DFTT-58/92

Dilaton Contributions To The Cosmic Gravitational Wave Background

M.Gasperini and M.Giovannini
Dipartimento di Fisica Teorica dell'Università,
Via P.Giuria 1, 10125 Torino, Italy,
and
Istituto Nazionale di Fisica Nucleare, Sezione di Torino

Abstract

We consider the cosmological amplification of a metric perturbation propagating in a higher-dimensional Brans-Dicke background, including a non trivial dilaton evolution. We discuss the properties of the spectral energy density of the produced gravitons (as well as of the associated squeezing parameter), and we show that the present observational bounds on the graviton spectrum provide significant information on the dynamical evolution of the early Universe.

———————————————-

To appear in **Phys.Rev.D**

1. Introduction

It is well known that the transition from a primordial inflationary phase to a decelerated one, typical of our present cosmological evolution, is associated with the production of a cosmic background of relic gravity waves[1-6]. The spectral distribution of their energy density may provide direct information on the very early history of our Universe, and can be used, in particular, to reconstruct the time dependence of the Hubble parameter[7].

Deflation, however, is not the only violent process typical of primordial evolution able to amplify a metric fluctuation. Although less known (or less studied, up to now at least), it is a fact that gravitons can be produced from the vacuum also as a consequence of a phase of dynamical dimensional reduction[8,9], in which a given number of "internal" dimensions shrink down to a final compactification scale. Another possible process which may lead to a cosmological graviton production, and which (to our knowledge) has not yet been discussed in the literature, is the time variation of the effective gravitational coupling constant G.

The main purpose of this paper is to compute the expected spectrum of the cosmic gravitons background, by including both the contributions of dimensional reduction and of $\dot{G}$ among the possible sources (besides inflation), and by using a Brans-Dicke-like graviton-dilaton coupling as a dynamical model of variable G. We are led to this choice, in particular, by the models of early universe evolution based on the low energy string effective action [10-12], which suggest that the standard radiation-dominated cosmology is preceeded by a dual, "string-driven" phase, in which the effective gravitational coupling changes just because of the time-dependence of the dilaton background. The possibility of looking for tracks of such a string phase in the properties of the cosmic graviton spectrum provides indeed one of the main motivations of the present work.

The paper is organized as follows. In Section II we deduce the linearized equation for a gravitational wave perturbation in a Brans-Dicke background, and, in Section III, this equation is used to compute the spectral distribution of the gravitons, produced by the cosmological background transitions. We shall take into account the dilaton-driven variation of G in a higher dimensional framework in which also the scale of the internal spatial dimensions is allowed to vary, and in which the matter-dominated and radiation-dominated evolution of the external space follows a phase of accelerated (i.e. inflationary) expansion. The squeezing parameter[13] corresponding to this scenario will be given in Section IV.

In Section V the present bounds on the energy density distribution of the relic gravitons are used to obtain information, and constraints, on the value of the curvature scale at the transition between the inflationary and the radiation-dominated era, versus the parameters characterizing the background kinematics. The predictions of some string-inspired cosmological models (and of related Kaluza-Klein scenarios) will be compared with these bounds in Section VI. The main conclusions of this paper will be finally summarized in Section VII.

2. Gravitational perturbations of a Brans-Dicke background.

The starting point to discuss the production of gravitons, induced by a cosmological background transition, is the linearized wave equation for a gravitational perturbation propagating freely in the given background. In order to include the effect of a changing gravitational coupling, such an equation will be obtained by perturbing (at fixed sources) the Brans-Dicke field equations, around a background configuration which includes a time-dependent dilaton field.

It should be perhaps recalled that, in a general relativity context and in a spatially flat Friedmann-Robertson-Walker manifold, a gravity-wave perturbation obeys the same equation as a minimally coupled massless scalar field[1,14,15]. In a Brans-Dicke context, however, the graviton wave equation is different from the covariant Klein-Gordon equation, because the gravitational perturbations are coupled not only to the background metric tensor, but also to the scalar dilaton background $\phi(t)$ representing the G variation.

