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Abstract

It is stressed that the kinematical problems of the standard cosmological model can be solved by a phase of accelerated contraction of the cosmic scale factor. Such a behaviour is only a particular case of a more general "pre-big-bang" inflationary scenario, arising naturally in a string cosmology context, and driven asymptotically by the free dilaton field without any self-interaction potential and/or vacuum contribution.

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THE INFLATIONARY ROLE OF THE DILATON IN STRING COSMOLOGY

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ABSTRACT

It is stressed that the kinematical problems of the standard cosmological model can be solved by a phase of accelerated contraction of the cosmic scale factor. Such a behaviour is only a particular case of a more general "pre-big-bang" inflationary scenario, arising naturally in a string cosmology context, and driven asymptotically by the free dilaton field without any self-interaction potential and/or vacuum contribution.

1. Introduction: Inflation Without Inflation

It is well known that various problems of the standard cosmological scenario can find a natural explanation [1], if we assume that the phase of radiation dominance is preceeded in time by a phase of accelerated expansion (the so-called inflation), characterized by $H > 0$, $\ddot{a} > 0$, where a is the scale factor of an isotropic metric, $H = \dot{a}/a$, and the dot denotes differentiation with respect to cosmic time. What seems to be presently less known, however, is the existence of another possible solution of the standard kinematical problems (suggested by recent studies of string cosmology [2,3,4]), based on the simple assumption that the evolution of our universe is time-symmetric with respect to some given past curvature scale $H_1 \sim 1/t_1 > 0$.

Suppose, in particular, that our universe is radiation-dominated up to H_1, i.e. $a_+(t) \sim (t+t_1)^{1/2}$ for $t > 0$. By time symmetry, the scale factor is to be continued to negative times as

$$a_-(t) \sim (t_1 - t)^{1/2}, \qquad t < 0 \tag{1.1}$$

This metric describes accelerated contraction, $\dot{a} < 0$, $\ddot{a} < 0$, and growing curvature scale, $|\dot{H}| > 0$, up to H_1, and it is still an exact solution of the radiation-dominated cosmological equations (with negligible spatial curvature), as the Einstein equations are invariant under time-reversal transformations. Of course, near $t = 0$, a suitable modification of both $a_\pm$ and of the source stress tensor is to be expected, in order to guarantee the presence of a bounce at $|H| \sim H_1$, and a continuous matching of the two branches of the radiation- dominated evolution. As a consequence, $a_\pm$ are to be regarded only as the asymptotic form of the solution, for $|H| << H_1$. This asymptotic form is already enough, however, to show that the horizon and flatness problems [1] are absent in a radiation-dominated universe, if its evolution is time-symmetric.

In fact, in the primordial contracting phase (1.1) the proper size of a causally connected region shrinks in time like the scale factor, while the proper size $d_e(t)$ of the event horizon,

$$d_e(t) = a(t) \int_t^{t_1} \frac{dt'}{a(t')} = \frac{1}{2}(t_1 - t) \tag{1.2}$$

shrinks linearly in cosmic time, i.e. faster than $a(t)$. This means that causally connected scales are "pushed out" of the horizon (just like in conventional inflationary expansion). The horizon problem may thus be solved, provided the ratio

$$\frac{a(t)}{d_e(t)} \sim \eta^{-1} \tag{1.3}$$

(η is the conformal time coordinate, defined by $dt = a d\eta$) grows enough during the contraction ($t < 0$), so as to compensate the subsequent decrease of $a(t)/t$ during the phase of decelerated expansion ($t > 0$). To obtain, in particular, a causally connected universe at the end ($\eta = \eta_2$) of the radiation-dominated epoch, we must require according to (1.3) that the contracting phase is at least as long as the subsequent expanding one, namely

$$\frac{|\eta_i|}{|\eta_1|} \gtrsim \frac{|\eta_2|}{|\eta_1|} \tag{1.4}$$

where η_i marks the beginning of the contraction, and η_1 is the fixed point of the time-reversal symmetry, corresponding to the maximum scale H_1. This condition is certainly satisfied if the evolution is time-symmetric, irrespective of the value of η_1.

With similar arguments one can show that also the flatness problem is solved by a time-symmetric cosmological evolution. Indeed, the solution of this problem requires that the ratio between the spatial curvature term, k/a^2, and the other terms of the cosmological equations,

$$\frac{k}{a^2 H^2} = \frac{k}{\dot{a}^2} \sim \eta^2 \tag{1.5}$$

at the end of the contracting phase ($\eta = \eta_1$) is tuned to a value small enough, so that the subsequent decelerated evolution eventually leads to a value of the ratio $\lesssim 1$ at $\eta = \eta_2$. This implies

$$\left(\frac{\eta_1}{\eta_i}\right)^2 \lesssim \left(\frac{\eta_1}{\eta_2}\right)^2 \tag{1.6}$$

which is equivalent to the condition (1.4), and which is again satisfied by a time-symmetric cosmological evolution.

