega_1 \tag{8}$$

For modes crossing the horizon in the dilaton phase ($\omega < \omega_s$) we have instead, from eqs.(5,6,7),

$$r(\omega) \simeq \frac{g_1^2}{16\pi^2}\left(\frac{\omega}{\omega_1}\right)^{4-\sqrt{3}} z_s^{-\sqrt{3}} e^{-\Delta\phi_s} \qquad , \quad \omega < \omega_s \tag{9}$$

The above electromagnetic spectrum has been obtained by using a homogeneous and isotropic model of metric (and dilaton) background. It is thus valid provided the amplified fluctuations remain, at all times, small perturbations of a nearly homogeneous configuration, with a negligible back-reaction on the metric. This requires $r(\omega)\lesssim 1$, at all ω. This bound is satisfied, for $(g_1/4\pi)\lesssim 1$, provided

$$(g_s/g_1) \gtrsim z_s^{-2} \tag{10}$$

which restricts the allowed region in the two-dimensional parameter space (z_s, g_s) of our background model.

Consider now a length scale ω^{-1} reentering the horizon in the radiation era. The associated electromagnetic perturbation represents, at the time of reentry, a coherent field over the horizon scale which, consistently with the bound (10), could be strong enough to seed the galactic dynamo mechanism, or the galactic magnetic field itself [8]. Soon after reentry, however, the perturbation may be expected to thermalize and homogenize rapidly since, unlike metric (scalar and tensor) perturbations, photons are not decoupled from matter in the radiation era, and the shape of their spectrum remains frozen only outside the causal horizon.

For all scales reentering the horizon after the decoupling epoch, however, the electromagnetic perturbations can contribute to the inhomogeneity of the CMB radiation, with a spectral distribution $r(\omega)$ determined by eqs.(8,9). Such a spectrum grows with frequency, with the position of its peak fixed in the plane (ω, r) (i.e. $r \simeq g_1^2/16\pi^2$ at $\omega \simeq \omega_1$). The perturbation amplitude at a given scale ω can thus be uniquely determined as a function of the unknown duration and slope of the "stringy" branch (8) of the spectrum, namely in terms of the two parameters z_s, g_s.

A large enough perturbation to match COBE's observations [1, 2] ($\Delta T/T \simeq 10^{-5}$ at the present horizon scale ω_0) would require a spectral energy density such that, in terms of our variable $r(\omega)$,

$$r(\omega_0) \simeq 10^{-6}, \qquad\qquad \omega_0 \simeq 10^{-18} Hz \tag{11}$$

(the proof of this statement is quite non-trivial, and will be given in detail elsewhere. The contribution to the anisotropy is measured by $\delta\rho/\rho$, according to the standard Sachs-Wolfe effect, and turns out to be quadratic in terms of our stochastic electromagnetic variable). If the scale ω_0 crossed the horizon during the dilaton phase (i.e. if $z_s \lesssim 10^{29}$), the condition (11) is compatible with eq.(10) only in a very small region of parameter space. In such case an electromagnetic origin of the CMB anisotropy is possible, but requires a certain degree of fine-tuning. If, on the contrary, the horizon crossing of ω_0 occurred during the string phase ($z_s \gtrsim 10^{29}$), and we define as usual the spectral index n for eq.(8) as $n-1 = 4-2\beta$, then the present electromagnetic contribution to the anisotropy at the scale ω_0 can be written as

$$\log_{10} r(\omega_0) \simeq -29(n-1) + \frac{1}{2}(5-n)\log_{10}(\frac{g_1}{4\pi}), \qquad\qquad z_s \gtrsim 10^{29} \tag{12}$$

This equation (which is the main result of this paper) relates the perturbation amplitude at the scale ω_0 to the spectral index n, and provides a condition on the parameter space (z_s, ϕ_s) which is always compatible with eq.(10), as $(g_1/4\pi) \leq 1$. The requirement (11), in particular, is satisfied for

$$n \simeq \frac{35 + \frac{5}{2}\log_{10}(g_1/4\pi)}{29 + \frac{1}{2}\log_{10}(g_1/4\pi)} \tag{13}$$

Typical values of $(g_1/4\pi)^2$ range from 10^{-1} to 10^{-3} [16]. The COBE observations are thus accounted for, in this context, for values of the spectral index that are typically in the range $n \simeq 1.11 - 1.17$. This spectral index is certainly flat enough to be well consistent with the analysis of the first two years of the COBE DMR data [2]. Such a slightly growing ($n > 1$, also called "blue" [17]) spectrum is also consistent with the upper bound $n < 1.5$, recently obtained by using the COBE FIRAS limits on the CMB spectral distortions [18]. It should be noted that, according to eq.(13), n depends very weakly on the precise value of g_1, so that our estimate is quite stable, in spite of the rather large theoretical uncertainties about g_1. The above value of n corresponds in particular to an average value of $\dot{\phi}/2H \simeq \beta$ of about 1.9, which is not far from the value 2.37 characteristic of the dilaton-driven era.