Our background field dynamics is assumed to be described, in D dimensions, by the following scalar-tensor action

$$S = -\frac{1}{16\pi G}\int d^D x\ e^{-\phi}\ \sqrt{|g|}(R - \omega g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi) + S_m \tag{2.1}$$

where ω is the usual Brans-Dicke parameter, and S_m represent the possible contribution of matter sources, with $\sqrt{|g|}T_{\mu\nu} = 2\delta S_m/\delta g^{\mu\nu}$. The variation of this action with respect to ϕ provide the dilaton equation

$$R + \omega(\nabla\phi)^2 - 2\omega\Box\phi = 0 \tag{2.2}$$

where ∇ denotes the Riemann covariant derivative, and $\Box = g^{\mu\nu}\nabla_\mu\nabla_\nu$. The variation with respect to $g_{\mu\nu}$, combined with (2.2), provides the equation

$$R_\mu{}^\nu + \nabla_\mu\nabla^\nu\phi + (\omega+1)[\delta_\mu^\nu((\nabla\phi)^2 - \Box\phi) - \nabla_\mu\phi\nabla^\nu\phi] = 8\pi G_D e^\phi T_\mu{}^\nu \tag{2.3}$$

Note that we are using an exponential parametrization for the dilaton field, to make contact with the string cosmology models. For $\omega = \infty$, $\phi = const$ we recover general relativity, while for $\omega = -1$ eq (2.1) reduces indeed to the truncated low energy string effective action with phenomenological matter sources[10−12].

The free linearized wave equation for a metric fluctuation $\delta g_{\mu\nu} = h_{\mu\nu}$ is now obtained by perturbing eqs.(2.2) and (2.3), keeping all sources (dilaton included) fixed, $\delta T_\mu^\nu = 0 = \delta\phi$. It should be stressed that we have not explicitly included in the action a possible dilaton potential term, $V(\phi)$, as its contribution to the perturbation is vanishing for $\delta\phi = 0$. It is true that, in a class of duality-symmetric string cosmological models[10−12,16], the dilaton self interactions may also occur trough a coupling to the metric, and lead to a two loop potential of the form $V = V_0[\exp(2\phi - 2\ln\sqrt{|g|})]$, for which $\delta V \propto V\delta g$. This potential, however, is expected to affect in a significant way only the transition region between the inflationary and the radiation dominated regime[12]. Its contribution to the perturbation equations may then be neglected for the purpose of this paper where, as discussed in the following section, we shall evaluate the graviton spectrum in the "sudden" approximation, namely in the approximation in which the kinematic details of the transition regime are ignored, and the rapid exponential decay of the high frequency tail of the spectrum is replaced by a suitable high frequency cutoff.

We perform then the transformation $g_{\mu\nu} \to g_{\mu\nu} + \delta g_{\mu\nu}$, with

$$\delta g_{\mu\nu} = h_{\mu\nu} \qquad , \qquad \delta\phi = 0 = \delta T_\mu{}^\nu \tag{2.4}$$

By neglecting corrections of order higher than first in $h_{\mu\nu}$ (so that, for instance, $\delta g^{\mu\nu} = -h^{\mu\nu}$), we are led to the following variational expressions

$$\begin{gathered}
\delta(\nabla\phi)^2 = -h^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi \\
\delta(\Box\phi) = -h^{\mu\nu}\nabla_\mu\nabla_\nu\phi - g^{\mu\nu}\partial_\alpha\phi\delta\Gamma^\alpha_{\mu\nu} \\
\delta R = -h^{\mu\nu}R_{\mu\nu} + g^{\mu\nu}\delta R_{\mu\nu} \\
\delta(\nabla_\mu\nabla^\nu\phi) = -h^{\nu\alpha}\nabla_\mu\nabla_\alpha\phi - g^{\nu\alpha}\partial_\beta\phi\delta\Gamma^\beta_{\mu\alpha} \\
\delta(\nabla_\mu\phi\nabla^\nu\phi) = -h^{\nu\alpha}\partial_\mu\phi\partial_\alpha\phi
\end{gathered} \tag{2.5}$$

Here

$$\delta\Gamma^\alpha_{\mu\nu} = \frac{1}{2}g^{\alpha\beta}(\nabla_\mu h_{\nu\beta} + \nabla_\nu h_{\mu\beta} - \nabla_\beta h_{\mu\nu}) \tag{2.6}$$

and $\delta R_{\mu\nu}$ is the linearized expression for$R_{\mu\nu}(\delta g)$ (note that all covariant derivatives, as well as all operations of raising index on $h_{\mu\nu}$, are now to be understood as performed with the help of the background metric $g_{\mu\nu}$).

