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CONSTRAINTS ON INFLATION AT THE PLANCK SCALE FROM THE RELIC GRAVITON SPECTRUM

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ABSTRACT

Constraints on the maximum allowed scale for de Sitter inflation can be evaded if the de Sitter phase is preceded by an epoch of growing curvature and accelerated expansion (superinflation). In this case the universe can inflate up to a Planckian curvature scale. The duration of the final de Sitter phase, however, turns out to depend crucially on the power-law behaviour of the former superinflationary epoch, and is significatively constrained by the limit imposed by pulsar-timing data on the presently surviving background of cosmic gravitons.

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1. Introduction

It has been repeatedly stressed [1-4] that the energy distribution of relic gravitons, produced by the cosmological expansion, can provide us with significative information on the behaviour of the Hubble parameter H (and thus on the curvature) of the very early universe. Indeed, gravity waves decoupled from matter earlier than any other field interacting with the background curvature [3,4]; the shape of their spectrum could thus survive nearly unchanged up to present times, presumably since the Planck era.

In the interesting case of a transition from de Sitter inflation to the standard radiation-dominated expansion, the graviton spectrum was first calculated by Starobinsky [5] and later discussed in detail by many authors. By imposing on the energy density of the produced gravitons the bound following from the isotropy of the cosmic-microwave radiation (CMBR), one finds in that case that the Hubble parameter during de Sitter inflation must satisfy the constraint [6]

$$H_{dS} \leq 10^{-4} M_P \tag{1.1}$$

(M_P is the Planck mass). The same constraint applies if the de Sitter phase ($\dot{H} = 0$) is replaced by a phase of power-law ($\dot{H} < 0$) inflationary expansion [7] (a dot denotes differentiation with respect to the cosmic time t).

To reach this conclusion, however, one usually assumes for the de Sitter phase a time extension which is unbounded towards the past, or which is finite but long enough to provide (possibly together with an additional period of power-law inflation) a solution to the standard cosmological problems. The aim of this paper is to show that the constraint (1.1) can be evaded, if the de Sitter phase is preceded by an epoch of superinflationary ($\dot{H} > 0$) accelerated expansion (we are following the terminology of [8]).

Our argument is very simple. The spectral energy density of the gravitons, produced in the transition from inflation to radiation- dominated expansion, is frequency-independent if during inflation $\dot{H} = 0$, while it grows or decreases with frequency if, during inflation, $\dot{H} > 0$ or $\dot{H} < 0$ respectively. If the de Sitter phase follows a phase of superinflation, the resulting graviton energy spectrum turns out to be depressed, at low frequencies, with respect to the pure de Sitter case. Since the constraints following from CMBR isotropy applies to the low-frequency tail of the spectrum, it becomes thus possible to raise the spectrum up to a level corresponding, in the high frequency sector, to a de Sitter inflation at the Planck scale (and higher), without conflicting with phenomenological bounds at lower frequencies.

In this case, however, the duration of the de Sitter part of the total inflation is constrained by pulsar-timing data [9], and it turns out to depend crucially on the power-law behaviour of the former superinflationary expansion. We find, in particular, that a de Sitter phase at the Planck scale ($H_{dS} \simeq M_P$) may contribute at most with a factor $Z_{dS} \simeq 10^{18}$ to the total amount of inflation. Our results, therefore, may be of phenomenological interest for models of the universe like those obtained in a string cosmology context [10], where a (de Sitter) phase of constant curvature could possibly occur near the Planck (or Hagedorn) scale, and may be expected to separate a phase of decelerated evolution and decreasing curvature from a "dual" phase of increasing density and curvature.

2. Bogoliubov coefficients for the cosmological graviton production

In order to discuss our constraint on a Planck-scale-inflation, we shall first derive the graviton energy spectrum associated to a generic cosmological transition. We recall, to this aim, that a convenient method to deal with particle creation in a changing background metric is provided by the formalism of Bogoliubov transformations [11].