The presence of a long enough period of accelerated contraction, which may follow in particular from the hypothesis of time-symmetric cosmology (like in the previous example), has thus all the kinematical effects of inflation, with no need of introducing some "ad hoc" inflaton potential, slow-rolling conditions, fine-tuning, and so on, like in conventional inflationary models [5]. A possible objection to the complete equivalence of inflation and time-reversal symmetry is that the effective potential, $V(\eta) = (d^2a/d\eta^2)/a$, appearing in the scalar and tensor perturbation equations [6], is vanishing both for the expanding and contracting radiation-dominated evolution, in which $a \sim |\eta|$. This implies that no parametric amplification of the perturbations is possible, in such context. However, in a truly time- symmetric evolution the radiation-dominated contraction should be preceeded by the time-reversed of the dust-dominated evolution, which also describes accelerated contraction with $a \sim |t|^{2/3} \sim |\eta|^2$. The transition between the two contracting phases amplifies perturbations with just a flat Harrison-Zeldovich spectrum, even if only for those comoving scales k which are smaller than the "height" $|\eta_2|^{-1}$ of the effective potential barrier, namely for scales entering inside the horizon during our present matter-dominated epoch.

2. The Pre-Big-Bang Scenario

Another objection to the "inflationary" properties of a phase of accelerated contraction is that such a phase seems unable to dilute the possible production of topological defects, such as monopoles or strings. A possible answer to this objection is that strings shrinks in contracting backgrounds and, as a consequence, they are eventually diluted by the same amount as in the conformally related super-inflationary (expanding) backgrounds [3]. Moreover, if it is true that monopoles inflate together with the metric background in which they are embedded, as recently argued in [7], then they should shrink, just like strings, if the background is contracting. So accelerated contraction is not incompatible, in principle, with the dilution of unwanted topological relics.

However, what it should be stressed at this point is that the previous example of phase with accelerated contraction and growing curvature, obtained through a time-reversal transformation from the standard radiation-dominated cosmology, is only a particular case of "pre-big-bang" scenario [8,2-4]. Such a scenario, describing a universe which starts from a flat and vacuum initial state, and approaches a maximum curvature scale (identified with the big-bang) through a period of accelerated evolution, finds a natural motivation in the context of the equations obtained from the low energy string effective action (rather than in the context of Einstein's cosmological equations). In their minimal version such equations contain, besides the metric and the matter sources, also a non-minimally coupled scalar field (the dilaton). For a homogeneous and isotropic, spatially flat, $(d+1)$-dimensional background, with perfect fluid sources ($p/\rho = \gamma = const$), such equations can be written explicitly as [2,8]

$$(\dot\phi - dH)^2 - dH^2 = 8\pi G\rho e^{\phi}$$

$$2\dot H - 2H(\dot\phi - dH) = 8\pi G\gamma\rho e^{\phi}$$

$$(\dot\phi - dH)^2 - 2\ddot\phi + 2d\dot H + dH^2 = 0 \tag{2.1}$$

(where G is the d-dimensional analog of the Newton constant). They are invariant not only under the usual time-reversal transformation $t \to -t$, but also under the "duality" transformation [9,10]

$$a \to a^{-1}, \qquad \phi \to \phi - 2d\ln a, \qquad \gamma \to -\gamma \tag{2.2}$$

By combining duality and time reversal transformations it is thus possible, in a string cosmology context, to associate to any given standard "post-big-bang" solution, with decreasing curvature scale, a related "pre-big-bang" solution whose curvature has a specular behaviour with respect to the chosen scale of time-symmetry, but which is expanding, in an accelerated (inflationary) way. Consider, for instance, the general solution of eqs.(2.1), which becomes, at low enough curvature scales [2,3],

$$a(t) \sim |t|^{2\gamma/(1+d\gamma^2)}, \qquad \phi \sim \frac{d\gamma - 1}{\gamma}\ln a, \qquad \rho \sim a^{-d(\gamma+1)} \tag{2.3}$$