We choose, in particular, a time dependent background with $\phi = \phi(t)$, and a homogeneous diagonal metric describing a general situation of dimensional decoupling , in which d dimensions expand with scale factor $a(t)$, and n dimensions contract with scale factor $b(t)$. In a synchronous frame

$$g_{00} = 1 \quad , \quad g_{ij} = -a^2(t)\gamma_{ij}(x) \quad , \quad g_{ab} = -b^2(t)\gamma_{ab}(y)$$
$$g_{0\mu} = 0 = g_{ia} \qquad , \qquad \phi = \phi(t) \tag{2.7}$$

(conventions: $\mu, \nu = 1, ..., D = d+n+1$; $i, j = 1, ..., d$; $a, b = 1, ..., n$; t is the cosmic time coordinate, and γ_{ij}, γ_{ab} are the metric tensors of two maximally symmetric euclidean manifolds, parametrized respectively by "internal" and "external" coordinates $\{x^i\}$ and $\{y^a\}$). We are interested moreover, in a pure tensor gravitational perturbation, decoupled from sources, representing a gravitational wave propagating in the d-dimensional external space, such that $h_{\mu\nu} = h_{\mu\nu}(x, t)$, $h_{0\mu} = 0 = h_{a\mu}$, and which satisfies the transverse, traceless gauge condition

$$g^{\mu\nu} h_{\mu\nu} = 0 = \nabla_\nu h_\mu{}^\nu \tag{2.8}$$

In this case we have, for the background (2.7),

$$\delta R = 0 \quad , \quad \delta\Gamma^0_{\mu\nu} = -\frac{1}{2}\dot{h}_{\mu\nu} \tag{2.9}$$

(a dot denotes derivative with respect to t). The perturbation of eq.(2.2) is thus trivially satisfied, while the perturbation of eq.(2.3) provides for $h_{\mu\nu}$ the linearized wave equation

$$\delta R_\mu{}^\nu + \frac{1}{2}\dot{\phi}\dot{h}_{\mu\alpha} g^{\nu\alpha} - h^{\nu\alpha}\nabla_\mu\nabla_\alpha\phi = 0 \tag{2.10}$$

which, being ω independent, is remarkably the same for all Brans- Dicke models.

The non-vanishing components of the background Ricci tensor, for the metric (2.7), are given by

$$R_0{}^0 = -d(\dot{H} + H^2) - n(\dot{F} + F^2)$$
$$R_i{}^j = -\frac{1}{a^2}\tilde{R}_i{}^j(\gamma(x)) - \delta_i^j(dH^2 + \dot{H} + nHF)$$
$$R_a{}^b = -\frac{1}{b^2}\tilde{R}_a{}^b(\gamma(y)) - \delta_a^b(nF^2 + \dot{F} + dHF) \tag{2.11}$$

where $H = \dot{a}/a$, $F = \dot{b}/b$ and $\tilde{R}(\gamma)$ denotes the Ricci tensor for the n dimensional euclidean spaces computed from the metrics $\gamma_{ij}(x)$ and $\gamma_{ab}(y)$. By using the relations

$$\dot{g}_{ij} = 2Hg_{ij} \qquad , \qquad \dot{g}^{ij} = -2Hg^{ij} \tag{2.12}$$

one obtains[14,17], to the first order in $\delta g_{ij} = h_{ij}$,

$$\delta(\dot{g}_i{}^j) \equiv \delta(g^{jk}\dot{g}_{ik}) = \dot{h}_i{}^j \equiv (g^{jk}h_{ik})^{\cdot} \tag{2.13}$$

