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Relic Dilatons in String Cosmology

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Abstract

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RELIC DILATONS IN STRING COSMOLOGY

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1. Introduction

String theory has recently motivated the study of a cosmological scenario in which the Universe starts evolving from the string perturbative vacuum, namely from a cold and empty state with flat metric and vanishing gauge coupling. Because of the instability of this vacuum the Universe is necessarily driven, in a finite amount of time, to a state with high curvature and strong coupling, where the back-reaction of the quantum fluctuations becomes important, and the Universe eventually becomes hot and radiation-dominated as in the standard scenario. The big bang, in such a context, is no longer the starting point of the cosmological evolution, but only the intermediate stage corresponding to the transition from the high-curvature string phase to the standard radiation era. It thus seems appropriate to call "pre-big bang"[1,2,3] the whole cosmological epoch describing the evolution from the vacuum to the beginning of radiation-dominance, characterized by shrinking event horizons, growing curvature and string coupling, and naturally motivated by the duality symmetries of the string effective action.

By assuming, for simplicity, a non-trivial background configuration for the metric and the dilaton field only, the effective action[4] governing the dynamics of the pre-big bang cosmological scenario can be written, in the string frame:

$$S = -\int d^{d+1}x\sqrt{|g|}e^{-\phi}\Big[R + (\nabla\phi)^2 - \frac{\alpha'}{4}\left(R^2_{\mu\nu\alpha\beta} - 4R^2_{\mu\nu} + R^2 - (\nabla\phi)^4 + ...\right) + \\ + V(\phi)\Big] + \mathrm{loops}(g_s). \tag{1.1}$$

Here ϕ is the dilaton, V is a possible non-perturbative dilaton potential, $g_s(t) = e^{\phi/2}$ is the field-dependent (and thus time-dependent) string coupling, and the dots stand for higher-derivative terms, whose contribution to the effective action is controlled by the fundamental string mass parameter:

$$\alpha' \equiv \lambda_s^2 \equiv M_s^{-2}. \tag{1.2}$$

For an isotropic and spatially flat d-dimensional background,

$$g_{\mu\nu} = \mathrm{diag}\left(1, -a^2(t)\delta_{ij}\right), \quad \phi = \phi(t), \quad \dot{\overline{\phi}} = \dot{\phi} - dH, \quad H = \dot{a}/a, \tag{1.3}$$

the cosmological evolution determined by the action (1.1) can be schematically illustrated as in Fig. 1, in the two-dimensional space spanned by the convenient dynamical variables $\{\dot{\overline{\phi}}, \sqrt{d}H\}$.

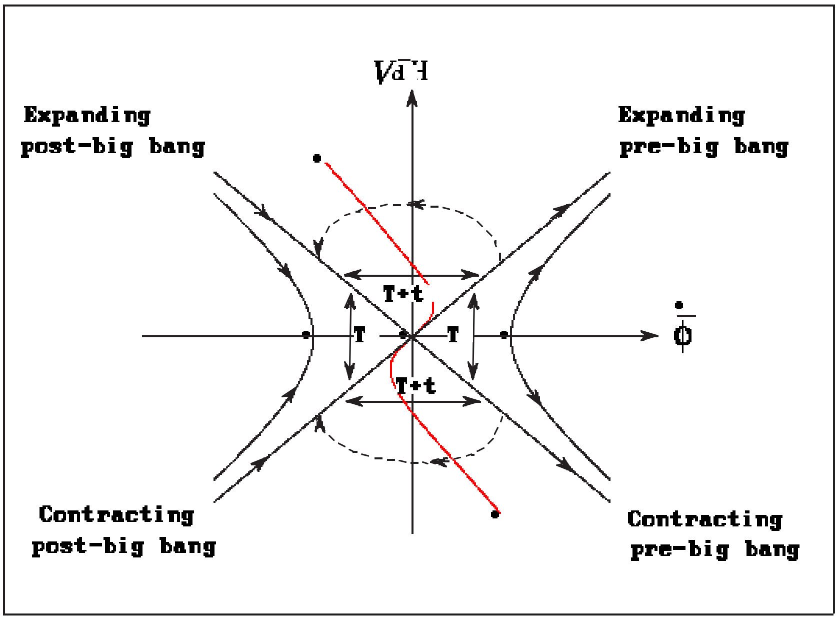

Fig. 1

Figure 1: *Cosmological evolution of the gravi-dilaton background according to the string effective action, at tree-level in the string-loop expansion.*

