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Relaxed Bounds on the Dilaton Mass In a String Cosmology Scenario

M.Gasperini
Dipartimento di Fisica Teorica dell'Università,
Via P.Giuria 1, 10125 Torino, Italy,
and
Istituto Nazionale di Fisica Nucleare, Sezione di Torino

Abstract

We discuss bounds on the dilaton mass, following from the cosmological amplification of the quantum fluctuations of the dilaton background, under the assumption that such fluctuations are dominant with respect to the classical background oscillations. We show that if the fluctuation spectrum grows with the frequency the bounds are relaxed with respect to the more conventional case of a flat or decreasing spectrum. If the growth of the spectrum is fast enough, the allowed range of masses may become compatible with models of supersymmetry breaking, and with a universe presently dominated by a relic background of dilaton dark matter.

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It is known that the coherent oscillations around the minimum of the potential of a cosmic scalar background [1-4], such as the dilaton [5-7], put severe cosmological bounds on the allowed value of the mass m of that scalar field. Such oscillations, however, become coherent, and then constrain the mass, only when the possible spatial inhomogeneities of the background are negligible.

Spatial inhomogeneities of the dilaton background can arise either because of thermal fluctuations, or because of the gravitational amplification of quantum fluctuations. If the spatial inhomogeneities of thermal origin are diluted by a subsequent inflationary expansion (or if they are absent simply because the maximum temperature scale is lower than the Planckian temperature required for thermal equilibrium), then their contribution to the energy density (in a mode expansion of the background oscillations) becomes negligible with respect to the mass contribution. The coherent oscillations of the background, with frequency m, can then dominate the dilaton energy density beginning at a scale $H \simeq m$.

The same happens for the energy density stored in the quantum fluctuations, if they are amplified with a spectrum which is flat or decreasing with frequency. Also in that case the mass coherent contribution begins to dominate (with respect to spatial inhomogeneities) the fluctuation energy at $H \simeq m$, thus avoiding a significant relaxation of the cosmological bounds, even for negligible amplitude of the classical background oscillations [2].

If, on the contrary, the quantum fluctuations are amplified with a spectrum which grows with frequency, then their energy density may stay dominated by the small scale inhomogeneities until values of H much lower than m. In that case the bounds on m can be alleviated, provided the classical background solution approaches enough the minimum of the potential before the scale drops to $H = m$.

A growing spectral distribution of the dilaton fluctuations is a typical outcome of the "Pre-big-bang" inflationary scenario [8,9], suggested by the duality properties of the string cosmology equations [10]. In such context, as we shall see in this paper, the relaxation of the bound has two important consequences: the allowed range of dilaton masses may become compatible with a) model of supersymmetry breaking able to provide a natural resolution of the gauge hierarchy problem [11], and with b) the possibility that our universe is presently dominated by a relic background of dilaton dark matter. (It should be mentioned that a growing scalar perturbation spectrum is also predicted by the "hybrid inflation" model proposed by Linde [12] and recently generalized to the class of "false vacuum inflation" [13] (see also [14]). Growing spectra, moreover, have been shown to be necessary for a simultaneous fit of the COBE anisotropies and of the observed bulk motion and large voids structures on a $50 Mpc$ scale [15]).

In view of its importance, we start by recalling the standard arguments leading to the bounds on the dilaton mass based on the coherent oscillations of the classical background [1-7]. For scales $H < H_1 \leq M_p$, where M_p is the Planck mass and H_1 the Hubble parameter at the time t_1 which marks the end of inflation (and which we assume to coincide with the beginning of the radiation-dominated era), a homogeneous dilaton background must satisfy the equation

