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Inflation in scalar-tensor gravity with quadratic Gauss-Bonnet self-interactions

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We present cosmological solutions for a graviton-plus-dilaton system coupled to a perfect fluid source, including quadratic curvature corrections. We show that, even in the absence of a dilaton self-interaction potential, isotropic superinflation ($\dot{H}>0$) can be realized for a wide range of equations of state, which includes in particular the ordinary cases of dust matter and radiation. It is found that in this context inflation can be produced more efficiently than in previous superinflationary models based on the contraction of internal dimensions.

It is known that there are difficulties for inflation in string-derived gravity models when quadratic curvature corrections are included [1,2]. The dilaton coupling, in particular, removes the maximally symmetric de Sitter solution permitted in its absence [3], unless an effective potential for the dilaton self-interactions is introduced [2] and suitably fine-tuned. Without such potential, exact solutions representing both power-law ($\dot{H}<0$) and super ($\dot{H}>0$) inflationary expansion are also forbidden.

In this paper we show that if the graviton-dilaton system is coupled to a phenomenological perfect fluid source, these difficulties are removed for a wide range of the fluid equation of state. For conformally coupled matter we find, in particular, that exact solutions representing superinflation are allowed provided, in four dimensions, $-\frac{2}{3}\leqslant p/\rho \lesssim 0.45$; even a larger p/ρ spectrum is compatible with inflation, moreover, if dilaton and matter sources are decoupled.

The scalar-tensor model of gravity we shall consider is described by the D-dimensional action density [1,2,4,5]

$$L=\frac{1}{16\pi G}\left[R^{ab}\wedge {}^*(V_a\wedge V_b)-\tfrac{1}{2}\mathscr{D}\phi\wedge {}^*\mathscr{D}\phi -\tfrac{1}{8}\lambda f(\phi)R^{ab}\wedge R^{cd}\wedge {}^*(V_a\wedge V_b\wedge V_c\wedge V_d)\right] +L(M,\phi)\,, \tag{1}$$

where $L(M,\phi)$ represents the contribution of all the effective matter sources (including their possible coupling to the dilaton ϕ), V^a is the vielbein one-form, $R^{ab}(\omega)$ the curvature two-form, an asterisk denotes the Hodge dual map and $\mathscr{D}$ the Lorentz exterior covariant derivative. The quadratic curvature correction is given in the form of the Euler-Gauss-Bonnet combination, which guarantees the absence of ghosts [6] and appears naturally in supersymmetric strings for the supercompletion of the multiplet with the Lorentz-Chern-Simons term [7]. Finally, in a string-derived model, λ^{-1} is the string tension and [1,2,4,8,9]

$$f(\phi)=\exp[-\phi\sqrt{2/(D-2)}]\,. \tag{2}$$

Note that we are working in the so-called Einstein frame, in which the scalar field is decoupled from the Einstein part of the action. The background field equations of a corresponding (sigma-model) string effective action, which exhibits a Brans-Dicke-like coupling of the dilaton to the Einstein lagrangian, can be recovered by a conformal rescaling and by an appropriate redefinition [8] of the metric and of the scalar field. Note also that we have not included in (1) a possible Lorentz-Chern-Simons correction, as it is known [4] that its contribution to the field equations is exactly vanishing for isotropic manifolds such as the cosmological solutions we shall consider in this paper.

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The variation of (1) with respect to ϕ provides the dilaton equation of motion

$$\mathscr{D}^*\mathscr{D}\phi=\tfrac{1}{8}\lambda f'R^{ab}\wedge R^{cd}\wedge{}^*(V_{abcd})-\frac{\delta L(M,\phi)}{\delta\phi}, \quad (3)$$

where a prime on f denotes differentiation with respect to ϕ (conventions: $V_{abc...}=V_a\wedge V_b\wedge V_c\wedge ...$). In the absence of matter sources (and of a dilaton potential), one thus immediately sees that a maximally symmetric de Sitter manifold, $R^{ab}=\Lambda V^{ab}$, with constant dilaton field, $\phi=\phi_0$, could be allowed as a solution only if $(f')_{\phi=\phi_0}=0$, which is not the case for the exponential coupling function (2) of string theory [1,2]. For such a coupling function, an allowed solution would seem to be an isotropic Friedman–Robertson–Walker (FRW) manifold

$$g_{\mu\nu}=\mathrm{diag}(1,-a^2(t)\delta_{ij}), \quad (4)$$

with power-law evolving scale factor, $a\sim t^\alpha$. In this case, eq. (3) is indeed solved by

$$\phi=\phi_0-\ln(kt)^2 \quad (5)$$

(k is an arbitrary constant fixing the time scale of units). However, the condition one obtains on α from eq. (3) is not compatible, if $L(M,\phi)=0$, with the other two gravitational equations following from the action (1).