Consider indeed a gravitational perturbation $h_{\mu\nu}$, propagating in a spatially flat cosmological background, with metric

$$g_{\mu\nu} = diag \quad a^2(\eta)(1,-1,-1,-1) \tag{2.1}$$

and satisfying, in this background, the linearized equations $\Box h_{\mu\nu} = 0$ (η is the conformal time such that $a = dt/d\eta$). After performing Fourier expansion one finds, for the Fourier component of the proper amplitude, $\mu(k,\eta) = a(\eta)h(k,\eta)$, (and for each polarization mode) the wave equation [1,2,4,7,12]

$$\mu'' + [k^2 - V(\eta)]\mu = 0 \tag{2.2}$$

A prime denotes differentiation with respect to the conformal time η, $k = a\omega$ is the comoving wave-number (ω is the proper frequency), and $V(\eta) = a''/a$.

Suppose now that the background metric evolves in such a way that $V(\eta)$ reaches a maximum and goes to zero as $\eta \to \pm\infty$ (typical behaviour for a universe which inflates and then decelerates). This effective potential barrier in eq.(2.2) leads to wave amplification (as first pointed out by Grishchuk [12]) or, equivalently, to particle production. Indeed, the positive frequency modes defining asymptotically the vacuum state to the left of the barrier ($\eta \to -\infty$), will be in general a linear superposition of modes which are of positive and negative frequency with respect to the vacuum at the right ($\eta \to \infty$ limit). The coefficients of the Bogoliubov transformation connecting the "left" and "right" vacuum, fixed by the matching conditions, will determine the spectral density of the produced gravitons [13].

In the model we consider here for illustrative purpose, the scale factor evolves in time like $\eta^{-\alpha}$ for $\eta < \eta_1$, and like $\eta^{-\beta}$ for $\eta > \eta_1$. The exact solution of eq.(2.2), representing positive frequency modes in the $\eta \to -\infty$ limit, can thus be written

$$N(\frac{\eta}{\eta_1})^{1/2} H^{(2)}_{\alpha+\frac{1}{2}}(k\eta) \tag{2.3}$$

where $H^{(2)}_\nu = J_\nu - iY_\nu$ are the Hankel functions (we follow the conventions of [14]), and N is a dimensionless coefficients to be fixed by the normalization conditions. Correspondingly, the positive frequency solution with respect to the "right" vacuum ($\eta \to \infty$) is given by

$$N(\frac{\eta}{\eta_1})^{1/2} H^{(2)}_{\beta+\frac{1}{2}}(k\eta) \tag{2.4}$$

The Bogoliubov coefficients $c_\pm(k)$ are thus defined, for each mode , by the expansion

$$N(\frac{\eta}{\eta_1})^{1/2} H^{(2)}_{\alpha+\frac{1}{2}}(k\eta) = (\frac{\eta}{\eta_1})^{1/2}[c_+ H^{(2)}_{\beta+\frac{1}{2}}(k\eta) + c_- H^{(1)}_{\beta+\frac{1}{2}}(k\eta)] \tag{2.5}$$

where $H^{(1)}_\nu = J_\nu + iY_\nu$ (recall, for $|\eta| \to \infty$, that $\sqrt{\eta} H^1(k\eta) \sim \frac{e^{ik\eta}}{\sqrt{k}}$, and that $\sqrt{\eta} H^2(k\eta) \sim \frac{e^{-ik\eta}}{\sqrt{k}}$).

The coefficients $c_\pm$ can be computed by performing an asymptotic matching of the solution around the $k \to 0$ limit (according to the procedure used also in [7]). In this limit eq.(2.2) becomes $\mu''/\mu = a''/a$, and the two sides of eq.(2.4) (which are both exact solution of eq.(2.2)) must reduce indeed to the general form [1,2,12]

$$aA(k) + aB(k)\int^\eta \frac{d\eta}{a^2}$$

where A and B are time-independent coefficients. By using the small argument limit of the Bessel functions, and the continuity of $a(\eta)$ in $\eta = \eta_1$, we can thus rewrite eq.(2.5) as

$$A_1 + B_1 \int \frac{d\eta}{\eta_1 a^2} = A_2 + B_2 \int \frac{d\eta}{\eta_1 a^2} \tag{2.6}$$

where

$$A_1 = \frac{i}{\pi} N\Gamma(\alpha + \frac{1}{2}) x_1^{-\alpha-1/2} \quad , \quad B_1 = \frac{2N}{\Gamma(\alpha+\frac{1}{2})} x_1^{\alpha+1/2}$$