The origin of the axes represents the perturbative vacuum, where $H = 0$ and $\phi = -\infty$. The bisecting lines represent the solutions of the lowest-order effective action[1,2,3], with $V = 0$ and without loops and α' corrections. The lines in the upper-half plane $H > 0$ correspond to expanding configurations, in the lower half-plane $H < 0$ to contracting configurations. For $\dot{\overline{\phi}} > 0$ they describe an accelerated evolution from the vacuum to the large curvature and coupling "stringy" regime (pre-big bang configurations); for $\dot{\overline{\phi}} < 0$ they describe a decelerated evolution from the stringy regime to the vacuum (post-big bang configurations). The four branches of the classical solutions are related by T-duality transformations

$$\dot{\overline{\phi}} \Longleftrightarrow \dot{\overline{\phi}} \qquad H \Longleftrightarrow -H, \tag{1.4}$$

and time-reversal transformations

$$\dot{\overline{\phi}} \Longleftrightarrow -\dot{\overline{\phi}} \qquad H \Longleftrightarrow -H, \tag{1.5}$$

as indicated in Fig. 1. Both transformations are required[5] for a monotonic (expanding or contracting) transition from pre- to post-big bang. In the case of a constant positive potential $V = \Lambda > 0$, the solutions[5] of the lowest-order action are represented by the two branches of the hyperbola plotted in Fig. 1. The perturbative vacuum is replaced in this case by a different configuration with flat metric and linearly evolving dilaton, $\dot{\phi} = \text{const}$, but the symmetry pattern is preserved.

In the absence of higher-order corrections, the four branches of the solutions are classically disconnected by a curvature singularity that cannot be removed by any realistic choice of the (local) dilaton potential[6]. With an appropriate potential, however, there are quantum transitions from pre- to post-big bang that preserve the monotonic evolution of the scale factor, and which are represented by the dashed curves of Fig. 1. The transition is allowed, in particular, between two states[7] with $\Lambda = 0$, between two states[8] with the same finite value of Λ, and also[9] from the vacuum to a state with $\Lambda > 0$. These processes are described by the scattering of the Wheeler–De Witt wave function that represents our cosmological configurations, in the two-dimensional minisuperspace spanned by the coordinates $\overline{\phi} = \phi - d \ln a$ and $\beta = \sqrt{d} \ln a$.

If we include the higher-derivative terms in the effective action, however, the transition from the pre- to the post-big-bang sector of Fig. 1 may become allowed even classically, and already to first order in α'. This effect is illustrated by the bold solid curve crossing the origin in Fig. 1, which is obtained by numerically integrating the equations following from the action (1.1), without potential and loop corrections, but with the four-derivative Gauss–Bonnet and dilaton terms included[10]. Starting from the perturbative vacuum, it is found in that case that the Universe necessarily evolves towards a final state with constant curvature, linearly running dilaton and $\dot{\overline{\phi}} < 0$, a fixed point of the cosmological equations.

Assuming that, with the inclusion of the appropriate loop corrections[11], the transition to the post-big bang phase is successfully completed towards a state of radiation-dominated evolution, we may wonder whether the phenomenological predictions of such a string cosmology scenario are or not significantly different from those of the standard scenario. The answer to this question is known, and it is affirmative. There are, in particular, three main effects worth mentioning: 1) the production of a relic graviton background[1,12,13,14], much stronger, at high frequency, that the one expected in the context of standard inflationary models (so strong that the associated energy density may be of the same order as the entropy stored in the present CMB electromagnetic radiation[15]); 2) the production of a cosmic background of relic dilatons[3,12,16]; 3) the amplification of the vacuum fluctuations of the electromagnetic field, and the consequent production of "seeds" for the galactic magnetic fields[17].

The status of our present knowledge about the graviton background has been reviewed and discussed in a recent paper[18]. In this paper I will thus concentrate my discussion on the general properties of the relic dilaton background.

2. Scalar perturbations and dilaton production

In a cosmological context there are various mechanisms of dilaton production: particle collisions at high temperature, coherent oscillations around the minimum of a scalar potential, amplification of the vacuum fluctuations. We shall concentrate here on the third possibility because, even if the temperature remains too low to allow thermal production, and oscillations are avoided through a symmetry that ensures the coincidence of the minima of the potential at early and late times[19], quantum fluctuations cannot be eliminated; also, they may be expected to represent the dominant source[20] when the inflation scale is not smaller than about 10^{16} GeV, as in the context of our scenario (another possible mechanism, dilaton radiation from cosmic strings, has recently been discussed in Ref. [21]).

