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Repulsive gravity in the very early Universe

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Abstract

I present two examples in which the curvature singularity of a radiation-dominated Universe is regularized by *(a)* the repulsive effects of spin interactions, and *(b)* the repulsive effects arising from a breaking of the local gravitational gauge symmetry. In both cases the collapse of an initial, asymptotically flat state is stopped, and the Universe bounces towards a state of decelerated expansion. The emerging picture is typical of the pre-big bang scenario, with the main difference that the string cosmology dilaton is replaced by a classical radiation fluid, and the solutions are not duality-invariant.

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The aim of this Essay is to discuss the possibility of avoiding the initial cosmological singularity through a phase of repulsive gravity occurring in the very early Universe. I will consider two mechanisms of repulsive gravity: spin-torsion interactions and spontaneous breaking of the local $SO(3,1)$ gauge symmetry. I will show that in both cases the condition of geodesic convergence [1] can be violated, and the cosmological equations may admit regular homogeneous and isotropic solutions for which the energy density and the curvature grow up to a maximum (finite) scale, and then decrease, with a smooth joining to the standard decelerated evolution.

The interesting aspect of such models is that they do not require any violation of the strong energy condition [1] in the conventional matter sector. Indeed, in both cases I will simply take a radiation-like equation of state for the sources (no vacuum energy term will be included). In spite of the fact that I will use classical generalization of the Einstein equations, the results obtained might be of some relevance for applications to string cosmology, where the present cosmological phase is expected to emerge from a phase of growing curvature, through a smooth transition that should avoid the initial singularity [2].

I will first discuss the case of spin-torsion interactions. Torsion is a natural ingredient of gauge theories of the Poincarè group [3], as it represents the field strength of local translations, and it is thus the required Yang-Mills partner of the curvature (the field strength of local Lorentz rotations). In addition, torsion couples minimally to the axial current of spinor matter, as required by local supersymmetry: simple supergravity, containing only the graviton and the gravitino, can indeed be formulated as an Einstein-Cartan theory for the Rarita-Schwinger field [4].

The Einstein-Cartan theory [3], which I will consider in this paper, is the simplest example of gravitational theory with torsion. In such a theory torsion does not propagate, and it can be non-vanishing only in the presence of an intrinsic spin density of matter. As a consequence, no significant effect is expected for macroscopic bodies at ordinary densities; torsion interactions may become important, however, in the regime of extremely high density and curvature of the early Universe.

Let us thus consider a cosmological application of the Einstein-Cartan theory, by taking a perfect gas of spinning particles as the effective matter source. In that case the connection is non-symmetric, $\Gamma_{[\mu\nu]}{}^{\alpha} \neq 0$, and besides the equation relating the Einstein tensor and the canonical (non-symmetric) energy-momentum tensor,

$$G_{\mu\nu}(\Gamma) = 8\pi G T_{\mu\nu}, \tag{1}$$

we have an additional algebraic relation [3] between the torsion, $Q_{\mu\nu}{}^{\alpha} = \Gamma_{[\mu\nu]}{}^{\alpha}$, and the canonical spin density tensor, $S_{\mu\nu}{}^{\alpha}$:

$$Q_{\mu\nu}{}^{\alpha} = 8\pi G \left(S_{\mu\nu}{}^{\alpha} + \frac{1}{2}\delta^{\alpha}_{\mu} S_{\nu\beta}{}^{\beta} - \frac{1}{2}\delta^{\alpha}_{\nu} S_{\mu\beta}{}^{\beta} \right). \tag{2}$$

Thanks to the above relation, torsion can be eliminated everywhere in eq. (1). By assuming a convective model of spinning fluid minimally coupled to the geometry of the Riemann-Cartan manifold [5], we can rewrite eq. (1) in the standard Einsteinian form for a symmetric connection, but with additional terms that are linear and quadratic in the spin tensor of the matter sources.

In the absence of some external polarizing field the spins are randomly oriented, and the linear terms are zero after an appropriate space-time averaging, $\langle S_{\mu\nu\alpha} \rangle = 0$; the quadratic terms, however, are non-vanishing also on the average, $\langle S_{\mu\nu\alpha} S^{\mu\nu\alpha} \rangle \neq 0$. Because of the spinning sources we are thus led to a modified set of cosmological equations, even for unpolarized matter, and in the averaged macroscopic limit. For a spatially flat metric $g_{\mu\nu} =$ diag $(1, -a^2\delta_{ij})$, in particular, the averaged cosmological equations can be written as [6]:

$$H^2 = \frac{8\pi G}{3} \left(\rho - 2\pi G \sigma^2 \right), \tag{3}$$

$$\dot{H} + H^2 = -\frac{4\pi G}{3} \left(\rho + 3p - 8\pi G \sigma^2 \right). \tag{4}$$

Their combination gives the conservation equation

$$\dot{\rho} - 2\pi G (\sigma^2)^{\cdot} + 3H \left(\rho + p - 4\pi G \sigma^2 \right) = 0, \tag{5}$$

where $H = \dot{a}/a$, and a dot denotes differentiation with respect to cosmic time. I have defined $\sigma^2 = \langle S_{\mu\nu\alpha} S^{\mu\nu\alpha} \rangle / 2$, and ρ, $p > 0$ are the energy density and the pressure of the fluid in the zero spin limit.