$$A_2 = (c_+ - c_-)\frac{i}{\pi}\Gamma(\beta+\frac{1}{2})x_1^{-\beta-1/2} \quad , \quad B_2 = (c_+ + c_-)\frac{2}{\Gamma(\beta+\frac{1}{2})}x_1^{\beta+1/2} \qquad (2.7)$$

and $x_1 = k\eta_1/2$ (we have supposed $\beta + 1/2 > 0$ in performing the $k \to 0$ limit). The equality $A_1 = A_2$ and $B_1 = B_2$ gives finally

$$c_\pm = \frac{N}{2}(x_1^{\alpha-\beta}\gamma_{\alpha\beta} \pm x_1^{\beta-\alpha}\gamma_{\alpha\beta}^{-1}) \qquad (2.8)$$

where $\gamma_{\alpha\beta} = \Gamma(\beta+1/2)/\Gamma(\alpha+1/2)$.

The normalization condition $|c_+|^2 - |c_-|^2 = 1$ (required for the interpretation of $c_\pm$ as Bogoliubov coefficients determining, at a second quantization level, the particle production from the vacuum), is thus satisfied provided N is an arbitrary phase factor, $|N| = 1$.

Note that the result (2.8) is valid for $|x_1| < 1$, namely for $k < |V(\eta_1)|^{1/2}$, i.e. only for those modes which "hit" the barrier (otherwise stated: that cross the horizon) before the transition to a different scale factor. For $|x_1| > 1$ there is no wave amplification [12], the mode mixing is adiabatically suppressed [11], so that the corresponding particle production will be ignored for the purpose of this paper ($c_+ \simeq 1, c_- \simeq 0$).

The procedure leading to eq.(2.8) can obviously be iterated an arbitrary number of times, to take into account a situation in which the scale factor is described (in conformal time) by a power α_1 up to the time η_1, by a power α_2 up to the time η_2, and so on. After n transitions the resulting Bogoliubov coefficients are

$$c_\pm^{(n)} = \frac{N}{2}[\prod_{i=1}^{n} x_i^{\alpha_i-\alpha_{i+1}}\gamma(\alpha_i,\alpha_{i+1}) \pm \prod_{i=1}^{n} x_i^{\alpha_{i+1}-\alpha_i}\gamma^{-1}(\alpha_i,\alpha_{i+1})] \qquad (2.9)$$

where $N^* = N^{-1}$ is an overall phase factor,$x_i = k\eta_i/2$, $|x_i| < 1$, and $\gamma(\alpha_i, \alpha_{i+1})$ are numerical factors of order unity which depend on the model of matching (continuity of a and a' at $\eta = \eta_i$), and on the asymptotic behaviour of the Hankel functions (namely on the sign of $\alpha_i + 1/2$).

We note, finally, that if the graviton field is in a vacuum state at the left of the barrier, the total energy density ρ_g of the gravitons produced by the cosmological transition (in the proper frequency interval $d\omega$) is given by

$$d\rho_g = \frac{\omega^3}{\pi^2}d\omega|c_-(\omega)|^2 \qquad (2.10)$$

The spectral energy density, $\rho(\omega)$, which is the convenient variable usually adopted in this context to characterize the graviton production [1,2,4,7], can thus be expressed in terms of the Bogoliubov coefficients as

$$\rho(\omega) \equiv \frac{d\rho_g}{dln\omega} = \frac{\omega^4}{\pi^2}|c_-(\omega)|^2 \qquad (2.11)$$

3. Graviton spectra for the inflation-radiation transition

Consider now the transition between a generic inflationary phase and the standard radiation-dominated expansion. According to the relation between conformal and cosmic time

$$a \sim \eta^{-\alpha} \sim t^{\frac{\alpha}{\alpha-1}} \tag{3.1}$$

inflation ($\ddot{a} > 0$) corresponds to $\alpha > 0$, while radiation-dominance ($a \sim t^{1/2}$) corresponds to $\alpha = -1$. We have, in particular, power- law inflation ($\dot{H} < 0$) if $\alpha > 1$, de Sitter inflation ($\dot{H} = 0$) if $\alpha = 1$, and superinflation ($\dot{H} > 0$) if $0 < \alpha < 1$.