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi} = 0 \tag{1}$$

(we neglect for the moment the friction term due to a possible dilaton decay, with width $\Gamma \simeq m^3/M_p^2$). For values of ϕ sufficiently near to the minimum ϕ_0 of V, we approximate the potential by $V = m^2(\phi - \phi_0)^2/2$, with $m < H_1$. When $H >> m$ one thus finds that ϕ approaches asymptotically a constant value ϕ_1,

$$\phi = \phi_1 + \phi_2(\frac{H}{H_1})^{1/2} \tag{2}$$

so that, for $H << H_1$, the distance from the minimum can be approximated by $|\phi - \phi_0| \simeq |\phi_1 - \phi_0| = const$. Typically, for interactions of gravitational strength, ϕ_0, ϕ_1 and ϕ_2 are all of order M_p but, without fine tuning, one expects that also $|\phi_1 - \phi_0| \sim M_p$. As a consequence, when the scale H drops to $H = m$, the homogeneous background ϕ begins to oscillate coherently with frequency m and initial amplitude of order M_p, which decreases in time like $a^{-3/2}$ (a is the scale factor of the isotropic metric background). The associated energy density, ρ_ϕ, decreases like a^{-3}, starting from an initial value $\rho_i \simeq m^2 M_p^2$ which is of the same order as the radiation energy density ρ_γ at that epoch. For $H < m$ the universe thus enters a phase of dust-like matter domination, in which $\rho_\gamma/\rho_\phi \sim a^{-1}$. This leaves two possible alternatives open. If

$$m \lesssim H_2 \simeq 10^{-27} eV \tag{3}$$

where H_2 is the usual equilibrium scale corresponding to the matter-radiation transition, then ρ_ϕ stays always smaller than the critical density, and the oscillating dilaton background could survive until tody. This possibility seems to be excluded by the present tests of the equivalence principle, which imply [16,17]

$$m \gtrsim m_0 = 10^{-4} eV \tag{4}$$

(see however [18]). As a consequence, the dilaton must have decayed, i.e. the energy stored in the coherent oscillations must have converted in radiation, at a decay scale

$$H_d = \Gamma \simeq \frac{m^3}{M_p^2} < m \tag{5}$$

However, since the dilaton begins to dominate much before the nucleosynthesis scale H_N, as

$$H_N \simeq \frac{(1MeV)^2}{M_p} << m_0 \lesssim m, \tag{6}$$

one must impose that the reheating temperature T_r associated to the dilaton decay, such that

$$T_r^4 \simeq M_p^2 H_d^2, \tag{7}$$

be large enough to allow a subsequent nucleosynthesis phase. This means $T_r \simeq (m^3/M_p)^{1/2} \gtrsim 1MeV$, namely

$$m \gtrsim 10^4 \ GeV. \tag{8}$$

Moreover, the radiation temperature T_d just before dilaton decay is

$$T_d = T_m(\frac{a_m}{a_d}) \simeq (mM_p)^{1/2}(\frac{H_d}{m})^{2/3} \simeq (\frac{m^{11}}{M_p^5})^{1/6} \tag{9}$$

(the index m means that the variable is to be evaluated at the scale $H = m$). The reheating from T_d to T_r will thus produce an entropy increase

$$\Delta S = (\frac{T_r}{T_d})^3 \simeq \frac{M_p}{m} \tag{10}$$

In order to preserve any pre-existing baryon-antibaryon asymmetry, the condition $\Delta S \lesssim 10^5$ should be satisfied [5-7]. Such a condition implies

$$m \gtrsim 10^{14} \ GeV \tag{11}$$

This last requirement could be alleviated, however, in the case of low-energy (electro-weak, for instance) baryogenesis; in particular, in the case of baryogenesis associated to the dilaton decay itself [7,11], occuring at scales not much distant from nucleosynthesis.

These are the standard arguments, leading to bounds on m which are independent of the inflation scale H_1, and which are crucially grounded on the assumption that the asymptotic value ϕ_1, approached by ϕ during its evolution for $H >> m$, lies at a distance of order unity (in Planck units) from the minimum ϕ_0 of the potential.

One might thus be led to think that the bounds could be evaded if, owing to some mechanism, the initial amplitude of the classical background oscillations would be lowered to $|\phi - \phi_0| << M_p$. For instance, if the asymptotic value ϕ_1 would be fine-tuned to ϕ_0, then $\phi - \phi_0$ would be no longer constant but would become scale-dependent according to eq.(2). At a scale H, the shift from the minimum would be typically of order $|\phi - \phi_0| \sim M_p(H/H_1)^{1/2}$, thus leading to coherent oscillations for $H \leq m$ with initial energy density $\rho_1 \simeq (mM_p)^2(m/H_1) < (mM_p)^2$.

