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Massless (pseudo-)scalar seeds of CMB anisotropy

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Abstract

A primordial stochastic background of very weakly coupled massless (pseudo-)scalars can seed CMB anisotropy, when large-scale fluctuations of their stress-tensor re-enter the horizon during the matter-dominated era. A general relation between multipole coefficients of the CMB anisotropy and the seed's energy spectrum is derived. Magnitude and tilt of the observed anisotropies can be reproduced for the nearly scale-invariant axion spectra that are predicted in a particularly symmetric class of string cosmology backgrounds.

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In this letter we point out a possible new mechanism for generating large-scale CMB anisotropies. We will show that a cosmologically amplified stochastic background of massless (pseudo-)scalar perturbations, if primordially produced with a properly normalized nearly scale-invariant spectrum, can seed CMB temperature anisotropies in a way consistent with observations [1]. Axionic perturbations with the needed characteristics can be produced [2], [3], [4], for instance, in the so-called pre-big bang scenario [5] of string cosmology.

For the sake of generality we define a massless (pseudo-)scalar "seed" field σ through the way it enters the low-energy effective action:

$$S_{eff} = -\frac{1}{2}\int d^4x\ \sqrt{-g}\ A\ (\partial_\mu \sigma)^2, \tag{1}$$

and by the two additional conditions:

$$\langle\sigma\rangle = 0 \qquad , \qquad\qquad \Omega_\sigma \ll 1. \tag{2}$$

In Eq. (1), A is, in general, a σ-independent scalar combination of fields providing, together with the metric, the cosmological background. In Eq. (2), the brackets denote spatial average (or expectation value if perturbations are quantized), and Ω_σ is the fraction of critical energy density carried by the seed field. Such a fraction being small, seeds do not influence the background itself. The above conditions, together with the restriction to massless seeds, make it essentially mandatory for such seeds, if they exist, to consist of very weakly coupled pseudo-scalar (rather than scalar) particles [6]. An example would be the gravitationally coupled "universal axion" of string theory, on which we shall come back at the end of this letter.

We have in mind, typically, a situation in which, in the absence of σ, large-scale CMB anisotropies directly induced by quantum fluctuations of the metric are too small; we want to investigate under which conditions seeds may provide the dominant source for them. Seed vacuum fluctuations, after being amplified outside the horizon during inflation, re-enter during the standard Friedman-Robertson-Walker (FRW) era as stochastic Gaussian fields, and give rise to non-trivial – and not necessarily Gaussian – fluctuations of the energy-momentum tensor.

Within this general context, we derive a simple relation between the usual coefficients C_l of the multipole expansion of the CMB temperature fluctuations,

$$\left\langle \frac{\delta T}{T}(\mathbf{n})\frac{\delta T}{T}(\mathbf{n}')\right\rangle_{(\mathbf{n}\cdot\mathbf{n}'=\cos\vartheta)} \quad = \quad \frac{1}{4\pi}\sum_\ell (2\ell+1)C_\ell P_\ell(\cos\vartheta)\ , \tag{3}$$

and the fraction of critical density in the seeds. The relation reads:

$$C_\ell = K \quad \int_0^\infty d\log k \quad |j_\ell(k\eta_0)|^2 \quad \Omega_\sigma^2(k,\eta_{re})\ , \tag{4}$$

where K is a numerical fudge factor $O(1)$, $\Omega_\sigma(k,\eta) \equiv \rho_{cr}^{-1} d\rho_\sigma/d\log k$ is the seed fraction of critical energy density per logarithmic interval of frequency, evaluated at the conformal time η, η_0 is the present (conformal) time and $\eta_{re}(k)$ indicates, for each comoving mode k, its time of re-entry, $\eta_{re} \sim k^{-1}$. For the relevant (large) scales, re-entry occurs during the matter-dominated era. A crucial aspect of (4) is the appearance of Ω_σ at a k-dependent time (i.e. at re-entry), rather than at a common (e.g. at recombination) time. This is because, in the interesting cases, the so-called "integrated" Sachs-Wolfe (SW) contribution [7] turns out to dominate over the "ordinary" SW term.

In order to prove Eq. (4) we will proceed as follows. We start from a general formula expressing the spectrum of primordial seed fluctuations in terms of the early, inflationary evolution of the background. We then compute the inhomogeneities induced by this stochastic field in the energy-momentum tensor as well as the Bardeen potentials. Finally, we estimate the large-scale temperature anisotropy using the (total) SW effect. The result (4) will thus implicitly connect the observed CMB anisotropy to the very early (possibly to the pre-big bang) history of the universe.