When $8\pi G \sigma^2 > \rho + 3p$ the condition of geodesic convergence is violated,

$$R_{\mu\nu} u^{\mu} u^{\nu} = -3 \left(\dot{H} + H^2 \right) < 0, \tag{6}$$

even if the pressure satisfy the strong energy condition, $\rho + 3p > 0$. In a previous paper this repulsive contribution of the spin density was used to discuss the possibility of spin-dominated inflation [6]. Here it will be used for a possible regularization of the initial curvature singularity.

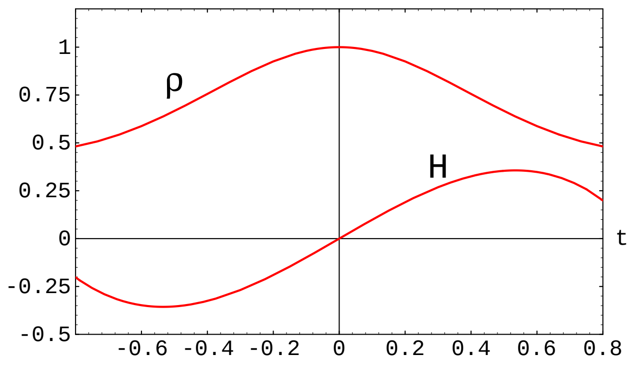

FIG. 1. *Time evolution of the Hubble factor and of the energy density according to eq. (7). I have put* $c_1 = c_2 = 1$*, and time is measured in units of* $(3/8\pi)^{1/2} l_p$.

The spin contribution to the geometry depends, of course, on the particular model of fluid. In order to show that this repulsive interaction can be strong enough to allow a smooth cosmological evolution, I will consider a spinning liquid of unpolarized fermions [7], with equation of state $p = \gamma\rho$, and averaged squared spin tensor $\sigma^2 \propto \rho^{2/(1+\gamma)}$. In this case the equations can be integrated exactly. For relativistic fermions, in particular, we have $\gamma = 1/3$, the conservation equation (5) gives $\rho \propto a^{-4}$, and the integration of eq. (3) leads to

$$\frac{t}{l_p}\sqrt{\frac{8\pi}{3}} = \frac{a}{2}\sqrt{c_1 a^2 - c_2} + \frac{c_2}{2}\ln\left|a + \sqrt{c_1 a^2 - c_2}\right| \tag{7}$$

(c_1, c_2 are dimensionless positive constants, and we are measuring time in Planck length units, with $l_p = \sqrt{G}$).

A plot of the energy density and of the Hubble parameter for this solution is shown in Fig. 1. The curvature is everywhere regular, and the models describes a smooth evolution from a phase of accelerated contraction, growing curvature, to a phase of decelerated expansion, decreasing curvature. The scale factor contracts down to a minimal value $a_m = \sqrt{c_2/c_1}$, and then re-expands (like $a \sim t^{1/2}$, asymptotically). In string cosmology, this behaviour is typical of the pre-big bang scenario represented in terms of the Einstein frame metric [8].

It may be interesting to observe that a similar class of solutions can also be obtained from the string cosmology equations through a duality boost of the flat, two-dimensional Milne metric [9]. Indeed, this fact is more than a coincidence, as the global $O(3,3)$ duality group, used in [9], introduces a non-trivial antisymmetric tensor background, $H_{[\mu\nu\alpha]} \neq 0$, which is known to have a geometric interpretation as the torsion of an appropriate connection. The main difference is that in string cosmology the "matter source" is the scalar dilaton field, while in this example matter is more conventionally represented as a perfect fluid, and the duality symmetry of string theory is lost.

A second, possible mechanism for the generation of repulsive interactions in the early Universe is associated to the breaking of the local $SO(3,1)$ symmetry of the gravitational interaction [10]. This symmetry is part of the local gauge group of gravity: in the gauge approach to general relativity, the anholonomic Ricci connection ω_μ^{ij} represents in fact the Yang-Mills potential of local Lorentz rotations, which transforms as a covariant vector in the index μ under general reparametrizations, and as an antisymmetric tensor in the two "internal" indices i, j, under the action of the local $SO(3,1)$ group.

