DFTT-24/94
June 1994

Phenomenological Aspects of the Pre-Big-Bang Scenario in String Cosmology*

M.Gasperini
Dipartimento di Fisica Teorica, Universitá di Torino,
Via P.Giuria 1, 10125 Turin, Italy,
and INFN, Sezione di Torino, Turin, Italy

Abstract

I review various aspects of the pre-big-bang scenario and of its main open problems, with emphasis on the role played by the dilaton. Since the dilaton is a compelling consequence of string theory, tests of this scenario are direct tests of string theory and also, more generally, of Planck scale physics.

To appear in the **Proceedings of the 2nd Journée Cosmologie**, *Observatoire de Paris, June 2-4, 1994* (World Scientific P.C., Singapore)

---

* Based on talks given at *CERN* (June 1993), at the *First Int. Workshop on Birth of the Universe and Fundamental Physics* (Rome, May 1994), and at the *2nd Journée Cosmologie* (Paris, June 1994).

PHENOMENOLOGICAL ASPECTS OF THE PRE-BIG-BANG SCENARIO IN STRING COSMOLOGY

M. GASPERINI

Dipartimento di Fisica Teorica, Universitá di Torino,
Via P.Giuria 1, 10125 Turin, Italy,
and INFN, Sezione di Torino, Turin, Italy

ABSTRACT

I review various aspects of the pre-big-bang scenario and of its main open problems, with emphasis on the role played by the dilaton. Since the dilaton is a compelling consequence of string theory, tests of this scenario are direct tests of string theory and also, more generally, of Planck scale physics.

Table of Contents

1. Introduction: the Pre-Big-Bang Scenario.

The aim of this paper is to provide a short but self-contained introduction to the so-called "pre-big-bang" inflationary scenario [1], arising in the context of string cosmology, with emphasis on its phenomenological aspects. In particular, I will concentrate my discussion on the process of cosmological dilaton production [2], as an example of the fact that it is the dilaton which mainly differentiates string cosmology from other inflationary models of the early universe.

Let me recall, first of all, that what I refer to here as "string cosmology" is simply a model of the early universe based on the low-energy string effective action, possibly supplemented by matter sources. The sources may be represented, phenomenologically, even in a perfect fluid form, but always with an equation of state which consistently follows from the solution of the string equations of motion in the given cosmological background. This definition is probably not enough for a truly "stringy" description of the universe in the very high curvature regime (see Sect. 4), but it is certainly enough for suggesting the scenario that I have sketched in **Fig. 1**.

Indeed, already at the level of the low energy string effective action, there are motivations to expect that the present phase of standard cosmological evolution (decelerated, with three spatial dimensions, decreasing curvature, "frozen" Newton constant), is preceeded in time by a phase which is the "dual" counterpart of the present one (dual in the sense explained in Sect. 2). Such dual phase is characterized by accelerated expansion of the external dimensions, possible accelerated shrinking of the internal ones, growing curvature, growing dilaton. As a consequence, the entire time evolution of the curvature scale corresponds to the bell-like like curve of **Fig. 1**, instead of blowing up like in the standard model, or of approaching a constant value like in the case of de Sitter-type evolution. A similar bell-like behaviour is expected for the temperature and for the total effective energy density.

This cosmological picture was first sketched in some pioneer papers based on an application of the target-space duality of string theory in a thermodynamical context [3], and later independently re-discovered with a different approach, based on the solution of the string equations of motion in a curved background [4]. The thermodynamical approach was further developed in [5,6], while the dynamical approach led to the notion of "scale-factor" duality [7-9], subsequently applied to formulate a possibly realistic inflationary cosmology in [1,2,10-13].

In the context of such a scenario the "big-bang" is simply interpreted as the phase of maximal (but finite) curvature and temperature, marking the transition from the epoch of accelerated evolution, growing curvature and gravitational coupling, to the standard radiation-dominated evolution. Hence the name "pre-big-bang" for the primordial phase in which the curvature is growing, as illustrated in **Fig. 1**.

It should be stressed, however, that **Fig. 1** gives only a qualitative, very rough description of the whole scenario. In particular, there is no need for the evolution of the curvature scale to be time-symmetric, and a standard inflationary phase could be included also in the post-big-bang period (or in the transition epoch). Indeed, the pre-big-bang scenario should be regarded not as *alternative*, but as *complementary* to the standard (even inflationary) cosmological picture. At least complementary to those inflationary models which cannot be extended for ever towards the past, without running into a singularity [14].

In the following Section I will report, very briefly, some string theory motivation supporting a pre-big-bang cosmological scenario.

2. Motivations for the Pre-Big-Bang Scenario.

A first motivation relies on a symmetry property of the cosmological equations obtained from the low energy string effective Lagrangian [15],

$$\mathcal{L} = -\sqrt{|g|}e^{-\phi}\left[R + (\partial_\mu \phi)^2 - \frac{1}{2}(\partial_{[\mu} B_{\nu\alpha]})^2 + ...\right] \tag{2.1}$$

the so-called "scale-factor" duality [7,8] (here ϕ is the dilaton and $B_{\mu\nu} = -B_{\nu\mu}$ the antisymmetric (torsion) tensor). This symmetry implies that if the background fields are only time-dependent, and if $\{a, \phi\}$ are scale factor and dilaton of a given isotropic exact solution with $B_{\mu\nu} = 0$, then a new exact solution $\{\tilde{a}, \tilde{\phi}\}$ is obtained through the transformation (in d spatial dimensions)

$$a \to \tilde{a} = a^{-1}, \quad \phi \to \tilde{\phi} = \phi - 2d\ln a \tag{2.2}$$

This symmetry is a particular case of a more general $O(d,d)$-covariance [9,16] of the cosmological equations, which holds for spatially flat metric backgrounds, and which mixes non-trivially the spatial components of the metric, g_{ij}, and of the antisymmetric tensor, B_{ij}. This covariance holds even in the presence of sources, provided they represent "bulk" string matter, satisfying the string equations of motion in the given background [11]. In such case, the duality transformation (2.2) acting on the background is to be accompanied (in a perfect fluid approximation) by a reflection of the equation of state,

$$p/\rho \to \tilde{p}/\tilde{\rho} = -p/\rho \tag{2.3}$$

It should be noted, moreover, that a generalized version of duality symmetry can be extended also to the case of spatially curved manifolds invariant under some non-abelian isometry [17], though there are technical problems related to the dilaton transformation in case the isometry groups is non-semisimple [18].

What is important, in our context, is that by combining a duality transformation with time-reversal, $t \to -t$, we can always associate to any given decelerated, expanding solution, with decreasing curvature, characterized by the condition

$$\dot{a} > 0, \quad \ddot{a} < 0, \quad \dot{H} < 0 \tag{2.4}$$

($H = \dot{a}/a$, and a dot denotes differentiation with respect to the cosmic time t), an accelerated, expanding solution, with growing curvature,

$$\dot{a} > 0, \quad \ddot{a} > 0, \quad \dot{H} > 0 \tag{2.5}$$

of the pre-big-bang type [1]. Consider, as example of decelerated "post-big-bang" solution, the simple but important case of standard radiation-dominated evolution, with frozen dilaton,

$$a = t^{1/2}, \quad p = \rho/3, \quad \phi = const, \quad t > 0 \tag{2.6}$$

which is still an exact solution of the string cosmology equations. The associated pre-big-bang solution

$$a = (-t)^{-1/2}, \quad p = -\rho/3, \quad \phi = -3\ln(-t), \quad t < 0 \tag{2.7}$$

describes superinflationary [19] expansion, with growing dilaton. Owing to the properties of the low energy string effective action it is then possible, in particular, to find "self-dual" cosmological solutions [1], characterized by the condition

$$a(t) = a^{-1}(-t) \tag{2.8}$$

and connecting in a smooth way the two duality-related regimes. This is impossible in the context of the Einstein equations, where there is no dilaton, and this duality symmetry cannot be implemented.