This note is intended to give the essential points in the argument and their main consequences. For more details on the calculation in a specific case we refer the reader to our longer recent paper [8], where other kinds of seeds, as well as the case of massive seeds, are also discussed. Further generalizations of the massive case will be discussed in [9].

Our computation of the fluctuations of σ follows closely the general approach of [10]. In a conformally flat metric the effective action (1) becomes:

$$S_{eff} = \frac{1}{2}\int d^3x d\eta \; S \; \left[(\sigma')^2 - (\nabla\sigma)^2\right] \ , \tag{5}$$

where a prime stands for derivative with respect to conformal time η, and the so-called pump field S is simply $S \equiv a^2 A$, where a is the scale factor of the homogeneous, isotropic, spatially-flat metric resulting from a long inflationary phase[1]. The corresponding effective Hamiltonian reads

$$H_{eff} = \frac{1}{2}\int d^3x \; \left[S^{-1}\pi^2 + S(\nabla\sigma)^2\right] \quad , \tag{6}$$

where $\pi = S\sigma'$ is the canonical variable conjugate to σ. The Fourier modes of σ, when correctly normalized to the vacuum before they "exit" the horizon at the time $|k\eta_{ex}| \sim 1$, are given by

$$\sigma(k,\eta) = \frac{1}{\sqrt{kS}} \; e^{-ik\eta + i\varphi_k} \quad , \qquad \pi(k,\eta) = \sqrt{kS} e^{-ik\eta + i\varphi'_k} \quad , \qquad \eta < \eta_{ex} \sim -k^{-1}, \tag{7}$$

(φ_k, φ'_k are random phases, originating from the random initial conditions). Furthermore, as far as the computation of energy spectra goes, fluctuations on superhorizon scales can be consistently truncated to their frozen modes [10] through

$$\sigma(k,\eta) = \frac{1}{\sqrt{kS_{ex}}} \; e^{i\varphi_k} \quad , \qquad \pi(k,\eta) = \sqrt{kS_{ex}} e^{i\varphi'_k} \quad , \qquad \eta_{ex} < \eta < \eta_{re} \sim k^{-1}. \tag{8}$$

[1] It is important to note that the results that we will obtain here are not valid for conformally coupled fields, e.g for the electromagnetic field, since these do not couple to the scale factor.

The matching at re-entry finally gives, for $k\eta > 1$,

$$\begin{aligned}
\sigma(k,\eta) &= \frac{1}{\sqrt{kS}}\left[\left(\frac{S_{re}}{S_{ex}}\right)^{1/2}\cos(k\eta)\ e^{i\varphi_{\vec{k}}} + \left(\frac{S_{ex}}{S_{re}}\right)^{1/2}\sin(k\eta)\ e^{i\varphi'_{\vec{k}}}\right] , \\
\pi(k,\eta) &= \sqrt{kS}\left[\left(\frac{S_{ex}}{S_{re}}\right)^{1/2}\cos(k\eta)\ e^{i\varphi'_{\vec{k}}} - \left(\frac{S_{re}}{S_{ex}}\right)^{1/2}\sin(k\eta)\ e^{i\varphi_{\vec{k}}}\right] \quad . \qquad (9)
\end{aligned}$$

A nice feature of these results is their generality. They hold for any kind of background and irrespectively of whether a perturbation re-enters during the matter- or the radiation-dominated epoch. Furthermore, these equations respect an invariance [10] of cosmological perturbations under the duality transformation $S \to S^{-1}, \nabla\sigma \leftrightarrow \pi$. For the sake of simplicity, we shall consider here the case of a growing pump field, keeping only the leading terms (those proportional to $S_{re}/S_{ex} \gg 1$) in the fluctuations. This will result in simpler formulae at the price of losing manifest duality.