We note, finally, that in view of the above arguments a value of n slightly larger than one could be interpreted as a possible indirect signature of string cosmology (see however [19]).

In order to give some concrete estimate of the phenomenological bounds relevant for the problem we will set $g_1/4\pi \simeq 1$ in the following [if the value of $(g_1/4\pi)^2$ would be smaller the constraints which we will discuss will be satisfied even better, thanks to the flatter ensuing spectrum]. In such case, the CMB anisotropy can receive a complete electromagnetic explanation provided the parameters of our background are constrained to lie on (or near) the half-line $\log_{10} g_s = -1.90 \log_{10} z_s$, $\log_{10} z_s > 29$. The resulting perturbation spectrum

$$r(\omega) \simeq \left(\frac{\omega}{10^{11} Hz}\right)^{6/29}, \qquad \omega < 10^{11} Hz \tag{14}$$

gives then $r \simeq 10^{-5}$ at the intergalactic scale $\omega_G = (1Mpc)^{-1}$, which is large enough to seed not only the galactic dynamo, but also the cosmic magnetic field directly [8]. This results holds in general for any realistic value of g_1, and leads to consider a scenario in which the CMB anisotropy and the primordial magnetic fields have a common origin. The peak value of order unity of the electromagnetic spectrum can then easily explain the (otherwise mysterious, to the best of our knowledge) coincidence that the total energy density of our galactic magnetic field, $\rho_B = \rho_{cmb} \int^{\omega_1} r(\omega) d\omega$, is of the same order of magnitude as the CMB energy density. We also note that, at the scale corresponding to the end of nucleosynthesis ($\omega_N \simeq 10^{-12}$Hz), eq.(14) predicts $r \simeq 10^{-4.7}$. It is thus automatically consistent with the bounds following from the presence of strong magnetic fields at nucleosynthesis time [20], which impose $r(\omega_N) \lesssim 0.05$.

The above discussion refers to the case in which all scales inside our present horizon crossed the horizon, for the first time, during the string phase, i.e. for $z_s \geq 10^{29}$. Such a phase was characterized by a de Sitter-like metric evolution, with a curvature scale of Planckian order. In the standard inflationary context such a background configuration is forbidden, as it would lead to an overproduction of tensor perturbations: a phase of constant Planckian curvature can last only up to a total red-shift $z \leq 10^{19}$ [21], to be compatible with the bounds obtained from pulsar timing data [22]. In a string cosmology context, however, we must recall that the de Sitter-like evolution of the metric refers to the String frame, where tensor metric perturbations are also coupled to the dilaton background [10].

As a consequence, the spectral distribution $r_g(\omega)$ of tensor perturbation is growing with frequency (instead of being flat like in the standard de Sitter scenario), with a peak value which is again of order one around 10^{11}Hz. Moreover, the growth is so fast ($r_g \sim \omega^{n+1}$, where n is the spectral index of the electromagnetic perturbations), that the contribution of r_g is negligible at the scale ω_0, and it is also largely consistent with the pulsar bound [22] which requires $r_g(\omega_P) \lesssim 10^{-2}$ at $\omega_P \simeq 10^{-8}$Hz.

A further remark related to the long duration of the string phase concerns the validity of the spectrum (8), which has been obtained from the tree-level, low energy action (1). It is true that, in the string phase, we may have corrections coming both from higher loops (expansion in e^{ϕ}) and from higher derivative terms (α' corrections). However, in order to reproduce the large scale anisotropy we have to work in a range of parameters where the dilaton is deeply in his perturbative regime. Eq.(14) holds in fact for $g_s = e^{\phi_s/2} \lesssim 10^{-55}$. We thus expect our results to be stable against loop corrections, at least at all the scales which are relevant for the observed anisotropy and for the generation of primordial magnetic fields.

As to the α' corrections, they are instead crucial in the basic assumption that the dilaton driven era leads to a quasi de Sitter epoch when the curvature reaches the string scale. Such corrections can also modify, in principle, the equation determining the evolution of electromagnetic fluctuations (eq.(2)). We are expanding, however, our perturbations around the vacuum background $F_{\mu\nu} = 0$. Therefore, no higher curvature correction of the form $(\alpha' F_{\mu\nu} F^{\mu\nu})^m$, $m \geq 2$, may provide significant contributions as long as we work in the region of parameter space in which perturbations can be consistently treated linearly, namely in the region in which eq.(10) is satisfied.

We thus believe that the main conclusion of this paper, namely that an electromagnetic origin of the CMB anisotropy is allowed in a realistic string cosmology scenario, is not only compatible with the various phenomenological bounds, but is also quite independent of many (unknown) kinematic details of the high energy "stringy" phase, preceding the phase of standard cosmological evolution.

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