When matter sources are included, a de Sitter solution in general is still forbidden (unless the matter contribution is in the form of a cosmological constant), while power-law solutions become possible. To illustrate this possibility, we shall first derive the gravitational equations by varying the action (1) in the most convenient first order formalism, in which the vielbein V and the Lorentz connection ω are treated as independent variables. The V-variation gives

$$R^{ab}\wedge{}^*(V_{abk})-\tfrac{1}{8}\lambda f(\phi)R^{ab}\wedge R^{cd}\wedge{}^*(V_{abcdk})$$
$$=\theta_k(\phi)+16\pi G\theta_k(M,\phi), \quad (6)$$

where θ_k are the canonical stress–energy $(D-1)$-forms, derived respectively from the dilaton kinetic term and the matter part of the action (note that ϕ is dimensionless with our conventions). Moreover, by performing the ω-variation, we get the torsion equation

$$R^c\wedge{}^*(V_{abc})-\tfrac{1}{4}\lambda\mathscr{D}f\wedge R^{cd}\wedge{}^*(V_{abcd})$$
$$-\tfrac{1}{4}\lambda fR^{cd}\wedge R^k\wedge{}^*(V_{abcdk})=0, \quad (7)$$

where $R^c=\mathscr{D}V^c$ is the torsion two-form (we are assuming no ω-dependence in the matter lagrangian).

In order to combine these two equations in a single, second-order expression, we now separate the total connection ω into the riemannian part, $\bar\omega$, and the contortion one-form K, i.e.,

$$\omega^{ab}=\bar\omega^{ab}-K^{ab}, \quad (8)$$

where $\bar{\mathscr{D}}V^a\equiv dV^a+\bar\omega^a_b\wedge V^b=0$. The torsion and curvature thus become

$$R^a=-K^a_b\wedge V^b, \quad (9)$$

$$R^{ab}(\omega)=\bar R^{ab}-\bar{\mathscr{D}}K^{ab}+K^a_c\wedge K^c_b, \quad (10)$$

where $\bar R^{ab}$ is the usual riemannian curvature. The solution of eq. (7) gives for K, to first order in λ (we are indeed considering the $|\lambda R^{ab}|\ll 1$ limit in which higher than quadratic curvature corrections to our action (1) have been neglected)

$$\frac{K^{ab}\wedge V^{cd}}{(D-3)!}=-\frac{\lambda f'}{2(D-4)!}\bar R^{c[a}\partial^{b]}\phi\wedge V^d. \quad (11)$$

By inserting the total curvature (10) into eq. (6) we get finally, again to first order in λ (henceforth we shall omit explicitly the bar symbol, but curvature and covariant derivative are both referred, always, to the riemannian connection $\bar\omega$)

$$R^{ab}\wedge{}^*(V_{abk})$$
$$+\tfrac{1}{2}\lambda[f'(\mathscr{D}\partial^b\phi)+f''\partial^b\phi\mathscr{D}\phi]\wedge R^{ac}\wedge{}^*(V_{abck})$$
$$-\tfrac{1}{8}\lambda fR^{ab}\wedge R^{cd}\wedge{}^*(V_{abcdk})$$
$$=\theta_k(\phi)+16\pi G\theta_k(M,\phi). \quad (12)$$

In four dimensions, the last term in the left-hand side of eq. (12) does not contribute and we are led, in the explicit tensor notation, to the following generalized Einstein equations:

$$2G^{\mu}{}_{\nu}+\tfrac{1}{2}\lambda f'(R\phi^{\alpha}{}_{\alpha}\delta^{\mu}{}_{\nu}-R\phi^{\mu}{}_{\nu}-2R^{\mu}{}_{\nu}\phi^{\alpha}{}_{\alpha}$$
$$-2\phi^{\alpha}{}_{\beta}R_{\alpha}{}^{\beta}\delta^{\mu}{}_{\nu}+2\phi^{\alpha}{}_{\nu}R_{\alpha}{}^{\mu}$$
$$+2\phi^{\mu}{}_{\alpha}R_{\nu}{}^{\alpha}-2\phi^{\alpha}{}_{\beta}R_{\alpha\nu}{}^{\mu\beta})$$
$$+\tfrac{1}{2}\lambda f''(R\phi^{\alpha}\phi_{\alpha}\delta^{\mu}{}_{\nu}-R\phi^{\mu}\phi_{\nu}-2R^{\mu}{}_{\nu}\phi^{\alpha}\phi_{\alpha}$$
$$-2\phi^{\alpha}\phi_{\beta}R_{\alpha}{}^{\beta}\delta^{\mu}{}_{\nu}+2\phi^{\alpha}\phi_{\nu}R_{\alpha}{}^{\mu}$$
$$+2\phi^{\mu}\phi_{\alpha}R_{\nu}{}^{\alpha}-2\phi^{\alpha}\phi_{\beta}R_{\alpha\nu}{}^{\mu\beta})$$
$$=\phi_{\mu}\phi_{\nu}-\tfrac{1}{2}\phi_{\alpha}\phi^{\alpha}\delta^{\mu}{}_{\nu}+16\pi G T^{\mu}{}_{\nu}(M,\phi) \qquad (13)$$

(conventions: $G_{\mu\nu}$ is the Einstein tensor, and $\phi_{\mu}\equiv\nabla_{\mu}\phi$ denotes the Riemann covariant derivative). They are in perfect agreement (modulo different definitions) with the set of gravitational equations first derived by Wetterich [5] from the four-dimensional version of the action (1), and recently re-considered in ref. [4].

We now assume a perfect fluid model of source, with equation of state $p=\gamma\rho$, and with a coupling to the dilaton field described by the phenomenological parameter ϵ, namely

$$T_{\mu}{}^{\nu}(M,\phi)=T_{\mu}{}^{\nu}\exp(-\epsilon\phi)\,, \qquad (14)$$

where

$$T_{\mu}{}^{\nu}=\mathrm{diag}(\rho,\,-p\delta_i{}^j)\,. \qquad (15)$$

The dilaton equation (3) becomes, in tensor notation,

$$\nabla_{\mu}\nabla^{\mu}\phi=\tfrac{1}{8}\lambda f'(R^2_{\mu\nu\alpha\beta}-4R^2_{\mu\nu}+R^2)$$
$$-16\pi G\epsilon(\rho-3p)\exp(-\epsilon\phi)\,.$$

We have, in particular, $\epsilon=0$ for a fully decoupled dilaton–matter system, while $\epsilon=-\sqrt{\tfrac{1}{2}(D-2)}$ if the coupling is obtained through a conformal transformation, starting from a source lagrangian which is decoupled in the Brans–Dicke frame, defined by an "unrescaled" sigma-model metric [2,8].

We shall concentrate, in the following discussion, on the phenomenologically interesting $D=4$ case. For a string-derived model we have thus $f(\phi)=\exp(-\phi)$, and the coupled graviton–dilaton equations can be solved exactly by the FRW ansatz (4), with ϕ given by eq. (5), and with

$$a(t)=(kt)^{\alpha},\quad \rho(t)=\rho_0(kt)^{-2(\epsilon+1)}\,, \qquad (16)$$

provided the constant parameters ρ_0, ϕ_0, α are fixed by

$$24B\alpha^4-24B\alpha^3-48\alpha+16=-4A\epsilon(1-3\gamma)\,, \qquad (17)$$

$$12B\alpha^3+12\alpha^2(1-2B)-4=A\,, \qquad (18)$$

$$8B\alpha^3+12\alpha^2(1+B)-8\alpha(1+B)+4=-\gamma A\,. \qquad (19)$$

We have defined

$$A=32\pi G\rho_0 k^{-2}\exp(-\epsilon\phi_0)\,,$$
$$B=\lambda k^2\exp(-\phi_0) \qquad (20)$$

[the first condition (17) follows from the dilaton equation, while the other two conditions correspond, respectively, to the time and space components of the gravitational equation (13)]. Note that, in the $B\to 0$ limit, these equations are solved by $\gamma=-1$ and $\alpha=1$ (in agreement with results of ref. [2]). This solution corresponds, in the conformally transformed Brans–Dicke frame, to a flat metric and a linearly evolving dilaton, which is well known to satisfy, to all orders, the sigma-model background field equations [10] (indeed, if we neglect the Gauss–Bonnet correction, our action (1) is directly related by a conformal rescaling to the low-energy string effective action [11]).