The basic information to be extracted from the preceding formulae is the stochastic (spatial) average of σ:

$$\langle\sigma(\mathbf{k})\sigma^*(\mathbf{k}')\rangle = (2\pi)^3\delta^3(k-k')\Sigma(\mathbf{k},\eta) \qquad (10)$$

where, according to Eqs. (8) and (9),

$$\begin{aligned}
\Sigma(\mathbf{k},\eta) = (kS_{ex})^{-1} \quad &, \quad k\eta < 1 \ , \\
\Sigma(\mathbf{k},\eta) = (kS_{ex})^{-1}\frac{S_{re}}{S(\eta)} \quad &, \quad k\eta > 1 \ . \qquad (11)
\end{aligned}$$

Equations (10) and (11) allow us to compute the correlation functions of the seed energy-momentum tensor:

$$T_{\mu\nu}^{(\sigma)} \equiv T_{\mu\nu} = \frac{S}{a^2}\left[\partial_\mu\sigma\partial_\nu\sigma - \frac{1}{2}\ g_{\mu\nu}(\partial_a\sigma)^2\right] . \qquad (12)$$

Let us start with the average energy distribution $d\rho_\sigma(k)/d\log k = (k^3/a^4)\langle H\rangle$ which, after re-entry, can be computed from the Hamiltonian (6) as [10]:

$$\frac{d\rho_\sigma(k)}{d\log k} \simeq \left(\frac{k}{a}\right)^4\frac{S_{re}}{S_{ex}}(k)\theta(k_1-k). \qquad (13)$$

The end-point of the spectrum k_1 is the maximal amplified frequency (the frequency that re-entered just after exiting), for which just one quantum is produced per unit phase space. Above k_1 the spectrum is exponentially depressed and we thus neglect it. Below k_1, we can express ρ_σ in units of critical energy density, $\rho_c = 3H^2/(8\pi G)$ as

$$\Omega_\sigma(k,\eta) \simeq G\left(\frac{k^4\eta^2}{a^2}\right)\left(\frac{a_{re}}{a_{rad}}\right)^2\left(\frac{S_{rad}}{S_{ex}}\right) \simeq G\left(\frac{k}{a_{re}}\right)^2\left(\frac{a_{re}}{a_{rad}}\right)^2\left(\frac{S_{rad}}{S_{ex}}\right)\ \left(\frac{a_{re}}{a}\right) . \qquad (14)$$

We have denoted by rad the beginning of the radiation era, and we have assumed the background field A to be constant for $\eta > \eta_{rad}$. Also, we have limited our attention to scales relevant to the COBE DMR data, which re-enter during the matter-dominated era. The suppression factor (a_{eq}/a), naively expected for massless particles, is actually replaced by (a_{re}/a). This is due to the additional amplification of modes which are still outside the horizon during (part of) the matter-dominated phase. In particular, just at re-entry, we find:

$$\Omega_\sigma(k,\eta_{re}) \simeq G\omega^2 \left(\frac{a_{re}}{a_{rad}}\right)^2 \left(\frac{S_{rad}}{S_{ex}}\right) \ . \tag{15}$$

Clearly, some condition has to be imposed on the behaviour of S during inflation to ensure that $\Omega_\sigma \ll 1$ at all times.

Let us consider next the fluctuations of the various components of the energy-momentum tensor and, in particular, their power spectra P_μ^ν defined by (no sum over μ, ν being implied):

$$\langle T_\mu^\nu(x) T_\mu^\nu(x')\rangle - \langle T_\mu^\nu(x)\rangle\langle T_\mu^\nu(x')\rangle = \int \frac{d^3k}{(2\pi k)^3} e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')} P_\mu^\nu(k) \ . \tag{16}$$

One easily finds that all the relevant components of P_μ^ν behave similarly, and are controlled by a convolution of the form:

$$P_\mu^\nu(k,\eta) \sim \left(\frac{S}{a^2}\right)^2 \ \left(\frac{k^3}{a^4}\right) \int d^3p \ p^2 |k-p|^2 \Sigma(p)\Sigma(k-p). \tag{17}$$

Using Eqs. (11), it is not hard to analyse the various integration regions in p in the above integral while always keeping $k\eta \leq 1$. In the region $0 < p < \eta^{-1}$ the integrand is proportional to $dp \ p^4 \ S_{ex}^{-2}(p)$. Imposing that seeds never be dominant makes this integrand peaked at its *upper* end. On the other hand, in the region $\eta^{-1} < p < k_1$ the integrand behaves as $dp \ p^{-4} \ (d\rho_\sigma/d\log p)^2$. If the seed spectrum $\Omega_\sigma(p)$ grows with a small enough power of p, i.e. smaller than $3/2$, this part of the integral is dominated by its *lower* end. In the opposite case it is dominated by the cutoff region $p \sim k_1$ with uninteresting consequences [8]. In conclusion, in the interesting cases, the integral is dominated by the contribution around $p \sim \eta^{-1} \geq k$, giving the following white-noise spectrum for the energy density:

$$(P_0^0)^{1/2} \sim \frac{k^{3/2}}{a^4\eta^{5/2}} \frac{S(\eta)}{S_{ex}(p\sim\eta^{-1})} \quad \sim \quad (k/a)^4 \quad (k\eta)^{-5/2} \quad S(\eta)S^{-1}(-\eta) \ \ , \tag{18}$$

where positive and negative values of η correspond, respectively, to the standard decelerated and accelerated (inflationary) phases.