Even in that case, however, the scenario would not be free of problems. Indeed, as pointed out in Ref.[2], besides the classical oscillations one must always take into account also the quantum fluctuations of the background, $\delta\phi = \chi$, amplified by the inflationary evolution. The fluctuation modes χ_k satisfy the equation

$$\ddot{\chi}_k + 3H\dot{\chi}_k + (\frac{k^2}{a^2} + m^2)\chi_k = 0 \qquad (12)$$

and for $m < H_1$ the spectral distribution of the associated energy density, ρ_χ, is

$$k\frac{d\rho_\chi}{dk} \simeq (\frac{H_1 a_1}{a})^4(\frac{k}{k_1})^{n-1} \qquad (13)$$

Here $k_1 = a_1 H_1$ is the maximum amplified comoving frequency, and the spectral index, $n = 3 - 2\alpha$, is fixed by the power α determining the evolution (in conformal time η) of the scale factor during inflation, $a \sim |\eta|^{-\alpha}$.

Let us discuss, first of all all, the case of the scale-invariant, Harrison-Zeldovich spectrum ($n = 1$), corresponding to de Sitter-like inflation ($\alpha = 1$) with $H = H_1 = const$. In this case all the modes χ_k contribute to the energy density

$$\rho_\chi(t) \simeq (\frac{H_1 a_1}{a})^4 = H_1^2 H^2 \qquad (14)$$

with the same amplitude. (Note the condition $H_1 < M_p$ to be satisfied in order that ρ_χ does not overclose the universe during radiation dominance). A generic mode $k > am$ begins to oscillate at a scale $H_k = k/a_k$, with an amplitude χ_k which is initially of order H_1, and which decreases in time as $\chi_k \simeq H_1(H/H_k)^{1/2}$. When $H \simeq m$, the non-relativistic modes ($k/a \lesssim m$) begin to oscillate, with initial amplitude H_1 and frequency $\simeq m$, and they immediately become dominant with respect to the other modes, as their contribution to ρ_χ decreases like a^{-3} instead of a^{-4}. For $H \leq m$ we are thus in a situation where, beside the energy density stored in the possible coherent oscillations of the classical background, $\rho_\phi = \rho_m(a_m/a)^3$, we must have, necessarily, also some energy stored in the coherent oscillations of the quantum fluctuations, with

$$\rho_\chi \simeq m^2 H_1^2 (\frac{a_m}{a})^3 \tag{15}$$

In this paper we want to discuss how the cosmological bounds on the dilaton mass are relaxed when we move from a scenario in which the fluctuation spectrum is flat or decreasing ($n \leq 1$), to another scenario characterized by a growing ($n > 1$) spectrum. We shall thus assume, henceforth, that the classical oscillations (whose initial amplitude is model-dependent) are always negligible, $\rho_\phi < \rho_\chi$, and that all bounds on m follow from the cosmological amplification of the quantum fluctuations only. We will obtain, in this way, the *maximum* (approximately model-independent) allowed region in parameter space.

The energy density (15) is smaller than the radiation energy ρ_γ when $H = m$, but it grows with respect to ρ_γ as the curvature scale decreases in time, until it equals ρ_γ at an initial scale H_i. If $H_i < H_2$, which means

$$m < H_2 (\frac{M_p}{H_1})^4 \tag{16}$$

(recall that H_2 denotes the usual matter-radiation transition scale of eq.(3)), then ρ_χ stays always smaller than the critical density. If, however, $H_i > H_2$ then

$$H_i = m (\frac{H_1}{M_p})^4 \tag{17}$$

and the dilaton must have already decayed, $H_d > H_0$, to avoid contradictions with the presently observed matter density. This implies

$$m \gtrsim (M_p^2 H_0)^{1/3}, \tag{18}$$

where $H_0 \sim 10^{-61} M_p$ is the present curvature scale. The dilaton decay generates an entropy $\Delta S = (T_r/T_d)^3$, where T_r is the reheating temperature (7) and T_d the radiation temperature at the dilaton decay epoch, namely

$$T_d = T_i (\frac{a_i}{a_d}) = (M_p H_i)^{1/2} (\frac{H_d}{H_i})^{2/3} \simeq (\frac{m^{11}}{M_p H_1^4})^{1/6} \tag{19}$$