When $k>0$, $t>0$ and $\alpha>1$, the scale factor (16) describes the time evolution of a power-law inflating FRW model, i.e., $\ddot{a}>0$, $\dot{H}<0$, where $H=\dot{a}/a$ (for $0<\alpha<1$ one gets the decelerated expansion of standard cosmology). On the other hand, eq. (16) can also correspond to an expanding superinflationary model (in the terminology of ref. [12]), characterized by $\ddot{a}>0$, $\dot{H}>0$ and a curvature singularity at $t=0$, provided $k<0$, $t<0$ and $\alpha<0$. In order to see whether inflationary solutions (of both types) are possible, for realistic values of γ in the range $-1\leqslant\gamma\leqslant 1$, we first note that a physically acceptable solution must satisfy the constraints $A>0$ (positive energy density) and $B>0$ (positive string tension). By combining eqs. (18), (19), we get from these two conditions an allowed region in the (α,γ) plane (independent of ϵ), which is shown in fig. 1. We see that both positive and negative values of α are possible, however, power-inflation ($\alpha>0$) is only allowed for $\gamma<-\tfrac{2}{3}$, while superinflation ($\alpha<0$) for $\gamma>\tfrac{2}{3}$. Remarkably enough, decelerated expansion, $0<\alpha<1$, is always excluded.

For any assigned value of γ and ϵ, the corresponding value of α is fixed by eqs. (17)–(19), whose combination gives

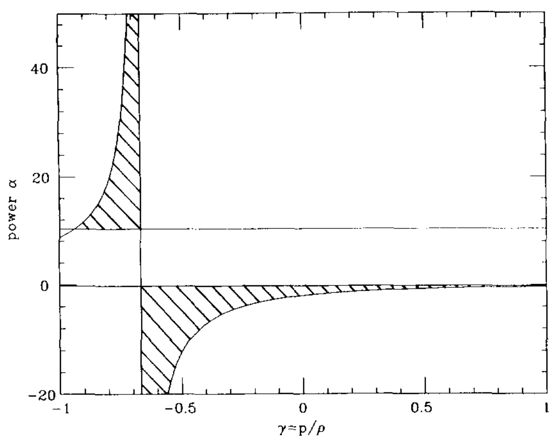

Fig. 1. The allowed part of the (α, γ) plane (hatched region), fixed by the conditions $A>0$ and $B>0$ [see eqs. (18), (19)]. The vertical line corresponds to $\gamma=-\frac{2}{3}$, the two horizontal lines correspond to $\alpha=10.2631$ and $\alpha=-0.212\,565$ respectively.

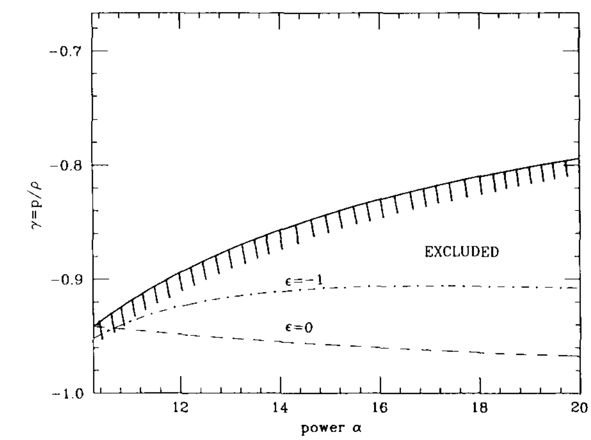

Fig. 2. Solutions of eq. (21) describing power-law inflation for $\epsilon=0$ (dashed curve) and $\epsilon=-1$ (dot-dashed curve). The area below the solid curve is excluded by the requirement of positive energy density.

$$
\begin{aligned}
&9(1+\gamma)\alpha^5+\alpha^4[6\epsilon(1-3\gamma)-15-9\gamma]\\
&\quad+\alpha^3[-66\epsilon(1-3\gamma)+21+15\gamma]\\
&\quad+\alpha^2[22\epsilon(1-3\gamma)+11-39\gamma]\\
&\quad+\alpha[-6\epsilon(1-3\gamma)-18+12\gamma]\\
&\quad-4\epsilon(1-3\gamma)+4\\
&\quad=0\,. \qquad (21)
\end{aligned}
$$

We shall consider, in particular, the two limiting cases corresponding to decoupled ($\epsilon=0$) and conformally coupled ($\epsilon=-1$) matter. For these two cases we find that there are no solutions of eq. (21) representing power-law inflation ($\alpha>1$), and falling inside the allowed sector of the (α, γ) plane. This situation is illustrated in fig. 2. Superinflation, however, is possible, and the solutions of eq. (21) with $\alpha<0$, included inside the allowed region, are plotted in fig. 3.