According to standard cosmological perturbation theory [11], the spectrum of the Bardeen potentials Φ, Ψ, parametrizing the scalar fluctuations of the metric in a gauge-invariant way, is related to P_0^0 by[2]

$$k^{3/2}|\Psi - \Phi|(k,\eta) \simeq G \ (a/k)^2 (P_0^0)^{1/2}. \tag{19}$$

[2]In general there can be additional "compensation" factors $(k\eta)^2$ appearing in this formula [8]. However, they will not matter since, in the end, we will evaluate everything at $k\eta \sim 1$.

Recalling that $S \sim a^2$ for $\eta > \eta_{rad}$, we obtain

$$\begin{aligned} k^{3/2}|\Psi - \Phi|(k,\eta) &\simeq G(k\eta)^{-5/2}\left(\frac{k}{a}\right)^2 \frac{S(\eta)}{S_{rad}}\, \frac{S_{rad}}{S(-\eta)} \simeq G(k\eta)^{-5/2}\left(\frac{k}{a_{rad}}\right)^2 \frac{S_{rad}}{S(-\eta)} \\ &\simeq (H_1/M_P)^2\, (k\eta)^{-5/2}(k/k_1)^2\, [S_{rad}/S(-\eta)] \ , \end{aligned} \tag{20}$$

where $H_1 \equiv k_1/a_{rad}$ is the Hubble parameter at the beginning of the radiation era. Equations (20) and (14) together provide the interesting relation:

$$k^{3/2}|\Psi - \Phi|(k,\eta) \simeq (k\eta)^{-5/2}\, [S_{ex}(k)/S(-\eta)]\, \Omega_\sigma(k,\eta_{re}) \tag{21}$$

and, in particular:

$$k^{3/2}|\Psi - \Phi|(k,\eta_{re}) \simeq \Omega_\sigma(k,\eta_{re}) \simeq (H_1/M_P)^2\, (k/k_1)^2\, [S_{rad}/S_{ex}(k)]\,. \tag{22}$$

At this point we insert the above result in the formula of the SW effect, which is known to dominate the temperature anisotropies at large angular scales, $\ell \ll 100$. Combining the so-called “ordinary” and “integrated” SW contributions, a standard analysis [8, 12] yields:

$$C_\ell^{SW} = \frac{2}{\pi}\int \frac{dk}{k}\left\langle\left[\int_{k\eta_{dec}}^{k\eta_0} k^{3/2}(\Psi-\Phi)(\mathbf{k},\eta) j'_\ell\,(k\eta_0 - k\eta)\, d(k\eta)\right]^2\right\rangle \ , \tag{23}$$

where j_l are the usual spherical Bessel functions and η_0, η_{dec} are, respectively, the present time and the time of decoupling between matter and radiation (a prime stands here for the derivative of the Bessel function with respect to its argument). We exploit the previously determined η-dependence of the Bardeen potentials, assuming that, after re-entry, these potentials are dominated by a cold dark matter (CDM) component and are therefore constant. We find [8] that the η integral in Eq. (23) is dominated by the region $k\eta \sim 1$, leading to:

$$C_\ell^{SW} \sim \int d\,(\log k)\left\langle\left[k^{3/2}(\Psi-\Phi)(\mathbf{k},\eta_{re}) j_\ell\,(k\eta_0)\right]^2\right\rangle. \tag{24}$$

Inserting (22) we immediately recover the desired result (4). Note that temperature fluctuations are controlled, for each scale k, by the value of the Bardeen potentials at the time it re-enters the horizon. Roughly:

$$(\Delta T/T)(k) \sim (\Phi - \Psi)(\eta)|_{\eta\sim k^{-1}} \sim \Omega_\sigma(k,\eta_{re}) \ . \tag{25}$$

In this way, the η dependence of $(\Phi - \Psi)$ gets translated into a k (or l) dependence of the temperature fluctuation spectrum. Thus a scale-invariant Ω_σ leads to scale-invariant Harrison-Zeldovich [13] spectrum of CMB fluctuations.

