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CAUSAL HORIZONS, ACCELERATIONS AND STRINGS *

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ABSTRACT

All the points of a string are always causally connected provided their relative acceleration is smaller than the critical value $a_c = (m\alpha')^{-1}$, where m is the mass and $1/\alpha'$ the string tension. It is pointed out that this limiting acceleration characterizes the transition to an unstable regime, in which an approximate description of the string motion around the classical path of a point particle is no longer consistent.

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Causal horizons, accelerations and strings.

It is well known, in the context of classical relativity, that for any massive extended object there is a critical value of the acceleration, determined by the size of the object itself and by the causal structure of the space-time manifold [1].

Consider, for example, a one-dimensional object of proper length λ, and suppose that one of its ends has a positive acceleration of constant proper strength a, with respect to the other end. A Rindler horizon will appear at a proper distance a^{-1} from the accelerated extremity, so that all the parts of the object may be causally connected only if $\lambda < a^{-1}$. This fixes a maximum allowed proper acceleration, $a < a_c = \lambda^{-1}$.

This boundary value obviously grows as λ decreases. However, it may be interesting to note that the maximum acceleration classically compatible with causality is finite even in the limit of a point-like object. In this limit, in fact, the proper size λ of the object becomes smaller than the "gravitational size" fixed by the Schwarzschild radius $r_s = 2GM$, so that the gravitational effects are no longer negligible. For a point-like gravitational source, on the other hand, there is a critical value of the proper acceleration, $a_c \simeq r_s^{-1}$ (up to a numerical factor of order unity), above which the Schwarzschild and Rindler horizon intersect, and a naked singularity is formed [2].

Causality breakdowns (which may occur in the field of a naked singularity) are thus in general avoided provided one requires, by combining the two bounds, that $a \leq Min\{\lambda^{-1}, r_s^{-1}\}$. It is worth noting that, if the mass of the object is inversely proportional to the extension, $M \simeq \lambda^{-1}$, this bound can be written in the suggestive form (measuring accelerations in units of Planck masses $M_p = L_p^{-1}$)

$$\frac{a}{M_p} \leq Min\{\frac{L_p}{\lambda}, \frac{\lambda}{L_p}\} \tag{1}$$

The Planck mass, for Planck-sized objects, is in this case the maximum allowed acceleration, in agreement with previous suggestions by various authors [3-7].

The above arguments relating critical acceleration and size are formulated in the context of classical relativity theory, and apply to the case of *classical* extended objects. In practice, however, the accelerations needed to produce the birth of a causal horizon at a distance comparable with the size of the object can be conceived only at a microscopic level, and then in the *quantum* regime (conceived, but not certainly achieved, up to now at least, in a terrestrial laboratory: the LEP accelerations, for example, correspond to horizons at a distance of about 10^9 Compton lengths from the accelerated particle [8]).

In a quantum context, the simplest extended object is a string. One is led, therefore, to wonder about the effects of accelerations on strings. Does it exist, at least in principle, a critical acceleration for strings like in the case of macroscopic extended bodies? and if it exists, how does it manifest itself? An answer to these questions could be useful to understand the interactions of strings with gravity.

The main purpose of this paper is to point out that the maximal size of a string, fixed by the string tension, seems just to define a critical acceleration, in the sense that when two ends of a string become causally disconnected because of the horizon associated to their relative acceleration, the string starts to become unstable [9]. In other words, above the critical acceleration determined by the inverse of the string's length, the string enters on a regime in which the oscillation modes develop imaginary frequencies [10], and the expansion of the solution to the string equations around the classical motion of a point particle turns out to break down [11], so that a different approximation scheme around a non-oscillating asymptotic configuration is required [12].

In order to display this effect, we have to couple a string to some accelerating external field. The simplest possibility to induce relative accelerations among the various parts of a string seems that of embedding the string into a curved background manifold. In the absence of interactions other than gravity, indeed, the string will be free falling. Different points, however, will fall along different geodesics, and if we consider the local free falling frame of one point (A) of the string, we find in this frame that another point (B), at a proper distance λ, has a non-vanishing local acceleration with respect to A given by the equation of geodesic deviation [13]

$$a^\mu = -R_{\nu\alpha\beta}{}^{\mu} z^\nu u^\alpha u^\beta \tag{2}$$

Here z^μ denotes the AB space-like separation vector, $z^\mu z_\mu = -\lambda^2$, u^μ is the geodesic velocity field, $u^\mu u_\mu = 1$, $u^\mu z_\mu = 0$, and $R_{\mu\nu\alpha\beta}$ is the curvature tensor of the background manifold.