Like every gauge symmetry, also this local Lorentz symmetry can be broken spontaneously when an appropriate (geometric) potential, generated by a self-interacting antisymmetric tensor, appears in the action [11]. This breaking leads to an effective "quasi-riemannian" theory [12], namely to a gauge theory of gravity invariant under general reparametrizations, but with a local tangent space group other than the Lorentz group. From a phenomenological point of view, the main consequences of such a breaking are the possible appearance of repulsive forces, [10,11], and the possible violation of the equivalence principle [13,14].

The violation of the weak equivalence principle, however, is not a necessary consequence of any Lorentz symmetry breaking. If we consider, for instance, a four-dimensional quasi-riemannian theory with local $SO(3)$ invariance, we find that the most general model contains four independent parameters in the gravitational part of the action, and three parameters in the matter action. By imposing four conditions on these seven parameters it is always possible to preserve the covariant conservation of the energy momentum tensor, in such a way that the motion of test particles remains geodesic [13].

In that case the causal structure of space-time is still determined by the metric tensor, the classical singularity theorems [1] still can be applied, and the violation of geodesic convergence is still a necessary condition for singularity prevention. Because of the modified dynamical equations, however, geodesic convergence and strong energy condition are no longer equivalent [15], so that a smooth and complete model of cosmological evolution can be implemented even with conventional matter sources, satisfying the strong energy condition.

As a particular example of this possibility I will consider here a one-parameter, $SO(3)$-invariant quasi-riemannian model of gravity, which for a closed, homogeneous and isotropic manifold is described by the action

$$S = 16\pi G S_m - \int dt a^3 \left[(1+\epsilon)\frac{6H^2}{N} - \frac{6k}{a^2}N\right]. \tag{8}$$

Here S_m is the action for perfect fluid matter, N is the lapse function, k is the spatial curvature (in Planck length units), and ϵ is a dimensionless constant parametrizing the breaking of the local Lorentz symmetry. All the other parameters have been fixed in such a way as to preserve the geodesic motion of the cosmological fluid [15]. In the limit $\epsilon \to 0$ the action reduces to the standard, general relativistic action.

The variation with respect to N and a, in the cosmic time gauge $N = 1$, leads to the equations

$$(1+\epsilon)H^2 + \frac{k}{a^2} = \frac{8\pi}{3}G\rho, \tag{9}$$

$$(1+\epsilon)\left(2\dot{H} + 3H^2\right) + \frac{k}{a^2} = -8\pi G p, \tag{10}$$

and their combination gives

$$\dot{\rho} + 3H(\rho + p) = 0, \tag{11}$$

in agreement with the weak equivalence principle, $\nabla_\nu T_\mu{}^\nu = 0$. Note that in the absence of spatial curvature this particular breaking of the gauge symmetry has no effect on a cosmological metric, apart from a trivial renormalization of the gravitational coupling constant.

The value of ϵ depends on the parameters of the antisymmetric tensor potential [11] that breaks spontaneously $SO(3,1)$ down to $SO(3)$. Today, and at a macroscopical level, a breaking of local Lorentz symmetry is strongly constrained by many experimental data [13,14]. In the regime of extremely high temperature and density of the very early Universe, however, such phenomenological constraints do not necessarily apply, and for $\epsilon < -1$ gravity may become repulsive enough to prevent the singularity, even if $\rho + 3p > 0$.

Consider in fact a radiation fluid, $p = \rho/3$, so that, from eq. (11), $\rho = \rho_0 a^{-4}$. The integration of eq. (9), for $k = +1$ and $\epsilon < -1$, gives then

$$a(t) = \left[\frac{8\pi}{3}\rho_0 l_p^4 + \frac{1}{|1+\epsilon|}\left(\frac{t}{l_p}\right)^2\right]^{1/2}, \tag{12}$$

where ρ_0 is a positive integration constant. For this solution, the plot of the Hubble parameter

$$H = \frac{t}{t^2 + |1+\epsilon|\frac{8\pi}{3}\rho_0 l_p^6} \tag{13}$$

and of the energy density is qualitatively the same as the plot of Fig. 1: the initial collapse of an asymptotically flat state is stopped, and the Universe bounces to a state of curvature-dominated, linear expansion. Note however that, unlike the Einstein-Cartan solution of the previous example, in this case the Universe does not become asymptotically radiation-dominated.

In conclusion, I would like to stress the fundamental role played by antisymmetric tensors in these two examples of regular cosmological models. In the first case the repulsive forces stopping the collapse are due to the coupling between the spin and the antisymmetric torsion field, in the second case they are due to a self-interacting antisymmetric tensor that provides the right "Higgs potential" for the breaking of the local $SO(3,1)$ symmetry. This suggests that a successful, singularity-free pre-big bang scenario might require a non-trivial antisymmetric tensor background, arising either from the NS (Neveu-Schwartz) or the RR (Ramond-Ramond) sector of the underlying string theory (or M-theory) effective action [16].

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