For a simple power-law behaviour of the pump field, $S_{rad}/S_{ex}(k) = (k/k_1)^{\alpha-2}$, Eq. (24) can be integrated analytically with the result:

$$C_\ell^{SW} \approx K(k_1\eta_0)^{-2\alpha}\left(\frac{H_1}{M_p}\right)^4 \frac{\Gamma(2-2\alpha)}{4^{(1-\alpha)}\Gamma(3/2-\alpha)}\frac{\Gamma(\ell+\alpha)}{\Gamma(\ell+2-\alpha)}. \tag{26}$$

Comparing (26) with the standard inflationary result for CDM [14], where the spectral index n is defined by [14]

$$C_\ell^{SW} \propto \frac{\Gamma(\ell - 1/2 + n/2)}{\Gamma(\ell + 5/2 - n/2)} \,, \tag{27}$$

leads to the identification $(n - 1) = 2\alpha$. More generally, we can relate an effective (i.e. k-dependent) spectral index n_{eff} to the behaviour of the pump field during inflation via the relation:

$$(n_{eff} - 1)/2 = \alpha_{eff} \equiv 2 - [d \log S_{ex}(k)/d \log k] \,. \tag{28}$$

The nearly scale-invariant spectrum, measured by the DMR experiment aboard the COBE satellite [15], requires

$$0.8 \leq n_{eff} \leq 1.4 \tag{29}$$

and thus, allowing for generous error bars, COBE's observations imply

$$-0.1 \leq \alpha_{eff} \leq 0.2 \tag{30}$$

in the very small k region. Thus, through the definition of α_{eff} in Eq. (28), one is able to relate COBE's data to the early-time evolution of the pump field.

In this paper we have concentrated our attention on scalar perturbations. However, since the seeds are of second order in the scalar field, we also expect the presence of vector and tensor perturbations with roughly similar amplitudes.

Turning to the absolute normalization, we see from Eq. (26) that it is controlled to a large extent by the crucial parameter $(H_1/M_P)^4$. The appearance of the fourth power of H_1/M_P rather than the (more usual) second power is precisely the consequence of using seeds — rather than first-order fluctuations of the scalar field — for generating anisotropies. Thus $\Delta T/T$ goes like the square of the original fluctuations. For the same reason, although the fluctuations of σ are expected to be Gaussian, some non-Gaussianity is expected in the fluctuations of $\Delta T/T$, since they are sourced by the seed energy-momentum tensor, which is quadratic in the Gaussian variable σ. Thus, we rather expect $\Delta T/T$ to obey a χ^2 statistics (note that this is one of the few non-Gaussian examples where we really have a handle on the statistics).

Let us finally turn our attention to a specific example of our new mechanism, that of the universal axion in pre-big bang (PBB) cosmology. The universal axion of superstring theory is just the (pseudo-scalar) partner of the dilaton in the string effective action, and is massless in perturbation theory because of a Peccei-Quinn (PQ) symmetry. While the dilaton is expected to acquire a mass as soon as supersymmetry is broken, the axion could remain massless, or almost massless, because of its Nambu-Goldstone origin. Although the PQ symmetry is broken by instantonic effects, in the presence of various axions coupled to the same topological current, a linear combination, mainly lying along the invisible axion's axis, is expected to remain very light or massless (for the purpose of this work a mass of order H_0 can be considered to be zero). Such a light particle, being a gravitationally coupled pseudo-scalar, should not lead to phenomenological difficulties.

The field A of Eq. (1) turns out to be, in this case, e^{ϕ} (where ϕ denotes the dilaton) and is related to the effective Newtonian constant, in the conventions used in PBB cosmology [5], by $G_N^{eff} \sim e^{\phi}$. The pump field S is thus $a^2 e^{\phi}$, and grows very fast during PBB inflation since both a and e^{ϕ} have accelerated behaviour (on the contrary, the pump field for dilaton and gravity-wave perturbations is $a^2 e^{-\phi}$, and the two factors tend to cancel out, giving too little power at large scales [16]). In the axion case the exponent α of Eq. (26) can be evaluated [2] from the known background solutions [5]. As noticed in [3], the desired value $\alpha = 0$ is reached, in particular, for a highly symmetric, ten-dimensional PBB background in which the six extra dimensions evolve like the three ordinary ones (up to an irrelevant T-duality transformation). Precisely in this case, a scale-invariant spectrum for $\Delta T/T$ will result.