By inserting $\beta = -1$ in eq.(2.8), and keeping only the dominant term for $|x_1| < 1$ (we neglect also unimportant numerical factors of order unity) , we obtain for this transition the spectral density

$$\rho_{i\to r}(\omega, t) \simeq \omega^4 x_1^{-(2+2\alpha)} \simeq \omega^{2-2\alpha}[a(t)\eta_1]^{-(2+2\alpha)} \tag{3.2}$$

where η_1 is the transition time, and t is the time at which the produced spectrum is observed. One can thus immediately see that $\rho(\omega)$ increases with frequency for superinflation ($\alpha < 1$), decreases with frequency for power-law inflation ($\alpha > 1$), while it is "flat" [5] for the de Sitter case ($\alpha = 1$).

We note that η_1 can be expressed in terms of the scale factor and of the Hubble parameter at the epoch of transition, i.e. $\eta_1 \simeq (H_1 a_1)^{-1}$. Analogously, a given comoving frequency k can be expressed in terms of the inflationary scale factor a_i and of the Hubble parameter, H_i, evaluated at the time $|\eta_k| \simeq k^{-1}$ at which the given mode enters the under-barrier region, i.e. $k \simeq |H_i(\eta_k)a_i(\eta_k)|$. Consequently, eq.(3.2) may be rewritten in the useful form

$$\rho_{i\to r}(\omega, t) \simeq H_i^2(\eta_k) H_1^2 [\frac{a_1}{a(t)}]^4 \tag{3.3}$$

(in agreement with previous results of Allen [4], Sahni [7] and Grishchuk [1]).

As previously stressed, this spectrum ranges in frequency up to a maximum value ω_M, determined by the height of the potential $V(\eta_1)$ (higher frequency modes are unaffected by the potential barrier, in the adiabatic approximation). This maximal frequency can be related to the minimal frequency $\omega_m \simeq 10^{-18} Hz$ of a gravitational wave crossing to-day the Hubble radius, and we obtain

$$\omega_M = \frac{|H_1 a_1|}{H_0 a_0} \omega_m \simeq 10^{11} Hz \tag{3.4}$$

where H_0 and a_0 are the present values of the Hubble parameter and scale factor (we have assumed an adiabatic evolution of the universe after the inflation era).Note, however, that this maximum value may turn out to be somewhat lowered, in practice, depending on the particular model of the reheating mechanism [2,4].

The spectrum (3.3) cannot be extended down to $10^{-18}Hz$, because low frequencies are affected by a further transition of our universe to the present matter dominated regime (in which $a \sim t^{2/3} \sim \eta^2$). Denoting by $\eta_2 > \eta_1$ the time of this second transition, for modes with $k < |\eta_2|^{-1} \simeq |H_2 a_2|$ the Bogoliubov coefficients can be obtained from eq.(2.9), putting $n = 2$, $\alpha_1 = \alpha$, $\alpha_2 = -1$, $\alpha_3 = -2$. Since, at fixed k, $|x_1| << |x_2| < 1$ (because of the fast decreasing of H during radiation-dominance), eq.(2.11) gives [4,7] (again neglecting numerical factors of order unity)

$$\rho_{i\to r\to m}(\omega, t) \simeq \omega^4 x_1^{-(2+2\alpha)} x_2^{-2} \simeq \rho_{i\to r}[\frac{H_2 a_2}{\omega a(t)}]^2 \sim \omega^{-2\alpha} \tag{3.5}$$

This behaviour of the spectral energy density holds from the minimal frequency ω_m, up to the frequency determined by the height of the potential barrier at $\eta = \eta_2$, i.e.

$$\omega_2 = \frac{H_2 a_2}{H_0 a_0}\omega_m \simeq 10^{-16}Hz \tag{3.6}$$

For higher frequencies eq.(3.3) applies.

