CERN-TH/95-85
DFTT-26/95
April 1995

Primordial Magnetic Fields from String Cosmology

M. Gasperini
Dipartimento di Fisica Teorica, Via P. Giuria 1, 10125 Turin, Italy
M. Giovannini and G. Veneziano
Theory Division, CERN, CH-1211 Geneva 23, Switzerland

Abstract

Sufficiently large seeds for generating the observed (inter)galactic magnetic fields emerge naturally in string cosmology from the amplification of electromagnetic vacuum fluctuations due to a dynamical dilaton background. The success of the mechanism depends crucially on two features of the so-called pre-big-bang scenario, an early epoch of dilaton-driven inflation at very small coupling, and a sufficiently long intermediate stringy era preceding the standard radiation-dominated evolution.

To appear in **Phys. Rev. Lett.**

It is widely believed that the observed galactic (and intergalactic) magnetic fields, of microgauss strength, are generated and maintained by the action of a cosmic dynamo [1]. The dynamo model, as well as any other model, requires however a primordial seed field; in spite of many attempts [2]-[5], it is fair to say that no compelling mechanism has yet been suggested, which would be able to generate the required seed field coherent over the Mpc scale, and with an energy density to radiation density ratio $\rho_B/\rho_\gamma \gtrsim 10^{-34}$ (possibly much greater, according to a careful analysis of the turbolence of the interstellar medium [6]).

A priori, an appealing mechanism for the origin of the seed field is the cosmological amplification of the vacuum quantum fluctuations of the electromagnetic field, the same kind of mechanism as is believed to generate primordial metric and energy density perturbations. The minimal coupling of photons to the metric background is, however, conformally invariant (in $d = 3$ spatial dimensions). As a consequence, a cosmological evolution involving a conformally flat metric (as is effectively the case in inflation) cannot amplify magnetic fluctuations, unless conformal invariance is broken. Possible attempts to generate large enough seeds thus include considering exotic higher-dimensional scenarios, or coupling non-minimally the electromagnetic field to the background curvature [2] with some "ad hoc" prescription, or breaking conformal invariance at the quantum level through the so-called trace anomaly [4].

In critical superstring theory the electromagnetic field $F_{\mu\nu}$ is coupled not only to the metric ($g_{\mu\nu}$), but also to the dilaton background (ϕ). In the low energy limit such an interaction is represented by the string effective action [7], which reads, after reduction from ten to four external dimensions,

$$S = -\int d^4x\sqrt{-g}e^{-\phi}\left(R + \partial_\mu\phi\partial^\mu\phi + \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right) + ... \tag{1}$$

where $\phi = \Phi - \ln V_6 \equiv \ln(g^2)$ controls the tree-level four-dimensional gauge coupling (Φ being the ten-dimensional dilaton field, and V_6 the volume of the six-dimensional compact internal space) and the dots refer to other moduli originating from the compactification.

In the inflationary models based on the above effective action [8, 9] the dilaton background is not at all constant, but undergoes an accelerated evolution from the string perturbative vacuum ($\phi = -\infty$) towards the strong coupling regime, where it is expected to remain frozen at its present value. In this context, the quantum fluctuations of the electromagnetic field can thus be amplified *directly* through their coupling to the dilaton, according to eq. (1). In the following we will discuss the conditions under which such a mechanism is able to produce large enough primordial magnetic fields to seed the galactic dynamo (a scalar-vector coupling similar to that of eq. (1) was previously discussed in [3], but ϕ was there identified with the conventional inflaton undergoing a dynamical evolution much different from the dilaton evolution considered here).