The production of dilatons through the amplification of the vacuum fluctuations of the dilaton background[3], $\phi \to \phi + \delta\phi$, is very similar to the process of graviton production through the amplification of tensor perturbations of the metric background[22], $g_{\mu\nu} \to g_{\mu\nu} + \delta h_{\mu\nu}$. Unlike the graviton case, however, the analysis of dilaton perturbations is complicated by their coupling to the component of the metric perturbations, and to the perturbations of the matter sources. The sources may even be absent in the pre-big bang phase, but they are certainly not absent when the dilatons re-enter the horizon, in the radiation and matter-dominated epoch.

The general system of coupled perturbation equations, for the background (1.3) and for a perfect-fluid model of sources, can easily be written down in the Einstein frame, in the standard longitudinal gauge[23], in terms of the variables ψ, χ, $\delta\rho$, δp, δu_i defined by:

$$\begin{aligned} &ds^2 = a^2\left[d\eta^2\,(1+2\psi) - (1-2\psi)\,dx_i^2\right], \qquad\qquad \delta\phi = \chi, \\ &\delta T_0^0 = \delta\rho, \qquad \delta T_i^j = -\delta p \delta_i^j, \qquad \delta T_i^0 = (p+\rho)\delta u_i/a. \end{aligned} \tag{2.1}$$

The vector-like equation[3] then obtained is:

$$Z_k'' + 2\frac{a'}{a}\mathcal{A}Z_k' + \left(k^2\mathcal{B} + \mathcal{C}\right)Z_k = 0, \tag{2.2}$$

where the components of the doublet $Z_k^\dagger = (\psi_k, \chi_k)$ are the Fourier modes of the metric and dilaton perturbations, and the prime denotes differentiation with respect to the conformal time η. The 2×2, time-dependent mixing matrices

$$\mathcal{A} = \begin{pmatrix} \frac{1}{2}(2d-3+d\epsilon), & -\frac{1-\epsilon}{4(d-1)}\beta \\ -(d-1)[\frac{cd}{2}(1-d\epsilon)+\beta], & \frac{1}{2}(d-1) - \frac{c}{4}(1-d\epsilon)\beta \end{pmatrix} \tag{2.3}$$

$$\mathcal{B} = \begin{pmatrix} \epsilon, & 0 \\ -c(1-d\epsilon)(d-1), & 1 \end{pmatrix} \tag{2.4}$$

$$\mathcal{C} = \begin{pmatrix} 2(d-2)\frac{a''}{a} + (d-2)[d-4+d\epsilon + \frac{1-\epsilon}{2(d-1)}\beta^2](\frac{a'}{a})^2, & \frac{\epsilon+1}{2(d-1)}V'a^2 \\ cd(d-2)(\epsilon-\gamma)[d(d-1) - \frac{\beta^2}{2}](\frac{a'}{a})^2 + 2(d-2)V'a^2, & [V'' - \frac{c}{2}(1-d\epsilon)V']a^2 \end{pmatrix} \tag{2.5}$$

where $c = \sqrt{2/(d-1)}$ and $V' = \partial V/\partial\phi$, depend on the matter equation of state, $\gamma(t) = p/\rho$, on the fluid model of perturbations, $\epsilon(t) = \delta p/\delta\rho$, on the dilaton potential $V(\phi)$, and on the explicit background solution through the parameter $\beta = \dot{\phi}/H$. Once eq. (2.2) is solved, for a given set of parameters $\{\beta, \gamma, \epsilon\}$, there are two additional independent equations[3] that determine the density and velocity contrast $\delta\rho$ and δu in terms of ψ and χ:

$$\partial_i \left[2(d-1)\left(\frac{a'}{a}(d-2)\psi + \psi' \right) - \chi\phi' \right] = (\rho + p)a\delta u_i, \tag{2.6}$$

$$\nabla^2\psi - d\frac{a'}{a}\psi' - \left[d(d-2)\left(\frac{a'}{a}\right)^2 - \frac{d-2}{2(d-1)}\phi'^2 \right]\psi = $$
$$= \frac{1}{2(d-1)}\left(\phi'\chi' + \frac{\partial V}{\partial\phi}a^2\chi + a^2\delta\rho \right). \tag{2.7}$$

An exact computation of the scalar perturbation spectrum thus requires an exact solution of the coupled equations (2.2). Also, the correct normalization of the spectrum needed to determine the amplification of the quantum vacuum fluctuations, requires the knowledge of the normal modes of oscillation of the system *gravi-dilaton background* $\oplus$ *fluid sources*, namely of the variables that diagonalize the perturbed action, and satisfy canonical commutation relations[23,24]. Such variables are known for the pure metric–fluid system[25], and for the pure metric–scalar field system[26], but not for the complete system (2.1), when the dilaton is coupled to matter.