This gives

$$\Delta S = \frac{H_1^2}{m M_p} \tag{20}$$

If $m < 10^4 GeV$ the reheating temperature is too low ($< 1 MeV$) to allow nucleosynthesis: we must impose that nucleosynthesis already occurred, $H_i < H_N$, and that [5-7] $\Delta S \lesssim 10$, in order not to destroy all light nuclei formed. If, on the contrary, $m > 10^4 GeV$, the nucleosynthesis scale is subsequent to dilaton decay, and the only possible constraint is [5-7] $\Delta S \lesssim 10^5$ in order to preserve primordial baryogenesis.

These are the conditions to be imposed if $m < H_1$. If $m > H_1 = k_1/a_1$, then all modes are always non-relativistic, and the spectral energy distribution becomes [19]

$$k\frac{d\rho_\chi}{dk} \simeq H_1^4(\frac{a_1}{a})^3(\frac{m}{H_1})^2(\frac{k}{k_1})^{n-1} \tag{21}$$

This gives, for $n = 1$,

$$\rho_\chi(t) \simeq m^2 H_1^2(\frac{a_1}{a})^3 \tag{22}$$

The scale $H = H_1$ marks the beginning of coherent oscillations with frequency m and initial energy $\rho_i = m^2 H_1^2$ (hence $m < M_p$ to avoid over-critical density). There are no further bounds on m, in this case, as the fluctuation energy is dissipated before a possible dilaton dominance. Indeed, the scale $H_i = H_1(m/M_p)^4$ corresponding to the equality $\rho_\chi = \rho_\gamma$ is always smaller than the decay scale, $H_d = m^3/M_p^2 > H_i$.

The values of (m, H_1) allowed by the previous constraints are shown in **Fig.1**. One can see that the bounds on m are relaxed but, as stressed in [2], too low values of H_1 are in general required to be compatible with an interesting range of masses. The preferred supersymmetry breaking scale $m \sim 1TeV$, for instance, is forbidden unless $H_1 \lesssim 10^{-8} M_p$. Similar values of H_1 ($\sim 10^{-6} - 10^{-9} M_p$) are required to be compatible with the possibility of a not yet decayed, and presently dominating, dilaton.

The situation becomes even worse for quantum fluctuation with a decreasing spectrum ($n < 1$). In that case the initial amplitude χ_i of the coherent oscillations become larger, $\chi_i \simeq H_1(H_1/m)^{(1-n)/4} > H_1$ and, as a consequence, a lower inflation scale H_1 is required to be compatible with the same given value of m.

String cosmology, however, suggests a scenario in which the standard radiation dominated era is preceeded by a so-called pre-big-bang phase, describing the evolution from a flat, weakly coupled initial state [8]. The universe super-inflates, bends up and heats up to a maximum scale H_1, after which curvature and temperature begin to decrease. The dilaton grows up to the strong coupling regime, and its settlement to a constant value marks the beginning of the standard cosmological evolution. The particle production associated with the transition between pre and post-big-bang regime is characterized by a growing spectrum [8,9], and imposes the constraint $H_1 < M_p$ to prevent an over-critical density of produced massless particles (such as gravitons). In that context, the quantum fluctuations of the dilaton background are also amplified with a growing spectrum, in particular with the same spectrum of tensor perturbations in the case of a vacuum, dilaton-driven, isotropic pre-big-bang [19].