Let us first define a few important parameters of the inflationary scenario (also called "pre-big-bang" scenario) discussed in [9]. The phase of growing curvature and dilaton coupling ($\dot{H} > 0$, $\dot{\phi} > 0$), driven by the kinetic energy of the dilaton field, is correctly described in terms of the lowest order string effective action only up to the conformal time $\eta = \eta_s$ at which the curvature reaches the string scale $H_s = \lambda_s^{-1}$ ($\lambda_s \equiv \sqrt{\alpha'}$ is the fundamental length of string theory). A first important parameter of this cosmological model is thus the value ϕ_s attained by the dilaton at $\eta = \eta_s$. Provided such a value is sufficiently negative, it is also arbitrary, since there is no perturbative potential to break invariance under shifts of ϕ. For $\eta > \eta_s$ high-derivatives terms (higher orders in α') become important in the string effective action, and the background enters a genuinely "stringy" phase of unknown duration. It was shown in [10] that it is impossible to have a graceful exit to standard cosmology without such an intermediate stringy phase. Such stringy phase eventually ends at some conformal time η_1, in the strong coupling regime. At this time, the dilaton, feeling a non-trivial potential, freezes to its present constant value $\phi = \phi_1$, and the standard radiation-dominated era starts. The total duration η_1/η_s, or the total red-shift z_s encountered during the stringy epoch (i.e. between η_s and η_1), will be the second crucial parameter (besides ϕ_s) entering our discussion. For the purpose of this paper, two parameters are enough to specify completely our model of background, if we accept that during the string phase the curvature freezes at the string scale, that is $H \simeq \lambda_s^{-1}$ for $\eta_s < \eta < \eta_1$. We will work all the time in the String (also called Brans-Dicke) frame, in which test strings move along geodesic surfaces. In this frame the string scale λ_s is constant, while the Planck scale $\lambda_P = e^{\frac{\phi}{2}}\lambda_s$ grows from zero (at the initial vacuum) to its present value, reached at the end of the string phase. We have explicitly checked, however, that all our results also follow in the more commonly used (but less natural in a string context) Einstein frame.

We shall now consider, in the above background, the amplification of the quantum fluctuations of the electromagnetic field, assuming that, at the very beginning, it was in its vacuum state. In a four-dimensional, conformally flat background, the Fourier modes A_k^μ of the (correctly normalized) variable corresponding to the standard electromagnetic field, and obeying canonical commutation relations, satisfy the equation

$$A_k'' + [k^2 - V(\eta)]A_k = 0 \quad , \quad V(\eta) = g(g^{-1})'' \quad , \quad g(\eta) \equiv e^{\frac{\phi}{2}} \quad . \tag{2}$$

where a prime denotes differentiation with respect to the conformal time η. This equation is valid for each polarization component, and is obtained from the action (1) with the gauge condition $\partial_\nu[e^{-\phi}\partial^\mu(e^{\frac{\phi}{2}}A^\nu)] = 0$, which, for backgounds depending just on time, is equivalent to the conventional radiation gauge for electromagnetic waves in the vacuum. The effective potential $V(\eta)$ grows from zero like η^{-2}, for $\eta \to 0_-$, in the phase of dilaton-driven inflation, is expected to reach some maximum value during the string phase, and then goes rapidly to zero at the beginning of the radiation-dominated era (where $\phi =$ const). The approximate solution of eq. (2), for a mode k "hitting" the effective potential barrier at $\eta = \eta_{ex}$, and with initial conditions corresponding to vacuum fluctuations, is given by:

$$\begin{aligned} A_k &= \frac{e^{-ik\eta}}{\sqrt{k}} \quad , & \eta < \eta_{ex} \\ A_k &= g^{-1}(\eta)[C_k + D_k \int^\eta d\eta' \;\; g^2(\eta')] \quad , & \eta_{ex} < \eta < \eta_{re} \\ A_k &= \frac{1}{\sqrt{k}}[c_+(k)e^{-ik\eta} + c_-(k)e^{ik\eta}] \quad , & \eta > \eta_{re} \end{aligned} \tag{3}$$

where η_{ex} and η_{re} are the times of exit and reentry of the comoving scale associated with k, defined by the conditions $k^2 = |V(\eta_{ex})| = |V(\eta_{re})|$ ($C, D, c_\pm$ are integration constants). We are following here the usual convention for which a mode in the underbarrier region is referred to, somewhat improperly, as being "outside the horizon". Moreover, we are considering a background in which the potential $V(\eta)$ keeps growing in the string phase until the final time η_1, so that a mode crossing the horizon during dilaton-driven inflation remains outside the horizon during the whole string phase, i.e. $\eta_{re} \geq \eta_1$.