A second string theory motivation for the pre-big-bang scenario is related to the fact that when the evolution is accelerated and the curvature scale is growing, according to eq.(2.5), then the proper size of the event horizon

$$d_e(t) = a(t)\int_t^{t_1} dt' a^{-1}(t') \tag{2.9}$$

(t_1 is the maximal allowed future extension of the cosmic time coordinate in the given manifold) shrinks linearly in cosmic time,

$$d_e(t) \sim (t_1 - t), \qquad t \to t_1 \tag{2.10}$$

instead of being constant like in case of de Sitter-like exponential expansion (see Table I and II of Ref.[1]). No problems for points, of course, but objects of finite proper size may become in such case larger than the horizon itself. Different points of the object, falling along different geodesics, tend to become asymptotically causally disconnected, because their proper spatial separation becomes larger than the proper size of the local horizon associated to their relative acceleration [20].

In such a situation the stress tensor $T_\mu{}^\nu$ of a gas of test strings satisfies the condition [4,21]

$$T_0{}^0 \simeq \sum_i T_i{}^i \tag{2.11}$$

which implies, in the perfect fluid approximation $T_0{}^0 = \rho$, $T_i{}^j = -p\delta_i{}^j$, a negative effective pressure $p < 0$. With such a negative pressure the string gas itself may sustain, self-consistently, the given background evolution [1]. This is impossible in the case of point-like objects, where there is no such asymptotic "stretching" regime, and then no equation of state compatible with the background evolution.

This means, in other words, that by using as model of sources a sufficiently diluted gas of classical strings, we can find self-consistent solutions to the full system of equations [2,22] including *both* the background field equations obtained from the Lagrangian (2.1),

$$R_\mu{}^\nu + \nabla_\mu \nabla^\nu \phi - \frac{1}{4} H_{\mu\alpha\beta} H^{\nu\alpha\beta} = 8\pi G_D e^\phi T_\mu^\nu \tag{2.12}$$

$$R - (\nabla_\mu \phi)^2 + 2\nabla_\mu \nabla^\mu \phi - \frac{1}{12} H_{\mu\nu\alpha} H^{\mu\nu\alpha} = 0 \tag{2.13}$$

$$\partial_\nu(\sqrt{|g|}e^{-\phi}H^{\mu\alpha\beta}) = 0 \tag{2.14}$$

with an effective source term given by a sum over all strings of the the stress tensor of each individual strings,

$$T^{\mu\nu}(x) = \frac{1}{\pi\alpha'\sqrt{|g|}}\int d\sigma d\tau(\frac{dX^\mu}{d\tau}\frac{dX^\nu}{d\tau} - \frac{dX^\mu}{d\sigma}\frac{dX^\nu}{d\sigma})\delta^D(X-x) \tag{2.15}$$

and the string equations of motion (plus constraints) in the same given background,

$$\frac{d^2X^\mu}{d\tau^2} - \frac{d^2X^\mu}{d\sigma^2} + \Gamma^\mu_{\alpha\beta}(\frac{dX^\alpha}{d\tau} + \frac{dX^\alpha}{d\sigma})(\frac{dX^\beta}{d\tau} - \frac{dX^\beta}{d\sigma}) = 0$$

$$g_{\mu\nu}(\frac{dX^\mu}{d\tau}\frac{dX^\nu}{d\tau} + \frac{dX^\mu}{d\sigma}\frac{dX^\nu}{d\sigma}) = 0, \quad g_{\mu\nu}\frac{dX^\mu}{d\tau}\frac{dX^\nu}{d\sigma} = 0 \tag{2.16}$$

Here G_D is the $D = (d+1)$-dimensional gravitational constant, $(\alpha')^{-1}$ is the string tension, $\Gamma^\alpha_{\mu\nu}$ and ∇_μ are connection and covariant derivative for the background metric $g_{\mu\nu}$, $H_{\mu\nu\alpha} = 6\partial_{[\mu}B_{\nu\alpha]}$ is the torsion field strength, X^μ are the background string coordinates, and τ and σ the usual world-sheet time and space variables (we are using the gauge in which the world-sheet metric is conformally flat).

If we impose, as initial condition, the flat and cold string perturbative vacuum (which is the most natural one in a pre-big-bang context, and requires no fine-tuning), supplemented by a constant but non-vanishing density of pressureless string matter [2],

$$a = const, \quad \phi = -\infty, \quad \rho = cont, \quad p = 0 \tag{2.17}$$

then the general solution describes the accelerated evolution of the weakly coupled initial regime towards a curved, dilaton-driven, strong coupling regime, characterized by a final Kasner-like configuration, possibly anisotropic,

$$a_i \sim (-t)^{\beta_i}, \quad \phi \sim (\sum_i \beta_i - 1)\ln(-t), \quad \sum_i \beta_i^2 = 1, \quad t < 0 \tag{2.18}$$

in which the expanding dimensions superinflate [2].

It should be stressed, in conclusion of this Section, that the initial condition (2.17) in not new, as it was previously reported also by the Bible. In fact, as clearly stated in the Genesis [23],

"In the beginning God created the Heaven and the Earth.
And the Earth was without form, and void;
and darkness was upon the face of the deep.

And the Breath of God

moved upon the face of the water..."

which clearly means, in a less metaphorical language,

"In the beginning God created the Background Fields and the Matter Sources.

And the Sources were pressureless and embedded in flat space;

and this dark matter had negligible interactions ($\phi = -\infty$).

And the Dilaton

fluctuated in the string perturbative vacuum..."

(For an explanation the next step, "And God said: *Let there be light*", work is still in progress). On account of the experience of Galilei, who got into juridical trouble for contradicting the words of the Bible, we prefer to follow very closely the cosmological prescriptions of the Genesis. Surprisingly enough, they seem to be in excellent agreement with the overall picture of a pre-big-bang scenario.