Motivated by string cosmology, we shall thus discuss how the previous bounds on the dilaton mass are to be changed if the fluctuation spectrum is growing with frequency. If $n > 1$ in eq.(13), the scale $H = m$ marks the beginning of coherent oscillations (with frequency m) of the mode $k_m = ma_m$, which provides a non-relativistic contribution to the fluctuation energy density,

$$\rho_\chi(t) \simeq m^2 H_1^2 (\frac{m}{H_1})^{(n-1)/2} (\frac{a_m}{a})^3 \tag{23}$$

This contribution grows in time with respect to ρ_γ, so that we can repeat exactly the same phenomenological discussion previously applied to the case of a flat spectrum, with the only difference that the energy density (15) is now replaced by eq.(23). As a consequence, we obtain bounds which depend on the spectral index n and which are, in particular, less and less constraining as n is growing (because ρ_χ decreases with n).

There is a limit, however, on such a relaxation, because for $n \geq 2$ the contribution (23) to the total fluctuation energy becomes subdominant with respect to the contribution of the maximum frequency mode k_1. Indeed, this mode becomes non-relativistic at a scale H_{nr} such that $k_1/a_{nr} = m$, namely

$$H_{nr} = \frac{m^2}{H_1} < m \tag{24}$$

For higher scales, k_1 dominates the relativistic part of the fluctuation energy, $\rho_\chi \simeq H_1^2 H^2$. Therefore, the contribution (23) of k_m may dominate the total fluctuation energy only if the equality

$$m^2 H_1^2 (\frac{m}{H_1})^{(n-1)/2} (\frac{m}{H})^{-3/2} = H_1^2 H^2 \tag{25}$$

occurs at a scale $H > H_{nr}$, namely for $(m/H_1)^{n-2} > 1$, which implies $n < 2$.

In order to discuss to which *maximum* extent the cosmological bounds on m are possibly relaxed in the case of growing dilaton spectrum, we shall thus concentrate, in what follows, on the case $n \geq 2$ (see [19] for a discussion of the "interpolating" case $1 < n < 2$). One may note that if such a "large" spectral index would apply also to the scalar part of the metric perturbations, it would be impossible to explain the anisotropy observed by COBE [20], which should then to be ascribed to other sources. It should be stressed, however, that dilaton, scalar and tensor perturbations of the metric background are not necessarily forced to have the same spectrum, if other gravitational sources, beside the dilaton, are present and the background is anisotropic like, for instance, during a phase of dynamical dimensional reduction [19].

For $n \geq 2$, ρ_χ is always dominated by the highest mode k_1, and the contribution to ρ_χ of the small scale inhomogeneities becomes negligible only for $H \lesssim H_{nr}$, when ρ_χ behaves as non-relativistic matter,

$$\rho_\chi \simeq (\frac{mH_1 a_1}{k_1})^4 (\frac{a_{nr}}{a})^3 = m^4 (\frac{a_{nr}}{a})^3 \tag{26}$$

corresponding to coherent oscillations with frequency m and initial amplitude m.

This energy density stays always smaller than the critical one if $H_{nr} < H_2$, and also if ρ_χ equals ρ_γ at a scale $H_i < H_2$, namely for

$$m < (H_1 H_2)^{1/2} (\frac{M_p}{H_1})^2 \tag{27}$$

The same happens if $H_i > H_2$, where

$$H_i = \frac{m^2}{M_p} (\frac{H_1}{M_p})^3 \tag{28}$$

but the dilaton decays before becoming dominant, $H_d > H_i$. If on the contrary, $H_i > H_2$ and $H_d < H_i$, the dilaton must have decayed (to avoid a present over-critical density) when it was dominant and, as before, we are left with two possible alternatives.