The Bogoliubov coefficients $c_\pm(k)$, determining the parametric amplification of a mode $k < |V(\eta_1)|$, are easily determined by matching these various solutions. One finds:

$$\begin{aligned} 2ike^{ik(\eta_{ex}\mp\eta_{re})}c_\pm = & \mp \frac{g_{ex}}{g_{re}}\left(-\frac{g_{re}{}'}{g_{re}} \mp ik\right) \pm \frac{g_{re}}{g_{ex}}\left(-\frac{g_{ex}{}'}{g_{ex}} + ik\right) \pm \\ & \pm \frac{1}{g_{ex}g_{re}}\left(-\frac{g_{ex}{}'}{g_{ex}} + ik\right)\left(-\frac{g_{re}{}'}{g_{re}} \mp ik\right)\int_{\eta_{ex}}^{\eta_{re}} g^2 d\eta \end{aligned} \tag{4}$$

Remembering that reentry occurs during the radiation epoch in which the dilaton freezes to a constant value ($g'_{re} \simeq 0, g_{re} \simeq 1$), it is easy to estimate the complicated-looking expression (4) and to obtain, for the leading contribution, the amazingly simple and intuitive result: $|c_-| \simeq g_{re}/g_{ex} \equiv \exp[-(\phi_{ex} - \phi_{re})/2]$. An important feature of this result is that, for perturbations which went out of horizon during the dilaton-driven phase, the final result does not depend upon the details of the background during the high curvature stringy phase. This is because the perturbation evolves in a purely kinematical way while outside the horizon. We are thus trusting the large wavelength part of our spectrum in spite of the present lack of understanding of the stringy phase.

The coefficient $|c_-|$ defines the energy density distribution ($\rho_B(\omega)$) over the amplified fluctuation spectrum, $d\rho_B/d\ln\omega \simeq \omega^4|c_-(\omega)|^2$, where $\omega = k/a$ is the red-shifted, present value of the amplified proper frequency. We are interested in the ratio

$$r(\omega) = \frac{\omega}{\rho_\gamma}\frac{d\rho_B}{d\omega} \simeq \frac{\omega^4}{\rho_\gamma}|c_-(\omega)|^2 \simeq \frac{\omega^4}{\rho_\gamma}\left(\frac{g_{re}}{g_{ex}}\right)^2 \quad , \tag{5}$$

which measures the fraction of electromagnetic energy stored in the mode ω (in particular, for the intergalactic scale, $\omega_G \simeq (1\text{Mpc})^{-1} \simeq 10^{-14}\text{Hz}$), relative to the background radiation energy ρ_γ. The ratio $r(\omega)$ stays constant during the phase of matter-dominated as well as radiation-dominated evolution, in which the universe behaves like a good electromagnetic conductor [2]. In terms of $r(\omega)$ the condition for a large enough magnetic field to seed the galactic dynamo is [2] $r(\omega_G) \gtrsim 10^{-34}$. Using the known value of ρ_γ and $e^{\phi_{re}}$ we thus find, from eqs.(5), the condition $g_{ex}(\omega_G) \lesssim 10^{-33}$.

In order to see whether or not the previous condition can be fulfilled, we go back to our two-parameter cosmological model. The discussion is greatly helped by looking at **Fig.1** where we plot, on a double-logarithmic scale against the scale factor a, the evolution of the coupling strength (i.e. of $e^{\phi/2}$) and that of the "horizon" size (defined here by $a|V|^{-1/2}$, whose behaviour coincides with that of the Hubble radius H^{-1} during the dilaton driven-epoch). The horizon curve has an inverted trapezoidal shape, corresponding to the fact that $V=0$ during the radiation era, that $\dot\phi$ and H are approximately constant during the string era, and that, during the dilaton-driven era [8, 9],

$$a=(-t)^{\alpha}, \qquad \alpha=-\frac{1}{\sqrt{3}}\sqrt{1-\Sigma}, \qquad a|V|^{-\frac{1}{2}}\simeq a(t)\int_t^0 dt' a^{-1}(t') \simeq a^{\frac{1}{\alpha}} \qquad . \tag{6}$$