3. Inflation and Accelerated Contraction.

The pre-big-bang picture described in the previous Section was explicitly based on the the so-called Brans-Dicke (BD) metric frame, in which the effective Lagrangian (truncated to the metric and dilaton kinetic terms) takes the form

$$\mathcal{L} = -\sqrt{|g|}e^{-\phi}\left[R + (\partial_\mu\phi)^2\right] \tag{3.1}$$

This is the natural frame in a string theory context [24], as its metric coincides with the sigma-model metric to which test strings are directly coupled. In the associated Einstein (E) frame the dilaton is minimally coupled to the metric, and the truncated Lagrangian is diagonalized in the canonical form

$$\mathcal{L} = -\sqrt{|g|}\left[R - \frac{1}{2}(\partial_\mu\phi)^2\right] \tag{3.2}$$

A possible difficulty for the pre-big-bang scenario follows from the observation that any isotropic superinflationary solution, describing in the BD frame growing curvature and *accelerated expansion*,

$$\dot{a} > 0, \quad \ddot{a} > 0, \quad \dot{H} > 0 \tag{3.3}$$

when transformed into the E frame through the conformal (d-dimensional) rescaling [2,13]

$$a \to a\, e^{-\phi/(d-1)}, \qquad dt \to dt\, e^{-\phi/(d-1)} \tag{3.4}$$

becomes an isotropic solution whose curvature scale is still growing, but describing *accelerated contraction*, characterized by the conditions

$$\dot{a} < 0, \qquad \ddot{a} < 0, \qquad \dot{H} < 0 \tag{3.5}$$

Consider indeed, as example of superinflationary background, the isotropic version of the solution (2.18), with $\beta_i = \beta = -1/\sqrt{d} < 0$. In the E frame it is transformed into the solution

$$a \sim (-t)^{1/d}, \quad \phi \sim -\sqrt{\frac{2(d-1)}{d}} \ln(-t), \quad t < 0 \tag{3.6}$$

which satisfies the conditions of eq.(3.5).

This result would seem to imply that the inflationary properties of the background are frame-dependent. This fact, however, is only an apparent difficulty, as it turns out that accelerated contraction is equally good to solve the kinematical problems of the standard model as accelerated expansion [2,13]. Consider for instance the so-called flatness problem, in an isotropic manifold with scale factor $a \sim |t|^\alpha$. This problem is solved if the ratio between the spatial curvature term, k/a^2, and the other terms of the cosmological equations decreases enough during inflation,

$$\frac{k}{a^2 H^2} = \frac{k}{\dot{a}^2} \sim \frac{k}{|t|^{2(\alpha-1)}} \to 0 \tag{3.7}$$

so as to compensate its subsequent growth during the phase of standard evolution. This condition is satisfied in three possible cases:

1) $t \to \infty, a \sim t^\alpha, \alpha > 1$, namely during a phase of power-law inflation, which obviously includes the limiting case of standard exponential expansion ($\alpha \to \infty$).

2) $t \to 0, a \sim (-t)^\alpha, \alpha < 0$, namely by what is called pole-like, or superinflationary expansion [19], and which corresponds to a pre-big-bang solution in the BD frame.

3) $t \to 0, a \sim (-t)^\alpha, 0 < \alpha < 1$, namely by a phase of accelerated contraction, satisfying eq.(3.5), which is just the form of the pre-big-bang solution when transformed to the E frame.

With similar arguments one can show that also the horizon problem can be solved by a (long enough) phase of accelerated contraction. Consider indeed a phase of accelerated evolution and growing curvature, with $a \sim (-t)^\alpha$ for $t \to 0_-$. In that case the event horizon shrinks linearly (irrespectively of the sign of $\dot{a}$ [1]), while the proper size of a causally connected region scales like the scale factor.

The horizon problem is solved if causally connected scales are "pushed out" of the horizon, namely if the ratio

$$\left(\frac{proper\ size\ event\ horizon}{proper\ size\ caus.\ conn.\ reg.}\right) \sim \frac{(-t)}{a(t)} \sim (-t)^{1-\alpha} \tag{3.8}$$

shrinks during inflation. Again, this may occur for $\alpha < 0$, superinflation, but also for $0 < \alpha < 1$, accelerated contraction, because in this last case the event horizon shrinks always faster than the scale factor itself.

It is important to stress that both the "horizon ratio" (3.8), and the previous "flatness ratio" (3.7), measure the duration of the accelerated epoch in terms of the conformal time coordinate η, defined by $d\eta = dt/a$, and that conformal time is the same in the E and BD frame (see eq.(3.4)). As a consequence, if superinflation is long enough to solve the standard kinematical problems in the BD frame, then the problems are solved also in the E frame by the phase of accelerated contraction [2,13]. One can show, moreover, that the number of strings per unit of string volume is diluted in the same way in both frames [2], in spite of the contraction. All these results, together with the fact that the scalar and tensor perturbation spectrum is also the same in both frames [2,13], assure the frame-independence of the inflationary properties of the pre-big-bang background.

This independence emerges as a non-trivial consequence of dilaton dynamics, because it is the dilaton which generates the conformal transformation connecting the two frames.

4. Main Open Problems.

The scenario so far described leaves out the possible effects of a dilaton potential. Indeed, at low enough energy scale, the dilaton potential $V(\phi)$ can appear at a non-perturbative level only, and it is expected to be negligible, $V(\phi) \sim \exp[-\exp(-\phi)]$. In that case, however, the growth of the curvature and of the dilaton coupling is unbounded, as

$$|H_i| \to \infty, \qquad e^{\phi} \to \infty \tag{4.1}$$

when $t \to 0_-$ in the asymptotic solution (2.18), and the background unavoidably run towards a singularity.

We are thus led to what is probably the main difficulty of the whole scenario, namely the possible occurrence of a smooth transition from growing to decreasing curvature, with associated dilaton freezing. Such a transition should include, moreover, some non-adiabatic process of radiation production (*"Let there be light"*), in order to solve also the entropy problem of the standard model.

Up to now, only a few examples of smooth transitions are known, with non-vanishing antisymmetric tensor in $D = 2+1$ dimensions [10], or with a non-local, repulsive dilaton potential [1], or with α' corrections and moduli fields contributions [25], which simulate loop corrections. An additional example of regular background [26], which is inhomogeneous, can be obtained by performing $O(d,d)$ transformations starting from the cosmological solution of Nappi and Witten [27].

All these examples, however, are not fully "realistic", for various reasons. A truly realistic model should include *both* α' and loop corrections (which become important in the high curvature, strong coupling regime), *and* a non-perturbative dilaton potential, related to supersymmetry breaking, which forces the dilaton to a minimum, gives it a mass, and freezes the value of the Newton constant (a possible motivated example of dilaton potential is discussed in [28]). A graceful exit from the phase of pre-big-bang evolution seems to be impossible without including both contributions [29]. Moreover, limiting temperature effects should be added near the Hagedorn scale, ad this can modify the effective equation of state [30,31]. New exact solutions [32] confirm the importance of the antisymmetric tensor $B_{\mu\nu}$ in generating interesting dynamics, but cannot avoid curvature singularities.

Recent progress on exact string solutions, to all orders in α', is encouraging [33]. The problem, however, is that the explicit form of the full corrections is not known, and difficult to find to all orders. In order to discuss possible phenomenological aspects of the pre-big-bang scenario we shall thus adopt here a sort of "sudden" approximation, in which we cut off the details of the transition regime, by matching directly the pre-big-bang-phase to the standard radiation-dominated evolution, at some given curvature scale H_1. This will affect the high-energy "tail" of our predictions, but not the predictions for scales much smaller than the string one, provided the transition is localized in the high-curvature region. Work is in progress to estimate the length of the transition regime [34].

We shall assume, moreover, that the dilaton potential is negligible in the pre-big-bang phase, and vanishing in the post-big-bang, where a constant, massive dilaton background is sitting exactly at the minimum of the potential, with negligible classical oscillations around it. But I will come back on this point later, when discussing dilaton production.