The amplification of the normalized vacuum fluctuation spectrum has been determined[3], up to now, for the simple transition from a $d = 3$, dilaton-dominated pre-big bang phase with negligible matter sources ($T_\mu^\nu = 0 = \delta T_\mu^\nu$), to a radiation-dominated phase with adiabatic fluid perturbations ($\gamma = \epsilon = 1/3$) and with the dilaton frozen ($\beta = 0$) at the minimum of the non-perturbative potential ($\partial V/\partial\phi = 0$). In such a phase the perturbation equations (2.2) are decoupled, the canonical variables are known, and the spectrum of scalar and dilaton perturbations (neglecting a possible mass term $\partial^2 V/\partial\phi^2 = m^2$) turns out to be the same as the graviton spectrum, with a slope that is cubic[3] modulo logarithmic corrections[27].

It should be stressed, however, that such a spectral distribution cannot be extrapolated[28] down to frequency scales re-entering the horizon after equilibrium, since in the matter era ($p = 0$) the perturbation equations are no longer decoupled, even if the dilaton background is frozen at the minimum of the potential. They remain coupled not only in the longitudinal gauge, but also in the uniform-curvature gauge[29], an off-diagonal gauge more appropriate to scalar perturbations when growing modes are present[27]. In addition, a cubic slope cannot be extrapolated up to the maximum amplified frequency scale, as the slope is expected to be different (in general flatter) in the highest frequency band for which the first horizon crossing occurs not in the initial dilaton-driven phase, but in the subsequent high-curvature string phase[10].

In this paper we shall discuss phenomenological bounds that apply to the total integrated dilaton spectrum. As the spectrum is generally a non-decreasing function of frequency (because the curvature scale is non-decreasing in the pre-big bang epoch), we may restrict our analysis to the high frequency sector of the spectrum, for which all modes re-enter the horizon in the radiation era. In that range the perturbation equations are decoupled, the canonical normalization in known, and the spectral distribution of the energy density in the relativistic regime $\omega \gg m$ can be parametrized as[3]:

$$\Omega_\chi(\omega,t) = \frac{\omega}{\rho_c}\frac{d\rho_\chi(\omega,t)}{d\omega} = \Omega_\gamma(t)\left(\frac{H_1}{M_p}\right)^2\left(\frac{\omega}{\omega_1}\right)^\delta,$$
$$\omega < \omega_1 = \frac{H_1 a_1}{a(t)}, \qquad \delta > 0, \qquad H_1 \simeq M_s. \tag{2.8}$$

Here $\rho_c = 3M_p^2H^2(t)/8\pi$ is the critical density, M_p is the Planck mass, $\Omega_\gamma \simeq (H_1/H)^2(a_1/a)^4$ is the CMB electromagnetic energy density in critical units; H_1 is the curvature scale at the time t_1 of the inflation–radiation transition (which in the present scenario is of the same order as the string mass scale M_s); ω_1 is the maximal frequency of the spectrum undergoing parametric amplification (I have used the fact that the peak value of the dilaton spectrum has to be[30,31] of the same order as the peak of the graviton spectrum[14]). Finally, δ is a growing spectral index, whose exact value is at present unknown for the reasons mentioned above.

Fortunately, the bounds that we shall consider here are only weakly dependent on δ, and become completely δ-independent for $\delta \gtrsim 1$. In the following section I will thus assume $\delta \gtrsim 1$ to simplify the discussion, but the analysis can be easily extended[3,16] to any value of δ.