Concerning the overall normalization, controlled by $(H_1/M_P)^2$, we note that in the PBB scenario the inflationary scale H_1 is typically of order of the string scale M_s, usually taken to be around 5×10^{17} GeV. Typically, $(H_1/M_P)^2$ thus varies between 10^{-2} and 10^{-4}. We have to add the fudge factor K, which is hard to evaluate precisely, but is expected to contain factors like $(16\pi^2)^{-1}$. Thus, amusingly enough, the right order of magnitude [17] for C_2 ($C_2 \sim 10^{-10}$) may come out naturally from $K(M_s/M_P)^4$ (taking also into account the possibility that the spectrum be slightly tilted, see [8] for a quantitative discussion).

In conclusion, irrespectively of its possible model-dependent origin, we believe that a cosmic background of massless pseudo-scalar fluctuations may provide a consistent and interesting explanation of the anisotropies observed in the CMB temperature, at large angular scales. It is unclear, at present, whether such an axion-induced anisotropy may lead to significant differences in the acoustic peak structure of the CMB anisotropy spectrum at smaller angular scales. If it does, this (plus possibly some non-Gaussianity of the fluctuations) should allow tests of our axionic-seed mechanism through the high-precision measurements planned for the near future [18]. The discussion of this possibility is postponed to further work.

References

[1] G. F. Smoot et al., *Ap. J.* **396**, L1 (1992); C. L. Bennett et al., *Ap. J.* **430**, 423 (1994).

[2] E. J. Copeland, R. Easther and D. Wands, *Phys. Rev. D* **56**, 874 (1997); E. J. Copeland, J. E. Lidsey and D. Wands, *Nucl. Phys. B* **506**, 407 (1997).

[3] A. Buonanno, K. A. Meissner, C. Ungarelli and G. Veneziano, *JHEP01*, 004 (1998).

[4] R. Brustein and M. Hadad, *Phys. Rev. D* **57**, 725 (1998).

[5] G. Veneziano, *Phys. Lett. B* **265**, 287 (1991); M. Gasperini and G. Veneziano, *Astropart. Phys.* **1**, 317 (1993); *Mod. Phys. Lett. A* **8**, 3701 (1993); *Phys. Rev. D* **50**, 2519 (1994). An updated collection of papers on the pre-big bang scenario is available at `http://www.to.infn.it/~gasperin/`.

[6] A. A. Anselm and N. G. Uraltsev, *Phys. Lett. B* **114**, 39 (1982).

[7] R. K. Sachs and A. M. Wolfe. *Ap. J.* **147**, 73 (1967).

[8] R. Durrer, M. Gasperini, M. Sakellariadou and G. Veneziano, *Seeds of large-scale anisotropy in string cosmology*, CERN-TH/98-69, gr-qc/9804076.

[9] M. Gasperini and G. Veneziano, *Constraints on pre-big bang models for seeding large-scale anisotropy by massive Kalb-Ramond axions*, CERN-TH/98-180.

[10] R. Brustein, M. Gasperini and G. Veneziano, *Duality in Cosmological Perturbation Theory*, Phys. Lett. B (1998), in press (hep-th/9803018).

[11] R. Durrer and N. Straumann, *Helv. Phys. Acta* **61**, 1027 (1988).

[12] R. Durrer, *Phys. Rev. D* **42**, 2533 (1990); *Fund. of Cosmic Physics* **15**, 209 (1994).

[13] H. R. Harrison, *Phys. Rev. D* **1**, 2726 (1970); Y. B. Zel'dovich, *Mon. Not. Roy. Ast. Soc.* **160**, 1 (1972).

[14] R. Bond and G. Efstathiou, *MNRAS* **226**, 655 (1987).

[15] G.F. Smoot and D. Scott, in L. Montanet *et al.*, *Phys. Rev. D* **50**, 1173 (1994) (1996 update).

[16] M. Gasperini and M. Giovannini, *Phys. Rev. D* **47**, 1519 (1993);
R. Brustein, M. Gasperini, M. Giovannini, V. F. Mukhanov and G. Veneziano, *Phys. Rev. D* **51**, 6744 (1995).

[17] A. J. Banday et al., *Astrophys. J.* **475**, 393 (1997).

[18] M. Bersanelli et al., *COBRAS/SAMBA, Report on the phase A study*, ESA Document D/SCI(96) 3 (1996).