5. Phenomenological Signature of the Pre-Big-Bang Scenario.

Summarizing the previous discussion, we can say that string theory suggests a cosmological pre-big-bang scenario which has still many unsolved problems. What is presently lacking, in particular, is a detailed model for the transition from the growing to the decreasing curvature regime, and for the process of non-adiabatic radiation production. This scenario has also various interesting aspects, however, such as natural initial condition, natural inflation, the initial singularity of the standard model is eventually smoothed out, and so on.

The crucial question, however, is the following: is it possible to test observationally this scenario and, in particular, to distinguish it from other inflationary models of the early universe?

As far as I know, it is the dilaton which marks the main difference between string cosmology and other cosmological models, and which characterizes the main phenomenological aspects of the pre-big-bang scenario. At the present state of our knowledge, such aspects are:

1) *Scalar and tensor perturbation spectrum growing with frequency* (the so-called "blue" spectrum). This property is related to the superinflationary kinematics [35, 36,12] and, as such, is not peculiar of string cosmology. In a pre-big-bang context, however, superinflation is not necessarily associated to a phase of dynamical dimensional reduction [19], but it is typically a consequence of the dilaton dynamics.

2) *Squeezed vacuum*, and not squeezed thermal vacuum, *as the final state of the amplified perturbations.* This quantum property [37] is also not peculiar of string cosmology only, but of all those inflationary models with a cold ($T = 0$) initial state [38].

3) Possible existence of a *relic background of cosmic dilatons.* This is the new effect, peculiar of string cosmology, where in addition to the metric there is also a dilaton field. The parametric amplification of the dilaton fluctuations leads to dilaton production [2,39], just like the amplification of the tensor part of the metric perturbations leads to graviton production.

5.1 Growing Perturbation Spectrum.

The first point to be stressed, when considering a growing perturbation spectrum, is that besides the usual bounds on the spectrum obtained from large scale physics one must include, in general, an additional constraint imposed by the observed closure density.

Indeed, let me recall that for a background evolution of the type

inflation $\rightarrow$ *radiation-dominance* $\rightarrow$ *matter-dominance*

the spectral energy density of the amplified perturbations (in units of $\rho_c = H^2/G$) can be parametrized in terms of the inflation-radiation transition scale H_1 as [12,36,40]

$$\begin{aligned}
\Omega(\omega, t) \equiv & \frac{\omega}{\rho_c}\frac{d\rho}{d\omega} \simeq \\
\simeq & G H_1^2 \left(\frac{H_1}{H}\right)^2 \left(\frac{a_1}{a}\right)^4 \left(\frac{\omega}{\omega_1}\right)^{n-1}, \quad \omega_2 < \omega < \omega_1 \\
\simeq & G H_1^2 \left(\frac{H_1}{H}\right)^2 \left(\frac{a_1}{a}\right)^4 \left(\frac{\omega}{\omega_1}\right)^{n-1} \left(\frac{\omega}{\omega_2}\right)^{-2}, \quad \omega_0 < \omega < \omega_2
\end{aligned} \tag{5.1}$$

Here n is th spectral index, $\omega_1 = a_1 H_1/a \sim 10^{11}\sqrt{H_1/M_p}$ Hz is the maximum amplified proper frequency (M_p is the Planck mass), $\omega_0 \sim 10^{-18}$ Hz is the minimum amplified frequency corresponding to a mode crossing today the Hubble radius H_0^{-1}, and $\omega_2 = a_2 H_2/a \sim 10^2 \omega_0$ is the frequency corresponding to the matter-radiation transition scale $H_2 \sim 10^6 H_0$. The high frequency part of the spectrum ($\omega > \omega_2$) is determined by the first background transition only (from inflation to radiation), while the low frequency part is affected by both transitions.

The (approximate) large scale isotropy [41] of the cosmic microwave background (CMB) imposes on the perturbations the condition

$$\Omega(\omega_0, t_0) \lesssim 10^{-10} \quad \Longrightarrow \quad \log_{10}\left(\frac{H_1}{M_p}\right) \lesssim \frac{2}{5-n}(29n - 39) \tag{5.2}$$

The critical density bound reads

$$\Omega(t) = \int_{\omega_0}^{\omega_1} \frac{d\omega}{\omega} \Omega(\omega, t) \lesssim 1 \tag{5.3}$$

and implies, for a growing spectrum ($n > 1$),

$$H_1 \lesssim M_p, \quad \Omega(t_0) \lesssim 10^{-4} \tag{5.4}$$

If the growth is fast enough, the perturbations are thus mainly constrained by the critical density bound, rather than by CMB isotropy. This is illustrated in **Fig. 2**, where the spectral density of tensor perturbations is plotted versus the proper frequency, for three different values of the spectral index n. Also plotted in **Fig. 2** is the constraint obtained from pulsar-timing data [42], $\Omega(10^{-8}Hz) \lesssim 10^{-6}$, and also (very roughly) the planned sensitivity of LIGO [43], just to stress that there are more chances to observe a cosmic graviton background in case of growing spectral distribution.

For a very fast growth of the spectrum it becomes impossible, of course, to explain the observed COBE anisotropy [44], which should then be ascribed to other causes (topological defects, etc.). What is to be remarked however is that, even if constrained by CMB, higher inflation scales become compatible with the isotropy bound in the case of a growing spectrum, as illustrated in **Fig.3**, again for tensor perturbations.

For flat or decreasing spectra ($n \leq 1$) one has well known maximum allowed scale $H_1 \sim 10^{-5} M_p$, while for a fast enough growing spectrum ($n \gtrsim 1.35$) scales as high as Planckian are allowed (not higher, otherwise the produced gravitons would overclose the universe). This is important in a string cosmology context, where the natural scale for the inflation-radiation transition is indeed the string scale, $H_1 \sim (\alpha')^{-1/2} \sim 10^{-1} M_p$, of nearly Planckian order, which becomes in this case compatible with the phenomenological bounds.

It should be stressed, finally, that for Planckian values of the final inflation scale H_1, the "coarse graining" entropy ΔS associated to the cosmological amplification of the perturbations [45, 46]

$$\Delta S \sim \left(\frac{H_1}{M_p}\right)^{3/2} \times (CMB\ entropy) \tag{5.5}$$

becomes comparable with the entropy stored in the cosmic black-body spectrum. Moreover, in such case the (pre-big-bang) $\rightarrow$ (post big-bang) transition could be even responsible for the generation of the presently observed thermal background radiation, according to the mechanism proposed by Parker [47]. Indeed, the high frequency part ($\omega > \omega_1$) of the radiation produced in that transition is characterized by a Planck distribution, typical of thermal equilibrium, at a proper temperature T_1 which today is given by

$$T_1(t_0) \simeq \frac{a_1 H_1}{a(t_0)} = H_1 \left(\frac{H_2}{H_1}\right)^{1/2} \left(\frac{H_0}{H_2}\right)^{2/3} \sim 1^o K \left(\frac{H_1}{M_p}\right)^{1/2}$$

For $H_1 \sim M_p$ this is exactly of the same order as the observed CMB temperature.