It is thus simple to show (in the gauge $g^{ij}h_{ij} = 0$) that

$$\delta H = \frac{1}{2d}\,\delta(g^{ik}\dot{g}_{ik}) = 0 = \delta\dot{H}$$

$$\delta H^2 = \frac{1}{4d^2}\,\delta(g^{ik}\dot{g}_{ik})^2 = 0$$

$$\delta(H\delta_i{}^j) = \frac{1}{2}\dot{h}_i{}^j$$

$$\delta(\dot{H}\delta_j{}^i) = \frac{1}{2}\ddot{h}_i{}^j$$

$$\delta(H^2\delta_i{}^j) = \frac{1}{2}H\dot{h}_i{}^j \tag{2.14}$$

(the corresponding perturbations of the F terms are all vanishing, since $\delta g_{ab} = 0$). Therefore

$$\delta R_0{}^0 = 0 = \delta R_a{}^b$$

$$\delta R_i{}^j = -\delta(\frac{\tilde{R}_i{}^j}{a^2}) - \frac{d}{2}H\dot{h}_i^j - \frac{1}{2}\ddot{h}_i{}^j - \frac{n}{2}F\dot{h}_i{}^j \tag{2.15}$$

We shall consider, in particular, a flat euclidean metric $\gamma_{ik} = \delta_{ik}$, so that $\Gamma_{ij}^k(x) = 0 = \tilde{R}_i{}^j(\gamma)$. The gauge condition $\tilde{\nabla}(\gamma)h_i{}^j = 0$ reduces to $\partial_j h_i{}^j = 0$, and implies[14,17]:

$$\delta\tilde{R}_i{}^j = -\frac{1}{2}\nabla^2 h_i{}^j \tag{2.16}$$

with $\nabla^2 = \delta^{ij}\partial_i\partial_j$. We thus recover the usual result

$$\delta R_i{}^j = -\frac{1}{2}[\ddot{h}_i{}^j + (dH + nF)\dot{h}_i{}^j - \frac{1}{a^2}\nabla^2 h_i{}^j] \equiv -\frac{1}{2}\Box h_i{}^j \tag{2.17}$$

valid whenever the background is isotropic in the polarization plane, orthogonal to the direction of propagation of the wave[18].

On the other hand we have, for the background (2.7),

$$\nabla_i\nabla_j\phi = \frac{1}{2}\dot{\phi}\dot{g}_{ij} \tag{2.18}$$

Moreover, by using eq.(2.12),

$$g^{jk}\dot{h}_{ik} - h^{jk}\dot{g}_{ik} = \dot{h}_i{}^j \tag{2.19}$$

The linearised wave equation (2.10) thus reduces to

$$\Box h_i{}^j - \dot{\phi}\dot{h}_i{}^j = 0 \tag{2.20}$$

and, in terms of the eigenstates of the Laplace operator,

$$\nabla^2 h_i{}^j(k) = -k^2 h_i{}^j(k) \tag{2.21}$$

it takes the form

$$\ddot{h}_i{}^j + (dH + nF - \dot{\phi})\dot{h}_i{}^j + (\frac{k}{a})^2 h_i{}^j = 0 \tag{2.22}$$

For later applications, it is convenient to rewrite this equation in term of the conformal time coordinate η, defined by $dt/d\eta = a$. Denoting with a prime the differentiation with respect to η, and defining

$$\psi_i{}^j = h_i{}^j a^{(d-1)/2} b^{n/2} e^{-\phi/2} \tag{2.23}$$

we get finally, from eq.(2.22), that each polarization mode $\psi_i^j(k)$ must satisfy the equation:

$$\psi'' + (k^2 - V)\psi = 0 \tag{2.24}$$

where

$$\begin{aligned} V(\eta) = {} & \frac{(d-1)}{2}\frac{a''}{a} + \frac{n}{2}\frac{b''}{b} - \frac{\phi''}{2} + \frac{1}{4}(d-1)(d-3)(\frac{a'}{a})^2 + \\ & \frac{1}{4}n(n-2)(\frac{b'}{b})^2 + \frac{1}{4}\phi'^2 + \frac{1}{2}n(d-1)\frac{a'b'}{ab} - \frac{1}{2}(d-1)\frac{a'}{a}\phi' - \frac{n}{2}\frac{b'}{b}\phi' \end{aligned} \tag{2.25}$$