In order to compare these results with the experimental constraints on the graviton spectrum, it is convenient to measure the spectral density in units of critical energy density ρ_c, defining

$$\Omega(\omega) \equiv \frac{\rho(\omega)}{\rho_c} = \frac{d\Omega_g}{dln\omega} \tag{3.7}$$

(where $\Omega_g = \rho_g/\rho_c$). Moreover, from the Einstein equations $H_1^2 \simeq G\rho_\gamma(\eta_1)$, where ρ_γ is the radiation energy-density (because η_1 is the beginning of the radiation era). The spectral density (in critical units) can thus be recast in the form

$$\Omega(\omega) \simeq GH_i^2\Omega_\gamma(t) \quad , \quad \omega \geq 10^{-16}Hz \tag{3.8}$$

$$\Omega(\omega) \simeq GH_i^2\Omega_\gamma(t)(\frac{10^{-16}Hz}{\omega})^2 \quad , \quad 10^{-18}Hz \leq \omega \leq 10^{-16}Hz \tag{3.9}$$

As pointed out by many authors, Ω is constrained by the present measurements of the CMBR isotropy, which imply [2,6] $\Omega < 10^{-8}$ at $\omega \simeq 10^{-18}Hz$, and by pulsar timing observations, which imply [2,9] $\Omega < 10^{-6}$ at $\omega \simeq 10^{-8} Hz$. In the case of power and de Sitter inflation, the spectral energy density of the produced gravitons decreases with frequency, so that the most effective bound comes from the measures of CMBR isotropy. By inserting the to-day fraction of radiation energy density, $\Omega_\gamma \sim 10^{-4}$, one thus obtain from (3.9) the bound on the inflation scale already reported in eq.(1.1), namely $\sqrt{G} H_i \leq 10^{-4}$.

The graviton spectral density obtained from superinflation, on the contrary, grows with frequency and so is not constrained neither by the isotropy nor by the pulsar bound. The energy density can be raised, in principle, up to $\Omega \sim 1$, which corresponds to inflation at a scale $H_i \simeq 10^2 M_P$ (from eq.(3.8)). There are indirect bounds, based on early nucleosynthesis [15], which seem to forbid at high frequency a density of massless particles higher than $\Omega \sim 10^{-4}$; this means , however, $GH_i^2 \leq 1$, so that even in this case the inflation can reach the Planck scale.

The maximum value of the spectral energy density, presently allowed by the isotropy limit, pulsar limit, and critical density limit, is shown in **Fig.1** for the de Sitter case, for a typical power-law case ($a \sim t^2$) and for a typical superinflationary case ($a \sim t^{-1/2}$). Also shown, for comparison, is the thermal blackbody spectrum at $1^o K$, and the region of maximal planned sensitivity for a possible future Laser Interferometer Gravity wave Observatory (LIGO) (see for instance [3]). The sensitivity of present cryogenic bar detectors does not seem to be significative at the level of a stochastic background of cosmic gravitons.

We see that pulsars, and CMBR isotropy, may constrain superinflation only in the limit $\alpha \to 1$ in which the spectrum approaches the flat de Sitter behaviour. The allowed power α for a superinflationary spectrum, compatible with both a given isotropy bound $\Omega \leq \overline{\Omega}_1$ at $\omega = 10^{-18} Hz$, and with the bound $\Omega \leq \overline{\Omega}_2$ at $\omega = 10^{11} Hz$, must satisfy the phenomenological constraint

$$\alpha \leq \frac{1}{58} Log(10^{54} \frac{\overline{\Omega}_1}{\overline{\Omega}_2}) \tag{3.10}$$

The present isotropy bound, $\overline{\Omega}_1 \simeq 10^{-8}$, allows superinflation to reach the maximum allowed scale $10^2 M_P$ (i.e. $\overline{\Omega}_2 = 1$), provided $\alpha \leq 23/29$. For superinflation ending at the Planck scale ($\overline{\Omega}_2 = 10^{-4}$) this limit is somewhat released to $\alpha \leq 25/29$. In both cases, the present bound from pulsar data at $\omega = 10^{-8} Hz$ is automatically satisfied.

It may be interesting to note that, in the cosmic time gauge where $a \sim t^{-\beta}$, the limiting power for superinflation at the Planck scale becomes $\beta < 25/4$.