If $T_r < 1MeV$ we must impose that nucleosynthesis already occurred, i.e. $H_N > H_i$ (with the scale H_i of eq.(28)), and that $\Delta S = (T_r/T_d)^3 \lesssim 10$. Since

$$T_d = T_i (\frac{a_i}{a_d}) \simeq (M_p H_i)^{1/2} (\frac{H_d}{H_i})^{2/3} = (\frac{m^{10}}{M_p H_1^3})^{1/6} \tag{29}$$

the produced entropy to be bounded is, in this case,

$$\Delta S = (\frac{H_1^3}{m M_p^2})^{1/2} \tag{30}$$

The other possibility, $T_r > 1MeV$, allows a nucleosynthesis phase subsequent to dilaton decay, so that the only bound is imposed by primordial baryogenesis,

$\Delta S \lesssim 10^5$. The case $m > H_1$, finally, provides the only bound $m < M_p$ exactly like in the previous case of a flat perturbation spectrum.

The maximal allowed region in the plane (m, H_1), for the case of quantum fluctuations with a growing spectrum ($n \geq 2$), is illustrated in **Fig.2**. The forbidden region describes a sort of funnel wedged between the lower limit $m = m_0 = 10^{-4} eV$ allowed by the equivalence principle, and the upper limit $m = M_p$ allowed by the closure density. The bounds are relaxed with respect to the previous case, in such a way that for $H_1 \lesssim 10^{-5} M_p$ practically all values of m are allowed. In particular, a dilaton mass in the TeV range (required by models of supersymmetry breaking [11]), becomes compatible with an inflation scale $H_1 \simeq 10^{-5} M_p$, which seems to be the value suggested by the observed COBE anisotropy [21]. Moreover, a universe presently dominated by a relic background of not yet decayed dilatons becomes possible for

$$1\ eV \lesssim m \lesssim 100\ MeV \tag{31}$$

and compatible with realistic (in a string cosmology context) inflation scales $H_1 \geq 10^{-5} M_p$.

In conclusion, our qualitative analysis shows that the bounds on the dilaton mass arising from the cosmological amplification of the quantum fluctuations are less constraining when their spectrum is growing, like in a string cosmology context, instead of being flat or decreasing like in other, more conventional inflationary models. The bounds of **Fig.2** define the *maximum* allowed region, obtained for a spectral index $n \geq 2$, and corresponding to the case in which the classical dilaton background seats exactly at the minimum of the effective potential, without oscillating or running at the scale $H = m$. Such a region would be of course reduced if classical background oscillations were to be added to the quantum fluctuations. It can be easily verified, however, that as long as one can arrange a scenario in which the distance of the background from the minimum decreases with the scale like $H^{1/2}$ (or faster), the bounds on m in the higher scales sector $H_1 \gtrsim 10^{-5}$ (which is the interesting sector for string cosmology) remain practically unchanged with respect to the bounds reported in **Fig.2**.

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Figure captions

Fig.1 Maximum allowed region (inside the full lines) for quantum fluctuations amplified with a scale-invariant spectrum, and dominant with respect to the classical background oscillations. The dilaton mass is given in units of $m_0 = 10^{-4}eV$. The lines marked by a, b, c, d, e, f, g correspond respectively to: a) $m = m_0$, lower bound on m from the equivalence principle; b) $H_1 = M_p$, upper bound on H_1 from the closure density; c) $T_r = 1MeV$, lower bound on the reheating temperature from nucleosynthesis; d) $m = M_p$, upper bound on $m > H_1$ from the closure density; e) $m = H_2(M_p/H_1)^4$, upper bound on m from the present matter-to-radiation energy density ratio; f) $H_1^2/mM_p = 10^5$, upper limit on entropy production from primordial baryogenesis; g) $H_1^2/mM_p = 10$, upper limit on entropy from nucleosynthesis.

Fig.2 Maximum allowed region (inside the full lines) in the case of quantum fluctuations amplified with a growing spectrum (with $n \geq 2$), and dominant with respect to the classical background oscillations. The lines marked by a, b, c, d are the same as in Fig.1. The other lines correspond to: e) $m = (H_1H_2)^{1/2}(M_p/H_1)^2$, upper bound on m from the present matter-to-radiation energy density ratio; f) $(H_1^3/mM_p^2)^{1/2} = 10^5$, upper limit on entropy production from primordial baryogenesis; g) $(H_1^3/mM_p^2)^{1/2} = 10$, upper limit on entropy production from nucleosynthesis.