This effective potential generalizes to a higher number of dimensions the four-dimensional equation, used by Grishchuk and collaborators[1,3,7], to study the cosmological amplification of the quantum fluctuations of the metric tensor. In addition, it takes into account the coupling of the metric perturbations to a possible time variation of the gravitational coupling constant ($\phi' \neq 0$), and to a possible variation of the scale of n "internal" compactified dimensions ($b' \neq 0$). It may be interesting to note that this potential can also be expressed in terms of the scale factors only, by eliminating the explicit dilaton dependence through the background equation (2.2), which implies

$$-\frac{\phi''}{2} + \frac{1}{4}\phi'^2 - \frac{1}{2}(d-1)\frac{a'}{a}\phi' - \frac{n}{2}\frac{b'}{b}\phi' =$$

$$\frac{1}{4\omega}[2d\frac{a''}{a}+2n\frac{b''}{b}+d(d-3)(\frac{a'}{a})^2+n(n-1)(\frac{b'}{b})^2+2n(d-1)\frac{a'b'}{ab}] \qquad (2.26)$$

In this way one can re-introduce the ω-dependence which is otherwise hidden in the particular choice of the dilaton background. For the purpose of this paper, however, it will be more convenient to work directly with the form (2.25) of the potential, in which ϕ appears explicitly.

3. Parametrization of the graviton spectrum for a general model of background evolution

As discussed in the previous section, the present day background of cosmic gravitational waves may include, among its sources, not only a metric transition (deflation, dynamical dimensional reduction), but also a dilaton transition between two (or more) regimes with different gravitational coupling.

In order to take all these contribution into account, we shall consider the background metric of eq.(2.7) (with flat maximally symmetric subspaces $\gamma_{ij} = \delta_{ij}$, $\gamma_{ab} = \delta_{ab}$), starting with an initial configuration in which, for $\eta < -\eta_1$, d dimensions inflate with scale factor $a(\eta)$, n dimensions shrink with scale factor $b(\eta)$, and the dilaton coupling is growing according to

$$a \sim \eta^{-\alpha} \quad , \quad b \sim \eta^{\beta} \quad , \quad \phi \sim \gamma \ln a \quad , \quad \eta < -\eta_1 \qquad (3.1)$$

(note that in this equation η ranges over negative values, so that α, β and γ are all positive). We shall assume that this phase is followed, at $\eta = -\eta_1$ and $\eta = \eta_2$ respectively, by the standard radiation-dominated and matter-dominated expansion of three spatial dimensions. During these two last epochs, however, the gravitational coupling and the compactification scale of the possible additional n_1 internal dimensions are not assumed to be frozen, but they are allowed to vary as

$$a \sim \eta \quad , \quad b \sim \eta^{-\beta_1} \quad , \quad \phi \sim \gamma_1 \ln a \quad , \quad -\eta_1 < \eta < \eta_2$$

$$a \sim \eta^2 \quad , \quad b \sim \eta^{-\beta_2} \quad , \quad \phi \sim \gamma_2 \ln a \quad , \quad 0 < \eta_2 < \eta \qquad (3.2)$$

According to this model of background evolution, the effective potential (2.25) becomes

$$V(\eta) = \frac{1}{4\eta^2}\left[[\alpha(d-1-\gamma) - n\beta + 1]^2 - 1\right] \quad , \quad \eta < -\eta_1$$

$$V(\eta) = \frac{1}{4\eta^2}[(n_1\beta_1 + \gamma_1 - 1)^2 - 1]] \quad , \quad -\eta_1 < \eta < \eta_2$$

$$V(\eta) = \frac{1}{4\eta^2}[(n_1\beta_1 + 2\gamma_2 - 3)^2 - 1] \quad , \quad \eta_2 < \eta \tag{3.3}$$

(note that it goes to zero as $\eta \to \pm\infty$). A particular solution of eq.(2.24) for $\psi(k)$ can thus be written in terms of the first and the second kind Hankel functions $H^{(1)}$ and $H^{(2)}$ (we follow the notation of Ref.(19)), $\psi(k,\eta) \sim \eta^{\frac{1}{2}} H_\nu^{(2,1)}(k\eta)$, which correspond to free oscillating modes in the $|\eta| \to \infty$ limit, as $\eta^{\frac{1}{2}} H^{(2,1)}(k\eta) \to e^{\mp ik\eta}/\sqrt{k}$ (the minus and plus sign corresponds, respectively, to $H^{(2)}$ and $H^{(1)}$).