Here $\Sigma \equiv \sum_i \beta_i^2$ represents the possible effect of internal dimensions, whose radii b_i shrink like $(-t)^{\beta_i}$ for $t \to 0_-$ (for the sake of definiteness we show in the figure the case $\Sigma = 0$). The shape of the coupling curve corresponds to the fact that the dilaton is constant during the radiation era, that $\dot\phi$ is approximately constant during the string era, and that it evolves like

$$g(\eta)=a^{\lambda}, \qquad \lambda=\frac{1}{2}\left(3+\frac{\sqrt{3}}{\sqrt{1-\Sigma}}\right) \tag{7}$$

during the dilaton-driven era [8, 9] ($\Sigma = 0$ is the case shown in the picture). Notice that, during the stringy phase, the dilaton keeps growing (at an approximately constant and large rate) so that, ultimately, one is lead into the strong coupling regime in which the dilaton potential becomes important.

We can now easily see when a sufficient amplification is achieved. The galactic scale of length ${\omega_G}^{-1}$ was about 10^{25} in string (or Planck) units at the beginning of the radiation era. By definition, at earlier times it evolves as a straight line of slope 1 on our plot and thus inevitably hits the horizon curve sometimes during the string or the dilaton-driven era. At that time, the value of g should have been smaller than 10^{-33}. One can easily convince her/himself that this is all but impossible provided: i) $z_s = \eta_s/\eta_1 = a_1/a_s$ is sufficiently large, and ii) the dilaton evolution during the string era is sufficiently fast. For the first condition a minimal red-shift z_s of 10^{10} is necessary; for the average ratio $\dot\phi/H$ during the string era, a value below but not too far from the one just before η_s is sufficient. The combination of (i) and (ii) also implies that the coupling at the onset of the string era has to be smaller that 10^{-20} or so, which thus supports the scenario advocated in [10] for the gracious exit problem.

We can express our results more quantitatively by showing the allowed region in the g_s-z_s plane in order to have sufficiently large seeds. Considering the possibility of galactic scale exit during the string or the dilaton-driven phase, we find from eq. (5) that $r(\omega_G)$ can be expressed, in the two cases, respectively as:

$$r(\omega_G) \simeq \left(\frac{\omega_G}{\omega_1}\right)^4 e^{-\phi_{ex}(\omega_G)} \simeq \left(\frac{\omega_G}{\omega_1}\right)^{4+\frac{\phi_s}{\ln z_s}} \qquad , \quad \omega_s < \omega_G < \omega_1 \tag{8}$$

and

$$r(\omega_G) \simeq \left(\frac{\omega_G}{\omega_1}\right)^{4-2\gamma} z_s^{-2\gamma} e^{-\phi_s} \qquad , \quad \omega_G < \omega_s \tag{9}$$

where $\gamma = \lambda\alpha/(\alpha - 1)$, $\omega_1 = H_1 a_1/a \simeq 10^{11}$Hz is the maximal amplified frequency, and $\omega_s = \omega_1/z_s$. In the previous formulae (8), (9) we used the fact that, according to our model of background, the transition scale H_1 has to be of the order of the Planck mass M_p, so that $\rho_\gamma(t) \simeq H_1^4[a_1/a(t)]^4 = {\omega_1}^4$.

The resulting limits obtained by imposing $r(\omega_G) > 10^{-34}$ are plotted in **Fig. 2**, where they provide the right-side border of the allowed region (the shaded area). The previous spectrum, however, has been obtained using a homogeneous model of background. It is thus valid provided the fluctuations remain, at all times, small perturbations of a nearly homogeneous configuration, with negligible back-reaction on the metric (see also [11]), namely for $r(\omega) < 1$ at all ω. This provides, according to eqs.(8) and (9), the condition $\log_{10} g_s > -2\log_{10} z_s$, which determines the left border of the allowed region. It should be mentioned that such allowed region is compatible with the bounds following from the presence of strong magnetic fields at nucleosynthesis time [12]. Moreover, in the part of the allowed region in which $r \gtrsim 10^{-8}$ the primordial fields can even seed directly the galactic magnetic field [2], thus avoiding the necessity of a dynamo and the related difficulties discussed in [6].