For $\gamma > 0$ and $t > 0$ the metric describes a decelerated expansion with decreasing curvature, characterized by

$$\dot a > 0, \qquad \ddot a < 0, \qquad \dot H < 0 \tag{2.4}$$

By performing the duality transformation (2.2), plus a time inversion, we are led to a metric describing accelerated expansion (of the so-called superinflationary type) and increasing curvature scale, characterized by

$$\dot{a} > 0, \qquad \ddot{a} > 0, \qquad \dot{H} > 0 \tag{2.5}$$

The particular case of the radiation-dominated solution ($\gamma = 1/d$) corresponds to a constant dilaton,

$$a \sim t^{2/(d+1)}, \qquad \phi = const, \qquad \rho \sim a^{-(d+1)}, \qquad t > 0 \tag{2.6}$$

and it is associated to a pre-big-bang phase dominated by a gas of stretched strings [11] (with $\gamma = -1/d$), characterized by superinflationary expansion, growing curvature and dilaton coupling,

$$a \sim (-t)^{-2/(d+1)}, \qquad \phi = 2d \ln a, \qquad \rho \sim a^{-(d-1)}, \qquad t < 0 \tag{2.7}$$

The particular case of pre-big-bang discussed in the previous section ($a \sim (-t)^{1/2}$, $\phi = const$) is thus only a trivial example obtained from the solution (2.6) through time-reversal, which is the only possible transformation in the context of the Einstein cosmological equations, where there is no dilaton and the duality symmetry cannot be implemented. In the string cosmology context, on the contrary, the dilaton is present, and the associated duality symmetry has various important consequences.

First of all, with a superinflating (instead of contracting) pre-big-bang scenario the scale factor evolves monotonically in time, and this leads to the interesting possibility of self-dual cosmological solutions [8], characterized by $a(-t) = a^{-1}(t)$, and connecting in a smooth way the two duality-related pre and post-big-bang regimes (see however [12] for non-monotonic but smooth solutions). Of course, a superinflating pre-big-bang metric may become contracting when transformed in the Einstein frame (where the dilaton is minimally coupled to the metric). In that frame, however, the contraction is accelerated and the curvature is still growing, so that the solution of the standard kinematical problems is preserved [2-4], as discussed also in Section 1.

The presence of a non-trivial dilaton background, moreover, contributes to the effective potential appearing in the linearized perturbation equation [13,2,3], and then to the parametric amplification of the perturbations. The potential corresponding to the solution (2.7), in particular, is no longer flat, unlike the case of the solution (2.6) (for $d = 3$). But the main consequence of a dilaton background is that, in the high curvature regime, the general solution of the equations (2.1) becomes dilaton-dominated (quite independently of the matter equation of state), and the universe enters a phase of dilaton-driven superinflation, with [2,3]

$$a \sim (-t)^{-1/\sqrt{d}}, \qquad \phi \sim (d + \sqrt{d}) \ln a \tag{2.8}$$

When the sources are, in particular, a diluted gas of non-interacting fundamental strings, and we impose as initial condition the flat and cold string perturbative vacuum ($a = const$, $\phi = -\infty$), then the general solution of the system of background string equations and string equations of motion shows that the beginning of the superinflationary evolution of the metric, and of the phase of dilaton dominance, are nearly coincident at the same time scale [3]. In that case the dilaton field is completely responsible for the phase of accelerated evolution.

3. Discussion

According to the string cosmology equations, a phase of dilaton-driven isotropic superinflation can thus be arranged with no need of a simultaneous shrinking of the internal dimensions (like in the conventional realizations of superinflation [14]) or of some fine-tuned dilaton potential. Indeed, at low energy scales (with respect to the string tension $(\alpha')^{-1/2} \simeq 10^{-1}\, M_p$) the dilaton potential can appear at a non-perturbative level only, and it is expected to be negligible, as $V(\phi) \sim \exp[-\exp(-\phi)]$.

A string-motivated dilaton potential must play a fundamental role, however, in the transition from the accelerated evolution with growing curvature of the pre-big-bang regime, to the subsequent standard cosmological evolution. As recently discussed in [15], a smooth transition seems to be impossible without including in the low energy string effective action higher curvature corrections and the contribution of a non-perturbative dilaton potential. The "graceful exit" from the pre-big-bang to the post-big-bang regime is indeed the main unsolved problem, at present, of this scenario, even if recent results on exact string solutions (to all orders in α') are encouraging [16]. The non-perturbative potential, in particular, is expected to force the dilaton to a minimum which gives it a mass, and freezes the value of the Newton constant to its present value.

In addition, the transition from the growing to decreasing curvature regime should give rise to a copious non-adiabatic production of radiation, in order to explain the large amount of entropy presently observed on a cosmological scale. An interesting possibility is that such entropy be produced in the decay of the dilatons (or of other massive particles) created from the vacuum by the expanding metric background [3,17]. In such case, the dilaton freezing is likely to occur not simultaneously to, but sensibly after the bounce of the curvature at a maximum scale (of Planckian order); moreover, the universe is expected to be dilaton-dominated even in the first period of the post-big-bang era, before the radiation production. But for what concerns these aspects of string cosmology work is still in progress [18], and I hope to be able to report more details in some future publication.

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