The effective potential barrier (3.3) leads to an amplification of the gravitational perturbations or, equivalently, to a graviton production from the vacuum[2,3,5−8]. Indeed, starting with incoming modes which are of positive frequency with respect to the vacuum at the left of the barrier ($\eta \to -\infty$), one has in general, for $\eta \to +\infty$, a linear combination of modes which are of positive and negative frequency, with respect to the vacuum at the right of the barrier. The superposition coefficients $c_\pm(k)$ define the Bogoliubov transformation[20] connecting the "left" and "right" vacuum, and determine the spectral distribution of the produced gravitons.

By assuming, in our case, the "in" states of the gravitational field correspond to the Bunch-Davies "conformal" vacuum[5,6,20], we can write the general solution of eq.(2.24), for each mode $\psi(k)$, in the three temporal regions as follows:

$$\psi_I(k) = C\eta^{\frac{1}{2}} H_\nu^{(2)} \quad , \quad \eta < -\eta_1$$

$$\psi_{II}(k) = \eta^{\frac{1}{2}}[A_+ H_\mu^{(2)}(k\eta) + A_- H_\mu^{(1)}(k\eta)] \quad , \quad -\eta_1 < \eta < \eta_2$$

$$\psi_{III}(k) = \eta^{\frac{1}{2}}[B_+ H_\sigma^{(2)}(k\eta) + B_- H_\sigma^{(1)}(k\eta)] \quad , \quad \eta > \eta_2 \tag{3.4}$$

where

$$\nu = \frac{1}{2}[\alpha(d-1-\gamma) - n\beta + 1]$$

$$\mu = \frac{1}{2}(n_1\beta_1 + \gamma_1 - 1)$$

$$\sigma = \frac{1}{2}(n_1\beta_2 + 2\gamma_2 - 3) \tag{3.5}$$

and C is a normalization constant. The Bogoliubov coefficients are given by $c_\pm(k) = B_\pm/C$, and can be fixed by the four conditions obtained matching ψ and ψ' at $\eta = -\eta_1$ and $\eta = \eta_2$.

The coefficients determined in this "sudden" approximation lead, however, to an ultraviolet divergence of the energy density of the produced particles. The reason is that, for modes of comoving frequency k^2 higher than the height of the potential barrier, the sudden approximation is no longer adequate, and the mixing coefficients should be computed by replacing the potential step with a smooth transition of $V(\eta)$. In this way one finds, indeed, that the mixing of the modes with $k > |V|^{\frac{1}{2}}$ is exponentially suppressed with respect to the other modes[8,20,21], and the ultraviolet divergence is avoided. In this paper, however, we are mainly interested in the general behaviour of the spectral distribution, and not in the details of the transition regime. We shall completely neglect, therefore, the frequency mixing of modes which never "hit" the potential barrier, by putting, for such modes, $c_+(k) \simeq 1$, $c_-(k) \simeq 0$. This replaces the exponential decay of the high frequency side of the spectrum with a cutoff, at an appropriate frequency $k \simeq |V|^{\frac{1}{2}}$.

Our potential barrier (3.3) has two steps, which satisfy $V(\eta_1) \simeq \eta_1^{-2} \gg \eta_2^{-2} \simeq V(\eta_2)$ (for realistic values of the parameters). The propagation of modes with $\eta_2^{-1} < k < \eta_1^{-1}$ will thus be affected, in our approximation, only by the first background transition at $\eta = \eta_1$. In this frequency band, the Bogoliubov coefficients are then defined by $c_\pm = A_\pm/C$; by matching ψ_I, ψ_{II} and their first derivatives at $\eta = \eta_1$, and by using the small argument limit of the Hankel functions, we obtain (for $k\eta_1 < 1$)

$$c_\pm = \frac{1}{2}\left[\gamma(\frac{k\eta_1}{2})^{\nu-\mu} \pm \gamma^{-1}(\frac{k\eta_1}{2})^{\mu-\nu}\right] \tag{3.6}$$

(here $\gamma = \Gamma(\mu)/\Gamma(\nu)$, where Γ is the Euler function, and we have supposed $\mu > 0$, $\nu > 0$ when performing the $k \to 0$ limit).