5.2 Cold Initial State.

In order to discuss the second phenomenological signature of the pre-big-bang scenario, I recall that the parametric amplification of the cosmological perturbations may be interpreted, from a quantum point of view, as a process of pair production in an external (gravitational) field, described by a Bogoliubov transformation connecting the *in* and *out* solutions of the linearized perturbation equation [40,48],

$$\psi_k'' + \left[k^2 - V(\eta)\right] \psi_k = 0 \tag{5.6}$$

Here a prime denotes differentiation with respect to conformal time η, ψ_k is the Fourier component of the perturbation, for a mode of comoving frequency $k = a\omega$, and $V(\eta)$ is the effective potential barrier associated to the transition of the background from accelerated to decelerated evolution. Such a transformation relates the *in* annihilation and creation operators, $\{b, b^\dagger\}$, to the *out* operators, $\{a, a^\dagger\}$,

$$a_k = c_+(k) b_k + c_-^*(k) b_{-k}^\dagger, \quad a_{-k}^\dagger = c_-(k) b_k + c_+^*(k) b_{-k}^\dagger \tag{5.7}$$

where $c_\pm(k)$ are the Bogoliubov coefficients satisfying $|c_-|^2 - |c_+|^2 = 1$. It can be rewritten as a unitary transformation

$$a_k = \Sigma_k b_k \Sigma_k^\dagger, \qquad a_{-k}^\dagger = \Sigma_k b_{-k}^\dagger \Sigma_k^\dagger \tag{5.8}$$

generated by the "two-mode" squeezing operator [37,49],

$$\Sigma_k = \exp(z_k^* b_k b_{-k} - z_k b_k^\dagger b_{-k}^\dagger) \tag{5.9}$$

where the complex number z parametrizes the Bogoliubov coefficients as

$$z_k = r_k e^{2i\theta_k}, \qquad c_+ = \cosh r_k, \quad c_- = e^{2i\theta_k} \sinh r_k \tag{5.10}$$

and depends thus on the dynamics of the external gravitational field leading to the process of pair creation.

The squeezing operator describes the evolution of the initial state of the fluctuations into a final "squeezed" quantum state. If we start from the vacuum $|0\rangle$ we obtain, for each mode, a final squeezed vacuum state [37], $|z_k\rangle = \Sigma_k |0\rangle$, with final number of particles N_k determined by the squeezing parameter $r = |z|$ or, equivalently, by the coefficient c_- which measures the content of negative frequency modes in the *out* solution of the perturbation equation,

$$N_k = \langle z_k | b_k^\dagger b_k | z_k \rangle = \sinh^2 r_k = \langle 0 | a_k^\dagger a_k | 0 \rangle = |c_-(k)|^2 \tag{5.11}$$

If we start, however, with a non-trivial number state $|n_k\rangle$ or, more generally, with a statistical mixture of number states [46, 50], we obtain a squeezed statistical mixture, with final expectation number of particles [51]

$$N = |c_-|^2(1 + \overline{n}) + \overline{n}(1 + |c_-|^2) \tag{5.12}$$

depending on the squeezing parameter *and* on the initial average number of particles $\overline{n} = \sum_n np_n$, where p_n are the statistical weights of the mixture (here, and in what follows, the mode index k is to be understood, if not explicitly written). Starting in particular from a state of thermal equilibrium, $\overline{n} = \left(e^{\beta_0\omega} - 1\right)^{-1}$, the cosmological evolution leads to a final "squeezed thermal vacuum", with expectation number of particles given, in the large squeezing limit ($r >> 1$), by

$$N(\omega) \simeq \sinh^2 r \coth\left(\frac{\beta_0\omega}{2}\right) \tag{5.13}$$

For all the inflationary models requiring, as initial condition, a state of thermal equilibrium, the process of amplification of the fluctuations starts *not* from the vacuum, but from the initial thermal bath. As a consequence, the final state of the perturbations is a squeezed thermal vacuum, instead of the pure squeezed vacuum, and the spectrum (5.1) is modified as follows [38]

$$\Omega(\omega, t) \simeq GH_1^2 \left(\frac{H_1}{H}\right)^2 \left(\frac{a_1}{a}\right)^4 \left(\frac{\omega}{\omega_1}\right)^{n-1} \coth\left(\frac{\beta_0\omega}{2}\right) \tag{5.14}$$

(with similar expression for the low frequency part, $\omega < \omega_2$). The new parameter here is $\beta_0 = 1/T_0$, namely the inverse of the temperature of the initial bath, rescaled down (adiabatically) to the present observation time t_0.

The evident effect of the initial temperature is that, because of stimulated emission, particle production is enhanced at low frequency, with respect to the spontaneous production from the vacuum. It is a sort of "more power on larger scales" effect, due to the temperature. Such effect is clearly illustrated in **Fig. 4** for the case of a flat ($n = 1$) spectrum, by comparing the vacuum case ($\beta_0 = \infty$)

with two cases at finite temperature (the rescaled initial temperature is given in units of ω_0^{-1}).

As a consequence of this enhanced production, the inflation scale has to be lowered, in order not to exceed the observed CMB anisotropy. This is illustrated in **Fig. 5**, where the upper bound on the final inflation scale H_1, obtained from the condition $\Omega(\omega_0) \lesssim 10^{-10}$ applied to the modified spectrum (5.14), is plotted versus the spectral index n, for different values of β_0. The zero temperature case gives the usual bound, which reduces to $10^{-5} M_p$ for $n = 1$. At finite temperature, the maximum allowed scale is shifted to lower values.

The importance of this thermal effect depends of course on the present value of the initial temperature β_0^{-1}, and if the duration of the inflationary phase is very long, the effect may be completely washed out by the inflationary super-cooling of the initial temperature. It turns out [38], however, that the thermal corrections may be significant for those inflationary models which predict just the "minimal" amount of inflation required to solve the standard kinematical problems. In that case, the effect of the thermal bath on the perturbation spectrum would represent a truly (hot) "remnant" of the pre-inflationary universe [52].

What I want to stress here is that, in principle, the experimental data should be fitted by including a possible thermal dependence on the spectrum. If observed, however, such a dependence would be *in contradiction* with the pre-big-bang scenario, in which the universe starts to inflate from a flat and *cold* initial state, in such a way that no thermal modification of the spectrum is predicted.

5.3 Dilaton Production.

The new effect, peculiar of string cosmology and of the pre-big-bang scenario, is the parametric amplification of the quantum fluctuations of the dilaton background, which leads to a process of cosmological dilaton production [2,39].

In order to discuss this effect we need, first of all, the classical equations determining the time evolution of the perturbations. Such equations are obtained by perturbing the string cosmology equations (2.12)-(2.14), and inserting the background solutions. It turns out, as expected, that the dilaton perturbations are coupled to the scalar part of the metric perturbations (in the linear approximation, tensor perturbations are decoupled, and evolve independently).