This bound is largely satisfied, in particular, by all Kaluza-Klein models in which the superinflation of three-dimensional space is induced by the contraction of an appropriate number of internal dimensions [16].

4. Constraints on a two-component inflation model

To discuss the possibility of de Sitter inflation at the Planck scale, we now consider an approximated model of cosmological evolution in which an initial superinflationary phase is followed, at $\eta = \eta_1$, by a de Sitter inflation, which subsequently decays at $\eta = \eta_2$ to a radiation-dominated expansion, and finally becomes at $\eta = \eta_3$ a matter-dominated expansion.

The Bogoliubov coefficients, for this threefold transition, can be obtained from eq.(2.9) by putting $\alpha_1 = \alpha$, $\alpha_2 = 1$ (de Sitter), $\alpha_3 = -1$ (radiation), $\alpha_4 = 2$ (matter). For a realistic model, the transition times must satisfy, at fixed k,

$$|x_2| << |x_1| << |x_3| < 1 \tag{4.1}$$

(the first inequality follows from the requirement that the duration of the de Sitter phase may be long enough to give a significative contribution to the total amount of inflation, i.e. $Z_{dS} = a_2/a_1 >> 1$). For values of k such that $|x_i| > 1$, c_- is vanishing in the adiabatic approximation.

Using the condition (4.1) to pick out the dominant contributions to the Bogoliubov coefficients (and neglecting factors of order unity), we get the following spectral density (in critical units).

For $k > |\eta_1|^{-1}$ (high frequency sector) we have the flat de Sitter spectrum

$$\Omega_{dS\to r}(\omega, t) \simeq \frac{\omega^4}{\rho_c} x_2^{-4} \simeq G H_{dS}^2 \Omega_\gamma(t) \simeq 10^{-4} G H_{dS}^2 \tag{4.2}$$

where H_{dS} is the constant value of the Hubble parameter during the de Sitter phase. In the intermediate band $|\eta_3|^{-1} \leq k \leq |\eta_1|^{-1}$ we obtain the spectrum

$$\Omega_{s\to dS\to r} \simeq \frac{\omega^4}{\rho_c} x_2^{-4} x_1^{2-2\alpha} \simeq \Omega_{dS\to r}[\frac{\omega a(t)}{H_1 a_1}]^{2-2\alpha} \sim \omega^{2-2\alpha} \tag{4.3}$$

which is growing like in the pure superinflationary case. Finally, for $k \leq |\eta_3|^{-1}$, we have

$$\Omega_{s\to dS\to r\to m} \simeq \frac{\omega^4}{\rho_c} x_2^{-4} x_1^{2-2\alpha} x_3^{-2} \simeq \Omega_{s\to dS\to r}[\frac{H_3 a_3}{\omega a(t)}]^2 \sim \omega^{-2\alpha} \tag{4.4}$$

De Sitter inflation may thus reach the maximal scale $H_{dS} \simeq 10^2 M_P$, provided the growth of the spectrum in the intermediate frequency range is steep enough to evade the low frequency bounds. Its duration, however, is constrained by the present isotropy limit if $H_{dS} > 10^{-2} M_P$, and also by the pulsar limit if $H_{dS} > 10^{-1} M_P$.

To discuss this effect let us call, as before, $\overline{\Omega}_1$ the maximum allowed value of Ω imposed by the CMBR isotropy at $10^{-18} Hz$; we call, moreover, $\overline{\Omega}_3$ the limit obtained from pulsar timing data at $\omega = 10^{-8} Hz$. First of all we note that de Sitter inflation at a scale higher than $10^{-1} M_P$ is compatible with both limits provided superinflation is slow enough, according to the constraint

$$\alpha \leq \frac{1}{20} Log(10^{16} \frac{\overline{\Omega}_1}{\overline{\Omega}_3}) \tag{4.5}$$

For the present values of $\overline{\Omega}_1, \overline{\Omega}_3$ this means $\alpha < 7/10$ (or, in the cosmic time language, $a \sim t^{-\beta}$ with $\beta < 7/3$).