3. Phenomenological bounds

The main bound on the background of relic dilatons follows from the fact that the dilatons cannot be massless, because they are coupled non-universally to macroscopic bulk matter[32], thus inducing an effective violation of the equivalence principle in the macroscopic limit of weak gravitational fields. This may be reconciled with the present tests of the equivalence principle[33] if the range of the dilaton force is smaller than about 1 cm, i.e. for a dilaton mass

$$m \gtrsim 10^{-4}\ \mathrm{eV}. \tag{3.1}$$

Because of the mass, the produced dilatons tend to become non-relativistic, as their proper momentum is red-shifted. When the dominant mode $\omega_1(t)$ becomes non-relativistic, the total integrated energy evolves in time like a^{-3}:

$$\Omega_\chi = \int^{\omega_1}\frac{d\omega}{\omega}\frac{m}{\omega_1}\Omega_\chi(\omega,t) \simeq \frac{mM_s}{M_p^2}\left(\frac{M_s}{H}\right)^2\left(\frac{a_1}{a}\right)^3, \quad \omega_1(t) < m, \tag{3.2}$$

and starts to grow in time with respect to $\Omega_\gamma \sim a^{-4}$. The transition to the non-relativistic regime necessarily occurs before the present epoch t_0, because the dominant mode has today a proper wave number smaller than the dilaton mass[3], $\omega_1(t_0) \simeq (M_s/M_p)^{1/2} 10^{-4}$ eV $< m$.

In the matter-dominated era, Ω_χ remains frozen at the constant value $\Omega_\chi(t_{eq}) \simeq (mM_s/M_p^2)(M_s/H_{eq})^2(a_1/a_{eq})^3$, where $H_{eq} \sim 10^6 H_0 \sim 10^{-55} M_p$ is the curvature scale at the time of matter–radiation equilibrium (I am discussing here an order-of-magnitude estimate, and I will neglect the dependence of the bounds on the precise value of the present Hubble parameter H_0). By imposing $\Omega_\chi(t_{eq}) < 1$, to avoid a Universe overdominated by the coherent oscillations of the produced dilatons[34], we obtain the bound

$$m \lesssim \left(H_{eq} M_p^4/M_s^3\right)^{1/2}, \tag{3.3}$$

which represents, in our context, the most restrictive upper bound if dilatons are not yet decayed. For 100 keV $\lesssim m \lesssim$ 100 MeV a more restrictive constraint on Ω_χ is provided by the observations of the diffuse γ-ray background[21], but this range of masses is excluded, in our case, by the allowed range of M_s (see next section).

Values of the dilaton mass higher than allowed by the critical density bound (3.3) can be reconciled with present observations, only if the energy stored in the coherent oscillations was dissipated into radiation before the present epoch, at the decay scale $H_d > H_0$, fixed by the decay rate Γ_d of dilatons into photons, $H_d \simeq \Gamma_d \simeq m^3/M_p^2$. The reheating associated to this decay leads to an entropy increase $\Delta S \simeq (T_r/T_d)^3$, where $T_r \simeq (M_p H_d)^{1/2}$ is the final reheating temperature, and T_d is the radiation temperature immediately before dilaton decay. This increase is significant ($\Delta S > 1$) provided dilatons decay when they are dominant, namely for $t > t_i$, where t_i is the time scale marking the beginning of dilaton dominance, $\Omega_\chi(t_i) = \Omega_\gamma(t_i)$. From eq. (3.2) we have $H_i \simeq m^2 M_s^3/M_p^4$ so that, for $H_d < H_i$, the radiation temperature before decay is $T_d = T_i(a_i/a_d) \simeq (H_i M_p)^{1/2}(H_d/H_i)^{2/3} \simeq (m^{10}/M_s^3 M_p)^{1/6}$, corresponding to an entropy increase

$$\Delta S \simeq \left(M_s^3/m M_p^2\right)^{1/2}. \tag{3.4}$$

This entropy injection can in principle disturb nucleosynthesis or baryogenesis[34], and we must consider two possibilities.

- If the reheating temperature T_r is too low to allow nucleosynthesis, i.e. $m \lesssim 10$ TeV, we must assume that nucleosynthesis occurred before, and we must impose $\Delta S \lesssim 10$ to avoid destroying the light nuclei already formed. A more precise bound can be determined through a detailed analysis of photodissociation[35] and hadroproduction[36] processes, but such an increase of precision is irrelevant in our context since, as we shall see, it refers to values of m outside the allowed range.

- If the reheating temperature is large enough to allow nucleosynthesis, i.e. $m \gtrsim 10$ TeV, the only possible constraint comes from primordial baryogenesis.