Two remarks are in order. The first is that superinflation is always allowed for $-\frac{2}{3}\leqslant\gamma<1$, and then, in particular, also for a conventional perfect gas with positive pressure, $0\leqslant p\leqslant\frac{1}{3}\rho$. The second is that, for conformal coupling ($\epsilon=-1$), the inflation becomes faster as γ is growing. Even if, in this case, $|\alpha|\lesssim0.8$, this model of superinflation is thus more efficient than previous Kaluza–Klein models in which the superinflationary expansion of three spatial dimensions was induced by the simultaneous contraction of a

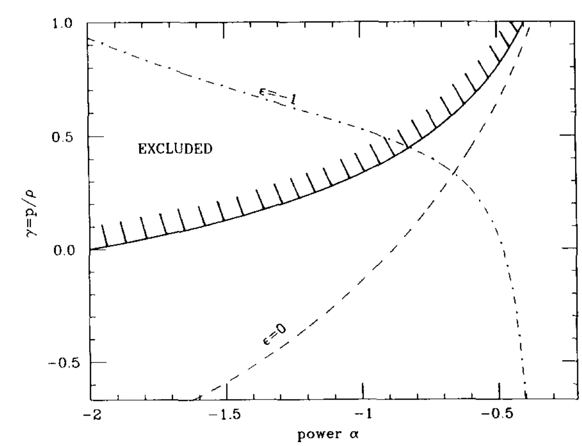

Fig. 3. Superinflationary solutions obtained from eq. (21) for $\epsilon=0$ (dashed curve) and $\epsilon=-1$ (dot-dashed curve). The physically allowed solutions are contained in the area below the solid curve.

large number of internal dimensions [13–18]. (Indeed, $\alpha=-0.358$ in ref. [16], $|\alpha|\leqslant1/\sqrt{3}$ in refs. [14,17], and $|\alpha|\leqslant\frac{1}{2}$ in ref. [18], where the limiting values correspond to a number of internal dimensions approaching infinity.) Values of $|\alpha|$ even larger than 1 are obtained, moreover, if dilaton and matter sources are decoupled ($\epsilon=0$). The possible impact of this model on the dynamics of a realistic inflationary scenario will be discussed elsewhere.

In conclusion, we have shown that a scalar–tensor model of gravity, with Gauss–Bonnet corrections, with a perfect fluid as a phenomenological source, and with a "string-inspired" graviton–dilaton mixing, can provide a mechanism to realize the isotropic superinflationary expansion of the whole space, without the contraction of some internal dimension (isotropic superinflation was recently found [19,20] also for the sigma-model metric of the tree-level string effective action, neglecting, however, quadratic and higher curvature corrections). Such a mechanism does not necessarily require the presence of negative matter pressure, nor the vacuum contribution of a cosmological constant term. In the $\epsilon=-1$ case, superinflation corresponds indeed to a phase of constant energy density $\rho=\rho_0$ [see eq. (16)], but with an equation of state different from the de Sitter one, as $\gamma\neq-1$.

It may be interesting to note, finally, that the superinflationary expansion of our solution $g_{\mu\nu}$ becomes a de Sitter-like exponential inflation when translated into the metric $\tilde{g}_{\mu\nu}$ corresponding to the Brans–Dicke form of the action. The two metrics are indeed related, in four dimensions, by the conformal transformation [8] $\tilde{g}_{\mu\nu}=\exp(\phi)\,g_{\mu\nu}$, or, in terms of the scale factors, $\tilde{a}(\tilde{t})=\exp(\frac{1}{2}\phi)\,a(t)$, where $d\tilde{t}/dt=\exp(\frac{1}{2}\phi)$ is the relation between the cosmic time coordinates of the two frames. One thus obtains

$$\tilde{a}(\tilde{t})=\exp(\tfrac{1}{2}\phi_0)\exp(\tilde{H}\tilde{t}) , \qquad (22)$$

where $\tilde{H}=k(|\alpha|+1)\exp(-\frac{1}{2}\phi_0)$ is the constant Hubble expansion rate in the $\tilde{g}$ frame and $-\infty\leqslant\tilde{t}\leqslant\infty$ (the curvature singularity at $t=0$ is removed). However, as previously stressed, in the presence of higher curvature corrections this conformally transformed Brans–Dicke metric differs (because of field redefinitions [8]) from the sigma-model metric appearing directly in the action of a tree-level string effective theory.

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