This acceleration defines, locally, a causal horizon [1] at a proper distance $d = (-a^\mu a_\mu)^{-1/2} \equiv |a|^{-1}$ from the accelerated point B. Of course, two points may become causally disconnected by such horizon, when $d < \lambda$, only if the relative acceleration (2) is positive (if, on the contrary, the acceleration of B points towards A, the two world-lines are always inside the causal horizon). This means, in other words, that in order to induce a causal separation the external field has to be repulsive along the AB direction. We may thus consider, as a simple example, an inflationary expanding cosmological background, spatially flat, which is described (in the cosmic time gauge) by the metric

$$g_{\mu\nu} = diag(1, -R^2(t)\delta_{ij}) \tag{3}$$

where $\ddot{R} > 0$, and a dot denotes differentiation with respect to cosmic time (other examples of repulsive configurations in gravitational backgrounds are considered in Ref.[9]).

In this background the relative acceleration (2) is always positive and, for two co-moving geodesics ($u^\mu = \delta_0^\mu$) with proper space- like separation $|z|$, it is given by

$$|a| = |z|\frac{\ddot{R}}{R} \tag{4}$$

On the other hand, the maximal allowed separation between two points on a string of mass m (and string tension $(\alpha')^{-1}$) is fixed by the string "maximal size", $m\alpha'$. The whole string will be always causally connected ($d > m\alpha' \geq |z|$), therefore, provided the relative acceleration (4) satisfies the bound $|a| < a_c = (m\alpha')^{-1}$, namely [9]

$$(m\alpha')^2 \frac{\ddot{R}}{R} < 1 \tag{5}$$

Remarkably, this is exactly the consistency condition required for the validity of an approximate description of the string motion, as an expansion around the classical geodesic evolution of its center of mass [11]. The string equations, in the gauge in which the world-sheet metric is conformally flat, can be written indeed

$$\Box X^\mu + \Gamma_{\alpha\beta}{}^{\mu}(X)\partial_+ X^\alpha \partial_- X^\beta = 0 \tag{6}$$

where $\Box \equiv \partial_\tau^2 - \partial_\sigma^2$, $\partial_\pm = \partial_\tau \pm \partial\sigma$ (σ and τ are the world-sheet space and time variables), and $\Gamma(X)$ is the Christoffel connection for the metric (3) (see Ref.[11] for a discussion of the fact that this is not a candidate string vacuum). By expanding the exact solution around a co-moving geodesic $q^\mu(\tau)$, i.e. by putting

$$X^\mu(\sigma,\tau) = q^\mu(\tau) + \eta^\mu(\sigma,\tau) \quad , \quad q^\mu = m\alpha'\tau\delta_0^\mu \quad , \quad |\eta| << |q| \tag{7}$$

and defining a "proper amplitude", $\chi^i = R\eta^i$, for the spatial components of the first order perturbations, one then finds that the n-th Fourier components χ_n satisfy the linearized equation [11]

$$\ddot{\chi}_n^i + (\frac{n^2}{m^2\alpha'^2} - \frac{\ddot{R}}{R})\, \chi_n^i = 0 \tag{8}$$

Therefore, as soon as the bound (5) is not satisfied, i.e. the relative acceleration between two ends of the string is large enough to make them causally disconnected, the string oscillators may develop imaginary frequencies. The approximation (7) is no longer valid as the perturbations start growing in time, and the whole string approaches asymptotically an unstable configuration in which proper amplitudes inflate like the scale factor $R(t)$, while co-moving amplitudes become "frozen" [11,12]. We can thus interpret $(m\alpha')^{-1}$ as the limiting acceleration signalling the transition between two different string regimes (stable and unstable), characterized, respectively, by oscillating and non-oscillating behaviour.

The same interpretation has been shown to apply also to the case of different gravitational backgrounds [9]. It is important to stress, moreover, that the existence of this limiting acceleration is not a peculiar feature of the gravitational interaction only. Indeed, in the case of a string with charge $\pm q$ on its ends, coupled to a background electric field E, it has been shown [14] that instability (i.e. negative frequencies) emerge for $E > (q\alpha')^{-1}$. Since, in this case, $a = qE/m$, again one finds that the stable regime corresponds to accelerations smaller than the causal bound, $a < a_c = (m\alpha')^{-1}$ (a similar conclusion, but with a slightly different value for the limiting acceleration a_c, has been independently obtained also in the context of a different model of accelerated string in flat space [15]).

Finally, as a conclusive remark, it may be interesting to point out that, in the context of a realistic string unification theory, the string Schwarzschild radius $r_s \simeq (m/M_p^2)$ should be always smaller than the maximal size $m\alpha'$. As a consequence, the formation of naked singularities corresponding to accelerated configurations with a Rindler horizon smaller than r_s [2] may become possible, at least in principle, only after a transition to the unstable regime.

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