The bound is model-dependent, but the constraint $\Delta S \lesssim 10^5$ seems to be sufficient[34] not to wash out any pre-existing baryon–antibaryon asymmetry (this bound could be evaded in the case of low-energy baryogenesis, occurring at a scale $H < H_d$).

The previous bounds refer to the case $m < M_s$. If $m > M_s$ then the produced dilatons are non-relativistic already from the beginning, their total integrated energy density is[3]

$$\Omega_\chi(t) \simeq \left(\frac{m}{M_p}\right)^2 \left(\frac{M_s}{H}\right)^2 \left(\frac{a_1}{a}\right)^3, \tag{3.5}$$

and the only bound to be imposed is $m < M_p$, to avoid overcritical density. There are no additional bounds, as the dilatons decay before becoming dominant.

4. Allowed mass windows

By intersecting the region allowed by the previous phenomenological bounds, with the allowed values of the string mass scale[37], $0.01 \lesssim M_s/M_p \lesssim 0.1$, we obtain for the dilaton mass the two windows illustrated in Fig. 2:

$$10^{-4}\ \text{eV} \lesssim m \lesssim 10\ \text{keV}, \qquad\qquad 10\ \text{TeV} \lesssim m. \tag{4.1}$$

They lie on the opposite sides of the 100 MeV decay line, corresponding to a dila-

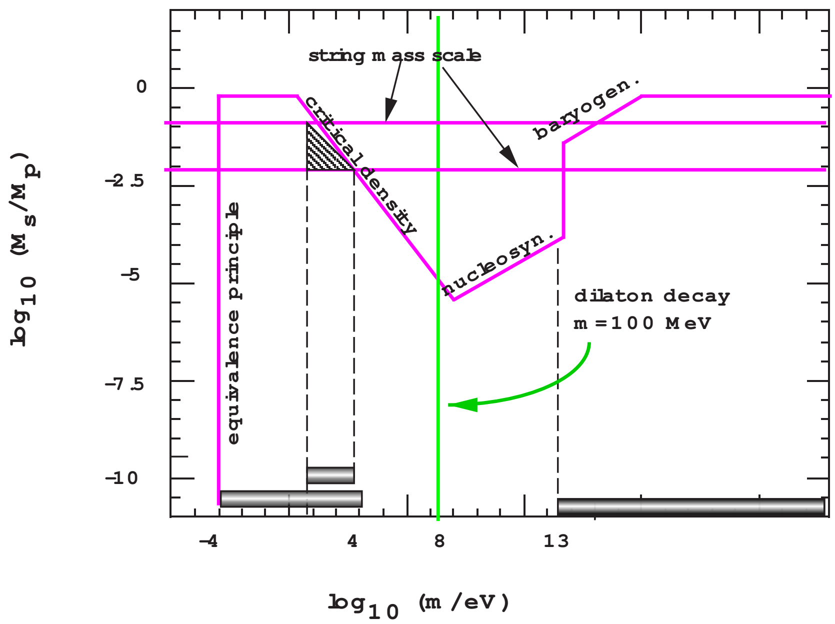

Figure 2: *Allowed dilaton mass windows for a spectral slope $\delta \geq 1$. The shaded triangle defines the region of parameter space compatible with a dominant contribution of relic dilatons to the present critical density.*

ton decay scale of the same order as our present Hubble scale, $H_d \simeq H_0$. If the mass is in the right window then all produced dilatons have already decayed, and no background is available today to direct observation. If, on the contrary, the mass is inside the left window, then the background is still around us and could be observed, in principle, by those experimental devices that are sensitive to scalar oscillations of gravitational strength, such as spherical gravity wave detectors[38]. Unfortunately, since dilatons today are non-relativistic, they should oscillate coherently at a frequency determined by their rest mass, and thus higher than about 100 GHz, according to eq. (3.1). This is clearly outside the typical frequency range of such detectors. Resonant microwave cavities, used in the search of cosmic axions[39], are also disfavoured because of the very small coupling of dilatons to photons, which is at least a factor 10^{-8} smaller than the corresponding coupling of axions.

In spite of the fact that a dilaton mass in the TeV range seems to be at present supported by supersymmetry-breaking motivations[40], it is important to recall that a dilaton mass in the left window is all but theoretically excluded, as shown for instance by models of supersymmetry breaking with light dilatons[41]. It is also worth stressing that, in the restricted range

$$100 \text{ eV} \lesssim m \lesssim 10 \text{ keV}, \tag{4.2}$$

the dilatons could saturate the critical density bound, as illustrated in Fig. 2, thus becoming an attractive dark matter candidate[3,16] with $0.01 \lesssim \Omega_\chi \lesssim 1$ according to eq. (3.2).