Because of the frame-independence of the spectrum we can work in the Einstein frame, where the field equations are simpler. In this frame, however, the dilaton is coupled to the fluid sources, with coupling functions determined by the conformal transformation connecting E and BD frame. It is convenient, moreover, to use the gauge-invariant Bardeen variables, and to work in the longitudinal gauge [48]. In the case of isotropic backgrounds we have then one independent variable ψ for the scalar perturbations of the metric,

$$ds^2 = (1+2\psi)dt^2 - (1-2\psi)a^2|d\vec{x}|^2 \tag{5.15}$$

two independent variables for the matter perturbations in the perfect fluid form,

$$\delta\rho, \qquad \delta p, \qquad \delta u_i \tag{5.16}$$

an, in addition, the dilaton perturbation

$$\delta\phi = \chi \tag{5.17}$$

(for the case of anisotropic backgrounds, more realistic in a pre-big-bang scenario, work is still in progress [34]).

By working out the perturbation equations, and performing a Fourier analysis, we end up with a system of four coupled differential equations. Two of these equations define the velocity (δu_i) and density ($\delta\rho$) perturbations in terms of the background, and of the metric and dilaton perturbations,

$$\begin{aligned} \delta u^i_k =& \delta u^i(a, \phi, \psi_k, \chi_k) \\ \delta\rho_k =& \delta\rho(a, \phi, \psi_k, \chi_k) \end{aligned} \tag{5.18}$$

The other two equations determine the coupled evolution of the metric and dilaton perturbations, ψ_k and χ_k, and can be written in compact vector form as

$$Z_k = \begin{pmatrix} \psi_k \\ \chi_k \end{pmatrix}, \qquad Z_k'' + 2\frac{a'}{a}AZ_k' + (k^2B + C)Z_k = 0 \tag{5.19}$$

where A, B, C are 2×2, time-dependent mixing matrices, determined by the background and by the equation of state. For any given background and equation of state one has then, in principle, a solution determining the classical evolution of the dilaton perturbations [2] (see also [53] for related work along these lines, with the possible difference that a scalar field model of sources is used, instead of perfect fluid matter).

The second step to be performed, in order to obtain the quantum dilaton spectrum, is to express the perturbations in terms of the correctly normalized variables satisfying canonical commutation relations, and reducing the action to the canonical form [48,54] (namely the variables determining the absolute magnitude of the vacuum fluctuations). Such canonical variable are known for the metric-scalar field system [55], or for the metric-fluid system [56], but not for the full system with scalar field and fluid sources. In the full case one can try, at present, a perturbative approach only (work is in progress along this direction).

The problem can be avoided, however, if we assume that the duration of the pre-big-bang inflation is much longer than the minimal duration required to solve the standard kinematical problems. In fact, the pre-big-bang solutions of the string cosmology equations (2.12)-(2.14) are characterized by an integration constant, T, which fixes the time scale at which the metric is no longer flat and starts to inflate, but also fixes the time scale at which the matter contributions become negligible with respect to the dilaton and metric kinetic energy [2],

$$\begin{array}{ll} H^2 << \dot{\phi}^2 \sim e^{\phi}\rho, & t << T \\ e^{\phi}\rho << \dot{\phi}^2 \sim H^2, & t >> T \end{array} \tag{5.20}$$

So, if inflation is much longer than minimal, all the modes contributing to the presently observed spectrum "hit" the effective potential barrier of the perturbation equation (5.6) when the pre-big-bang is already "dilaton-driven". Otherwise stated: all the scales inside our present horizon crossed the horizon during the phase of dilaton dominance.

In such case we can neglect the matter contribution to the perturbation equations, and we are left with the coupled metric and scalar field variables only, from which we know very well how to extract the spectrum [57]. By considering, in particular, a sudden transition between the simplest example of pre-big-bang background, three-dimensional, isotropic, dilaton-dominated, which in the E frame corresponds to

$$a \sim (-\eta)^{1/2}, \qquad \phi \sim -\sqrt{12}\ln a \tag{5.21}$$

and the standard radiation evolution, with frozen dilaton background, we find that the dilaton fluctuations are amplified with a growing spectral density, $\Omega_\chi \sim \omega^3$.

This is a very fast growth of the spectrum, but this special value of the spectral index should not taken as particularly indicative, because it could change substantially, while remaining growing, in a more realistic scenario in which one includes the contribution of string matter, dilaton potential and contracting internal dimensions [34].

6. Bounds on the Dilaton Spectrum.

For a phenomenological discussion of the consequences of a possible dilaton production, associated to the pre-big-bang scenario, I shall thus parametrize the spectrum in terms of a growing spectral index $\delta \geq 0$,

$$\Omega_\chi(\omega, t) \simeq G H_1^2 \left(\frac{H_1}{H}\right)^2 \left(\frac{a_1}{a}\right)^4 \left(\frac{\omega}{\omega_1}\right)^\delta \tag{6.1}$$

It turns out, as we shall see, that the phenomenological bounds are only weakly dependent on δ, and they become completely δ-independent for $\delta > 1$.

The dilaton spectrum is constrained by various phenomenological bounds. First of all we must require, according to eq.(5.3),

$$H_1 \lesssim M_p \tag{6.2}$$

in order to avoid that the produced dilatons overclose the universe in the radiation era, just like in the case of graviton production.

If dilatons would be massless, this would be, basically, the end of the story. The dilatons, however, cannot be massless, because they couple non-universally, and with at least gravitational strength, to macroscopic matter [58] (with a possible exception which requires however fine-tuning [59]). This may be reconciled with the present tests of the equivalence principle only if the dilaton range is smaller than about 1 cm, namely for a mass

$$m \gtrsim m_0 \equiv 10^{-4}\, eV \tag{6.3}$$

As a consequence we have, at low enough energy scales, also a non-relativistic contribution to the dilaton energy density [39],

$$\Omega_\chi(t) \simeq G m^2 \left(\frac{H_1}{H}\right)^2 \left(\frac{a_1}{a}\right)^3 \left(\frac{m}{H_1}\right)^{\frac{\delta-3}{2}} \tag{6.4}$$

due to non-relativistic modes, which start to oscillate at scales $H(t) \lesssim m$. Such a contribution grows in time with respect to the radiation energy density, and in the matter-dominated era, $H < H_2$, it remains frozen at a constant value which we must impose to be smaller than one, in order to satisfy the critical density bound. This provides the constraint

$$m \lesssim \left(H_2 M_p^4 H_1^{\delta-4}\right)^{1/(\delta+1)} \tag{6.5}$$

Higher values of mass may be reconciled with present observations only if the energy (6.4) stored in the coherent dilaton oscillations was dissipated into radiation before the present epoch, at a decay scale

$$H_d \simeq \frac{m^3}{M_p^2} > H_0 \tag{6.6}$$

The reheating associated to this decay generates an entropy [2,39]

$$\Delta S = \left(\frac{T_r}{T_d}\right)^3 = \left(\frac{H_1^{4-\delta} m^{\delta-2}}{M_p^2}\right)^{1/2} \tag{6.7}$$

(where T_d is the decay temperature and $T_r \simeq \sqrt{M_p H_d}$ the reheating temperature), and we are left with two phenomenological possibilities [60].

If $m < 10^4\ GeV$ then T_r is too low to allow a nucleosynthesis phase subsequent to dilaton decay. We must impose that nucleosynthesis occurred before, and that

$$\Delta S \lesssim 10 \tag{6.8}$$

in order not to destroy all light nuclei already formed. If, on the contrary, $m > 10^4\ GeV$, and nucleosynthesis is subsequent to dilaton decay, then the only possible constraint comes from primordial baryogenesis, and imposes

$$\Delta S \lesssim 10^5 \tag{6.9}$$

This last condition, however, could be evaded in case of low energy baryogenesis [61].