We want to recall, finally, that our results were obtained in the framework of the tree-level, string effective Lagrangian. We know that we could have corrections coming either from higher loops (expansion in e^ϕ) or from higher curvature terms (α' corrections). Since we work in a range of parameters where the dilaton is deeply in his perturbative regime ($\log_{10} g_s < -20$), we expect our results to be stable against loop corrections, at least for scales leaving the horizon during the dilaton driven phase. As to the α' corrections, they are instead invoked in the basic assumption that the dilaton-driven era ends when the curvature reaches the string scale λ_s^{-2}, and leads to a quasi-de Sitter epoch. It should be clear however that, once such an assumption is made, the detailed way in which it is implemented will not affect the behaviour of perturbations which stay outside of the horizon throughout the high-curvature phase. Such perturbations are frozen during that phase and their evolution is merely kinematical. In conclusion, our predictions for the large-wavelength part of the spectrum should be regarded as more rubust than those pertaining to shorter scales.

Acknowledgements

It is a pleasure to thank R. Brustein and V. Mukhanov for a fruitful collaboration on the spectral properties of metric perturbations in string cosmology, which inspired part of this work.

Note added

While this paper was being written, we received a paper by D. Lemoine and M. Lemoine, "Primordial magnetic fields in string cosmology", whose content overlaps with ours where the effects of dilaton-driven inflation on the amplification of electromagnetic perturbations are concerned. Their model of background does not include, however, a sufficiently long, intermediate stringy era whose presence is crucial to produce the large amplification discussed here.

References

[1] E. N. Parker, *Cosmical magnetic fields* (Clarendon, Oxford, 1979);

Y. B. Zeldovich, A. A. Ruzmaikin and D. D. Sokoloff, *Magnetic fields in astrophysics* (Gordon and Breach, New York, 1983)

[2] M. S. Turner and L. M. Widrow, Phys. Rev. D37, 2743 (1988).

[3] B. Ratra, Astrophys. J. Lett. 391, L1 (1992)

[4] A. D. Dolgov, Phys. Rev. D48, 2499 (1993).

[5] E. R. Harrison, Phys. Rev. Lett. 30, 188 (1973);

T. Vachaspati, Phys. Lett. B265, 258 (1991);

W. D. Garretson, G. B. Field and S. M. Carrol, Phys. Rev. D46, 5346 (1992);

B. Cheng and A. V. Olinto, Phys. Rev. D50, 2421 (1994).

[6] R. M. Kulsrud and S. W. Anderson, Astrophys. J. 396, 606 (1992)

[7] C. Lovelace, Phys. Lett. B135, 75 (1984);

E. S. Fradkin and A. A Tseytlin, Nucl. Phys. B261, 1 (1985);

C. G. Callan et al., Nucl. Phys. B262, 593 (1985)

[8] G. Veneziano, Phys. Lett. B265, 287 (1991);

M. Gasperini and G. Veneziano, Astropart. Phys. 1, 317 (1993) and Mod. Phys. Lett. A8, 3701 (1993).

[9] M. Gasperini and G. Veneziano, Phys. Rev. D50, 2519 (1994)

[10] R. Brustein and G. Veneziano, Phys. Lett. B329, 429 (1994)

[11] R. Brustein, M. Gasperini, M. Giovannini, V. F. Mukhanov and G. Veneziano, Phys. Rev. D51, 6744 (1995)

[12] B. Cheng, D. N. Schramm and J. W. Truran, Phys. Rev. D49, 5006 (1994);
D. Grasso and H. R. Rubinstein, Astropart. Phys. 3, 95 (1995).