The mass windows of Fig. 2 refer to a dilaton spectrum that grows linearly or faster with frequency, $\delta \geq 1$. If $\delta < 1$ the bounds become δ-dependent and slightly more constraining[3,16], and the allowed windows are further reduced. The critical density bound (3.3), in particular, becomes

$$m \lesssim \left(H_{eq} M_p^4 M_s^{\delta-4}\right)^{1/(\delta+1)}, \qquad \delta \leq 1, \tag{4.3}$$

and the left window disappears completely for $\delta \lesssim 0.5$.

In this sense, the mass windows of eq. (4.1) represent the maximally extended allowed range for the dilaton mass, at least in the context of a "minimal" pre-big bang scenario[18] in which the CMB radiation that we observe today is entirely produced at the end of the string phase at a scale $H_1 \simeq M_s$. It is not impossible, however, to imagine more complicated (but perhaps also more unnatural) models in which the phenomenological bounds determining the allowed mass window are relaxed, because of an additional reheating phase occurring before nucleosynthesis, and before the beginning of dilaton dominance. Such a reheating could be the consequence of a phase of "intermediate scale" inflation[42] or of "thermal" inflation[43], and is only constrained by the requirement of a negligible dilution of any pre-existing baryon number (but baryogenesis could be even produced by the "flaton" field[43] itself, whose decay is responsible for the additional reheating).

Any mechanism producing a significant amount of thermal radiation (associated or not to a phase of inflation) indeed dilutes the original dilaton density (2.8), with respect to Ω_γ, by the factor[14]

$$\Omega_\chi \to \Omega_\chi (1-\delta s)^{4/3} \, (n_f/n_b)^{4/3} \, , \qquad \delta s = (s_f - s_b)/s_f . \tag{4.4}$$

Here s_b, s_f, n_b, n_f are, respectively, the thermal entropy density of the CMB radiation and the number of particles species in thermal equilibrium, at the beginning (t_b) and at the end (t_f) of the reheating process. An efficient reheating, $s_f \gg s_b$, $\delta s \to 1$, reduces in a significant way the dilaton fraction of critical density Ω_χ: as a consequence, the scale of dilaton dominance H_i is lowered, the decay temperature T_d is raised, and the bounds on m following from the entropy constraint $\Delta S < 10$ and the critical bound $\Omega_\chi < 1$ are relaxed. We can easily estimate, by assuming in particular $n_f \sim n_b$, that the mass gap between the left and right windows of eq. (4.1) is completely filled for

$$1 - \delta s \lesssim 10^{-4}, \tag{4.5}$$

namely for an intermediate reheating phase producing more than 99.99% of the entropy at present stored in the thermal black-body background.

An appropriate reheating process can thus easily render a dilaton mass of the TeV order compatible with the string inflation scale M_s. The final allowed region should not be further reduced by other bounds applied to additional processes of production since, for an inflation scale of the order of M_s, the amplification of the vacuum fluctuations is expected to represent the dominant mechanism[20] of dilaton production.

5. Conclusion

The evolution from the string perturbative vacuum to the radiation era, described by pre-big bang models of the early Universe, is accompanied by the parametric amplification of the quantum fluctuations of the dilaton background, and leads to the production of a sea of relic cosmic dilatons. Their spectral distribution is in general non-decreasing with frequency, and is normalized to a peak value determined by the string mass scale, such as that of the relic graviton spectrum.

Our present ignorance of the kinematic details of the model at high curvature scale, and the complicated mixing with matter at the time of re-enter, have prevented so far a definite prediction for the spectral index, in both the high and the low frequency sectors. Using however the general properties of the spectrum, we can obtain a reliable estimate of the allowed range for the dilaton mass. The extension of such range depends only weakly on the unknown spectral slope, and becomes completely slope-independent for a spectrum that grows linearly or faster.

The analysis performed has produced, in particular, the following two results. 1) The allowed mass windows may include a range of values in which the cosmic dilatons are not yet decayed, and provide a dominant contribution to the present critical energy density. 2) With the introduction of an intermediate reheating stage, subsequent to the string-radiation transition, a non-minimal model can easily be made compatible with a dilaton mass in the TeV range, the present preferred value of the conventional supersymmetry-breaking scenario.

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