For a given scale H_{dS}, moreover, the duration of the phase (which can be expressed in terms of the inflation factor $Z_{dS} = a_2/a_1$) depends on the superinflationary power α as follows:

$$Z_{dS}(H_{dS}, \alpha, \overline{\Omega}_3) \leq 10^{\frac{21-19\alpha}{1-\alpha}} \overline{\Omega}_3^{\frac{1}{2(1-\alpha)}} (\frac{H_{ds}}{M_P})^{\frac{1}{\alpha-1}} \tag{4.6}$$

By inserting the present experimental value we get

$$Z_{dS}(H_{dS}, \alpha)|_{\overline{\Omega}_3 = 10^{-6}} \leq 10^{\frac{18-19\alpha}{1-\alpha}} (\frac{H_{ds}}{M_P})^{\frac{1}{\alpha-1}} \tag{4.7}$$

(for $0 \leq \alpha \leq 7/10$).

At a fixed scale $H_{dS} > 10^{-1} M_P$, the maximal duration of the de Sitter phase is thus obtained in coincidence with the $\alpha \to 0$ limit. For a given α, the duration decreases with the scale. The maximum allowed value of Z_{dS}, according to eq.(4.7), is plotted in **Fig.2** for $\alpha = 0, 1/3, 1/2$.

In **Fig.3** we show the maximum allowed spectral energy density of cosmic gravitons, for a model in which the final de Sitter phase settles down right at the Planck scale, and which is preceded by various types of superinflationary expansion, with $\alpha = 1/3, 1/2, 7/10$. They correspond, in the cosmic time language, to $t^{-1/2}, t^{-1}$ and $t^{-7/3}$ respectively. The limiting case $\alpha = 0$ (flat space) may be thought of as corresponding to a superinflationary phase so short to be approximately described as an almost instantaneous transition from the Minkowski to the de Sitter metric.

5. Conclusions

In this paper we have discussed the possibility that the phenomenological constraints on the spectral density of cosmic gravitons , produced in the transition from an inflationary to a decelerated phase , may allow inflation to occur at a Planck scale.

For a phase of accelerated expansion and growing curvature (superinflation), parametrized in conformal time by the scale factor $a \sim \eta^{-\alpha}$, $0 < \alpha < 1$, we have shown that the Planck scale may be reached, in spite on the bounds on the spectrum imposed by the CMBR isotropy and by the pulsar timing data. Superinflation must be slow enough, however, and according to the present values of the experimental bounds we have computed that superinflation may continue up to $H \simeq M_P$ provided

$$\alpha \leq 23/29 \tag{5.1}$$

if it is followed by a phase of radiation-dominated expansion, and provided

$$\alpha \leq 7/10 \tag{5.2}$$

if it is followed by a phase of de Sitter expansion.

It is not phenomenologically excluded, therefore, an early de Sitter configuration occuring at a Planck (and higher) curvature scale. Its duration (or, equivalently, the supplied amount of inflation Z_{dS}) is however bounded. The maximum inflation factor allowed by present constraints turns out to be, at the Planck scale,

$$Z_{dS} = 10^{18} \tag{5.3}$$

One may note that the same configuration seems to be possible even if the superinflationary phase before de Sitter is replaced by a decelerated expansion (or even a contraction). Both cases correspond indeed to $\alpha < 0$ in eq.(4.3), and both lead to a spectrum which is growing in the intermediate range of frequencies. In such cases, however , the inflation factor of the final de Sitter phase is not complemented by the superinflationary contribution, so that, according to eq.(5.3), the total amount of inflation cannot be large enough to solve the problems of the standard cosmological model.

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Figure captions

Fig.1

The maximal graviton spectral energy density, compatible with present experimental bounds, for three typical types of inflationary expansion. For comparison is also shown the thermal spectrum at $1^o K$, and the expected sensitivity of a LIGO interferometric system (the region inside the three dotted lines).

Fig.2

The maximum allowed inflation factor of the final de Sitter phase, plotted versus the inflation scale, for three different types of superinflationary evolution.

Fig.3

Maximum allowed graviton spectral density, corresponding to a final de Sitter inflation at the Planck scale, preceded by three different types of superinflationary evolution (the $\alpha \to 0$ limit corresponds to a phase of vanishing curvature). The intersection point corresponds to the present value of the bound on Ω obtained from pulsar timing data.