The bounds so far considered refer to the case $m < H_1$. If $m > H_1$ then all modes are always non-relativistic, with a total dilaton energy density [2]

$$\Omega_\chi(t) \simeq G m^2 \left(\frac{H_1}{H}\right)^2 \left(\frac{a_1}{a}\right)^3 \tag{6.10}$$

and the only constraint to be imposed is

$$m \lesssim M_p \tag{6.11}$$

again to avoid over-critical density.

The bounds reported here define an allowed region in the (m, H_1) parameter space. They have been already discussed in the past [62,60], with reference to the cosmological production of massive scalar particles with gravitational coupling strength, but always in the case of a flat $\delta = 0$ spectral distribution. The resulting allowed region is illustrated in **Fig. 6**.

Such an allowed region is source of problems for the dilaton, because realistic values of the inflation scale, say $H_1 \gtrsim 10^{-5} M_p$, are only compatible with very high values of the dilaton mass. In particular, a dilaton mass in the TeV range, suggested by models of supersymmetry breaking [63], turns out to be excluded for values of the inflation scale suggested by the observed COBE anisotropy [64].

In the case of a growing dilaton spectrum the allowed region "inflates" in parameter space, as illustrated in **Fig. 7**, which reports the previous bounds computed for a linearly growing spectrum, $\delta = 1$. There is a limit, however, on such a relaxation of bounds, because for $\delta > 1$ the spectral density is always dominated by the highest frequency mode ω_1, even in the non-relativistic regime, and the slope-dependence of the energy density disappears, after integration over all modes [2,39]. For all $\delta > 1$ the allowed region is thus the same as that of the linear case illustrated here. For $0 < \delta < 1$ the allowed region interpolates between the two limiting cases of **Fig. 6** and **Fig. 7**.

There are two interesting consequences of the the allowed region of **Fig. 7**.

The first is that, for fast enough growing spectra, a dilaton mass in the TeV range becomes compatible with realistic inflation scales as high as $10^{-5}\, M_p$.

The second is that for a "stringy" (nearly Planckian) inflation scale, the dilaton mass may be very small or very large. In the small mass range,

$$10^{-4}\, eV \lesssim m \lesssim 1\, eV \tag{6.12}$$

dilatons are not yet decayed as $m \lesssim 100\, MeV$ (see eq.(6.6)), and their present contribution to Ω must be very near the critical one. For the range (6.12) one has indeed, from the density (6.4) evaluated for $\delta = 1$ in the matter era,

$$10^{-4} \lesssim \Omega_\chi \lesssim 1 \tag{6.13}$$

This suggests the possibility of "dilaton dark matter" [2,39], with properties very similar to those of axion dark matter. My personal belief is that the dilaton should be heavy, but the possibility of a light dilaton cannot be ruled out on the grounds of the present discussion.

I should stress, however, that the bounds illustrated here refer to the cosmological amplification of the quantum fluctuations of the dilaton background. If there are, in addition, also classical oscillations of the background around the minimum of the effective potential [65,60], and/or other mechanisms of dilaton production, then other bounds are to be superimposed to the previous ones. Having neglected such additional constraints, the allowed region of **Fig. 7** is to be regarded as the *maximal* allowed region in parameter space. Work is in progress [34] to discuss dilaton production in a more realistic scenario in which the post-big-bang era is not immediately dominated by radiation, but includes a vacuum phase of dilaton dominance, dual to the phase of dilaton-driven pre-big-bang.

7. Conclusion.

The main conclusion of this paper is that a lot of work is still needed to clarify all the details of this cosmological scenario. However, I believe that such work is worth to be done, for the following, very important reason.

The dilaton is a compelling consequence of string theory.

The pre-big-bang scenario, and dilaton cosmology, are phenomenological consequences of string theory.

Tests of this scenario are thus direct tests of string theory (and also, more generally, of Planck scale physics).

Acknowledgements.

I am grateful to R. Brandenberger, V. Mukhanov, A. Starobinski and A. A. Tseytlin for helpful and clarifying discussions. I wish to thank also D. Boyanovsky, T. Damour, M. Gleiser, J. Lidsey, D. Lyth and D. Wands for their interest and for asking stimulating questions, and N. Sánchez and H. J. De Vega for their kind invitation to the *First* and *Second Journée Cosmologie* (Paris, May '93, June '94). Finally, I am greatly indebt to M. Giovannini, J. Maharana, N. Sánchez and G. Veneziano for a fruitful collaboration which led to most of the results reported here.

References.

1. M. Gasperini and G. Veneziano, Astropart. Phys. 1 (1993) 317
2. M. Gasperini and G. Veneziano, *Dilaton production in string cosmology*, Phys. Rev. D50 (1994) (in press), Preprint CERN-TH.7178/94 (gr-qc/9403031)
3. E. Alvarez, Phys. Rev. D31 (1985) 418;
   Y. Leblanc, Phys. Rev. D38 (1988) 3087;
   R. Brandenberger and C. Vafa, Nucl. Phys. B31 (1989) 391;
   E. Alvarez and M. A. R. Osorio, Int. J. Theor. Phys. 28 (1989) 940
4. M. Gasperini, N. Sánchez and G. Veneziano, Nucl. Phys. B364 (1991) 365
5. A. A. Tseytlin and C. Vafa, Nucl. Phys. B372 (1992) 443;
   A. A. Tseytlin, Class. Quantum Grav. 9 (1992) 979;
6. M. A. R. Osorio and M. A. Vasquez-Mozo, Preprint CERN-TH.7085/93
7. G. Veneziano, Phys. Lett. B265 (1991) 287
8. A. A. Tseytlin, Mod. Phys. Lett. A6 (1991) 1721
9. K. A. Meissner and G. Veneziano, Phys. Lett. B267 (1991) 33;
   Mod. Phys. Lett. A6 (1991) 3397
10. M. Gasperini, J. Maharana and G. Veneziano, Phys. Lett. B272 (1991) 277
11. M. Gasperini and G. Veneziano, Phys. Lett. B277 (1992) 256
12. M. Gasperini and M. Giovannini, Phys. Rev. D47 (1993) 1529
13. M Gasperini and G. Veneziano, Mod. Phys. Lett. A8 (1993) 3701
14. A. Vilenkin, Phys. Rev. D46 (1992) 2355
15. C. Lovelace, Phys. Lett. B135, (1984) 75;
    E. S. Fradkin and A. A. Tseytlin, Nucl. Phys. B261, (1985) 1;
    C. G. Callan, D. Friedan, E. J. Martinec and M. J. Perry, Nucl. Phys. B262, (1985) 593
16. A. Sen, Phys. Lett. B271 (1991) 295;
    S. F. Hassan and A. Sen, Nucl. Phys. B375 (1992) 103
17. X. C. de la Ossa and F. Quevedo, Nucl. Phys. B403 (1993) 377
18. M. Gasperini, R. Ricci and G. Veneziano, Phys. Lett. B319 (1993) 438;
    A. Alvarez, L. Alvarez-Gaumé and Y. Lozano, Preprint CERN-TH.7204/94 (1994)
19. D. Shadev, Phys. Lett. B317 (1994) 155;
    R. B. Abbott, S. M. Barr and S. D. Ellis, Phys. Rev. D30 (1994) 720;
    E. W. Kolb, D. Lindley and D. Seckel, Phys. Rev. D30 (1994) 1205;
    F. Lucchin and S. Matarrese, Phys. Lett. B164 (1985) 282
20. M. Gasperini, Phys. Lett. B258 (1991) 70;

Gen. Rel. Grav. 24 (1992) 219
21. M. Gasperini, N. Sánchez and G. Veneziano, Int. J. Mod. Phys. A6 (1991) 3853
22. M. Gasperini, M. Giovannini, K. A. Meissner and G. Veneziano, in preparation
23. The Holy Bible, *Genesis*, (Oxford University Press, London)
24. N. Sánchez and G. Veneziano, Nucl. Phys. B333 (1990) 253;
L.J. Garay and J. Garcia-Bellido, Nucl. Phys. B400 (1993) 416
25. I. Antoniadis, J. Rizos and K. Tamvakis, Nucl. Phys. B (1993) (in press)
26. M. Gasperini, J. Maharana and G. Veneziano, Phys. Lett. B296 (1992) 51
27. C. Nappi and E. Witten, Phys. Lett. B293 (1992) 309
28. N. Kaloper and K. Olive, Astropart. Phys. 1 (1993) 185
29. R. Brustein and G. Veneziano, Preprint CERN-TH.7179/94 (February 1994)
30. M. Gleiser and J. G. Taylor, Phys. Lett. B164 (1985) 36
31. N. Turok, Phys. Rev. Lett. 60 (1987) 549
32. E. J. Copeland, A. Lahiri and D. Wands, Preprint SUSSEX-AST-94/6
33. E. Kiritsis and C. Kounnas, Preprint CERN-TH.7219/94 (hep-th/9404092)
E. Kiritsis, these Proceedings
34. M. Gasperini, M. Giovannini and G. Veneziano, in preparation
35. F. Lucchin and S. Matarrese, Ref. 19
36. M. Gasperini and M. Giovannini, Phys. Lett. B282 (1992) 36
37. L. P. Grishchuk and Y. Sidorov, Class. Quantum Grav. 6 (1989) L161;
Phys. Rev. D42 (1990) 3413;
L. P. Grishchuk, Phys. Rev. Lett. 70 (1993) 2371
38. M. Gasperini, M. Giovannini and G. Veneziano, Phys. Rev. D48 (1993) R439
39. M. Gasperini, Phys. Lett. B327 (1994) 214
40. L. F. Abbott and M. B. Wise, Nucl. Phys. B244 (1984) 541;
L. P. Grishchuk, Sov. Phys. Usp. 31 (1988) 940;
B. Allen, Phys. Rev. D37 (1988) 2078;
V. Sahni, Phys. Rev. D42 (1990) 453;
L. P. Grishchuk and L. M. Solokhin, Phys. Rev. D43 (1991) 2566
41. G. F. Smooth et al., Astrophys. J. 396 (1992) 1
42. D. R. Stinebring et al., Phys. Rev. Lett. 65 (1990) 285
43. see for instance K. S. Thorne, in *"300 Years of Gravitation"*, ed. by S. W. Hawking and W. Israel (Cambridge Univ. Press, Cambridge, England, 1988)
44. D. H. Lyth, these Proceedings
45. B. L. Hu and D. Pavon, Phys. Lett. B180 (1986) 329;

B. L. Hu and H. E. Kandrup, Phys. Rev. D35 (1987) 1776;
M. Gasperini and M. Giovannini, Phys. Lett. B301 (1993) 334;
R. H. Brandenberger, V. Mukhanov and T. Prokopec, Phys. Rev. Lett. 69 (1992) 3606;
Phys. Rev. D48 (1993) 2443

46. M. Gasperini and M. Giovannini, Class. Quantum Grav. 10 (1993) L133
47. L. Parker, Nature 261 (1976) 20
48. V. Mukhanov, H. A. Feldman and R. Brandenberger, Phys. Rep. 215 (1992) 203
49. B. L. Schumaker, Phys. Rep. 135 (1986) 317
50. B. L. Hu, G. Kang and A. Matzac, Int. J. Mod. Phys. A9 (1994) 991;
B. L. Hu and A. Matzac, Phys. Rev. D (1994) (in press)
51. L. Parker, Phys. Rev. 183 (1969) 1057
52. M. Turner, Phys. Rev. D44 (1991) 3737
53. S. Mollerach and S. Matarrese, Phys. Rev. D45 (1992) 1961;
N. Deruelle, C. Gundlach and D. Langlois, Phys. Rev. D46 (1992) 5337;
T. Hirai and K. Maeda, Preprint WU-AP/32/93 (astro-ph/9404023);
D. Polarski and A. A. Starobinski, Preprint astro-ph/9404061
54. N. Deruelle, C. Gundlach and D. Polarski, Class. Quantum Grav. 9 (1992) 137
55. M. Sasaki, Prog. Theor. Phys. 76 (1986) 76;
V. F. Mukhanov, Sov. Phys. JEPT 67 (1988) 1297;
E. D. Stewart and D. H. Lyth, Phys. Lett. B302 (1993) 171
56. V. N. Lukash, Sov. Phys. JEPT 52 (1980) 807;
G. V. Chibisov and V. N. Mukhanov, Mon. Not. R. Astron. Soc. 200 (1982) 535
57. S. W. Hawking, Phys. Lett. B115 (1982) 295;
A. Linde, Phys. Lett. B116 (1982) 335;
A. A. Starobinski, Phys. Lett. B117 (1982) 175;
A. Guth and S. Y. Pi, Phys. Rev. Lett. 49 (1982) 1110;
J. M. Bardeen, P. S. Steinhardt and M. S. Turner, Phys. Rev. D28 (1983) 679
58. T. R. Taylor and G. Veneziano, Phys. Lett. B213 (1988) 459;
J. Ellis et al., Phys. Lett. B228 (1989) 264
59. T. Damour and A. M. Polyakov, Preprint IHES/P/94/1 (hep-th/9401069)
60. J. Ellis, D. V. Nanopoulos and M. Quiros, Phys. Lett. B174 (1986) 176;

J. Ellis, N. C. Tsamis and M. Voloshin, Phys. Lett. B194 (1987) 291
61. M. Gleiser, these Proceedings
62. A. S. Goncharov, A. D. Linde and M I. Vysotsky, Phys. Lett. B147 (1984) 279
63. T. Banks, D. V. Kaplan and A. Nelson, Preprint UCSD/PTH 93-26, RU-37, (hep-ph/9308292)
64. M. White, Phys. Rev. D46 (1992) 4198;
T. Souradeep and V. Sahni, Mod. Phys. Lett. A7 (1992) 3541;
A. R. Liddle, Preprint SUSSEX-AST 93/7-2 (July 1993)
65. G. D. Coughlan et al., Phys. Lett. B131 (1983) 59;
G. German and G. G. Ross, Phys. Lett. B172 (1986) 305;
O. Bertolami, Phys. Lett. B209 (1988) 277;
B. de Carlos et al., Phys. Lett. B318 (1993) 447