These coefficients satisfy correctly the Bogoliubov normalization condition, $|c_+|^2 - |c_-|^2 = 1$, and have been obtained in a more particular case[15], and also with a different procedure[6,22], in previous papers. For $k\eta_1 < 1$, we shall keep the dominant term only, ignoring corrections to the sudden approximation near th maximum frequency $k_1 = \eta_1^{-1}$, and neglecting also numerical factors of order unity, which depend on the model of background evolution (continuity of the scale factors and of the dilaton at the transition time), and which do not affect the qualitative behaviour of the spectrum. In the rest of the paper, therefore, we shall use the expression

$$|c_-(k)| = (k\eta_1)^{-|\mu-\nu|} \quad , \quad k_2 < k < k_1 \tag{3.7}$$

where $k_2 = 1/\eta_2$ is the frequency corresponding to the height of the barrier $V(\eta_2)$.

Lower frequency modes, $k < k_2$, are affected also by the second background transition, at $\eta = \eta_2$, from the radiation to the matter-dominated regime[3,5,6]. In this frequency sector the Bogoliubov coefficients are given by $c_\pm = B_\pm/C$, and the matching condition provide, for $k\eta_2 < 1$,

$$c_\pm(k) = \frac{1}{2}\left[\gamma_1(\frac{k\eta_1}{2})^{\nu-\mu}\gamma_2(\frac{k\eta_2}{2})^{\mu-\sigma} \pm \gamma_1^{-1}(\frac{k\eta_1}{2})^{\mu-\nu}\gamma_2^{-1}(\frac{k\eta_2}{2})^{\sigma-\mu}\right] \qquad (3.8)$$

where $\gamma_2 = \Gamma(\sigma)/\Gamma(\mu)$ for $\mu > 0$ and $\sigma > 0$. It may be useful to note that the expression can be easily generalized , by performing the product of n Bogoliubov transformations, to the case of n background transitions, at $\eta = \eta_i$, between the mode solutions H_{ν_i} and $H_{\nu_{i+1}}$, with $i = 1, 2, ..., n$. One finds, in general[22],

$$c_\pm^{(n)} = \frac{N}{2}\prod_{i=1}^{n}\left[\gamma_i(\frac{k\eta_i}{2})^{\nu_i-\nu_{i+1}} \pm \gamma_i^{-1}(\frac{k\eta_i}{2})^{\nu_{i+1}-\nu_i}\right] \qquad (3.9)$$

where γ_i are numerical factors of order unity, and $N^* = N^{-1}$ is an overall constant phase factor.

In order to keep only the dominant term of eq.(3.8), for $k < k_2 \ll k_1$, we have to note first of all that the phenomenological constraints on the time variation of the fundamental constant (including G), during the matter and radiation-dominated era, imply $\sigma - \mu < 0$ (see Sec.5). If $\mu - \nu < 0$ (as seems to be indeed the case for all the appropriate models of background evolution, see Sec.6), the second term on the r.h.s. of eq.(3.8) is the dominant one. If, on the contrary, $\mu - \nu > 0$, then the first term is dominant (for realistic values of η_1 and η_2). We shall thus use, for the graviton production at low frequencies,

$$|c_-(k)| \simeq (k\eta_1)^{-|\mu-\nu|}(k\eta_2)^{\mp|\sigma-\mu|} \quad , \qquad k_0 < k < k_2 \qquad (3.10)$$

where the $-(+)$ sign refers to $\mu - \nu < 0$ (> 0), and k_0 is the minimal amplified frequency[3,5] emerging to-day from the barrier (otherwise stated: crossing to-day the Hubble radius H_0^{-1}), namely $k_0 = a_0 H_0$.

The final number of produced gravitons, for each mode k, is given by $|c_-(k)|^2$. The corresponding energy density ρ_g, in the proper frequency interval $d\omega$, is obtained by summing over the two polarization states, and is related to c_- by[5,7]

$$d\rho_g = 2\omega|c_-|^2 4\pi\omega^2\frac{d\omega}{(2\pi)^3} \qquad (3.11)$$

The spectral energy density $\rho(\omega) = \omega d\rho_g/d\omega$, which is the variable usually adopted[3,5-7] to characterize the graviton energy distribution, turns out then to be parametrized as follows

$$\rho(\omega) \simeq \omega^4(k\eta_1)^{-2|\mu-\nu|} \quad , \qquad k_2 < k < k_1$$

$$\rho(\omega) \simeq \omega^4 (k\eta_1)^{-2|\mu-\nu|} (k\eta_2)^{\mp 2|\sigma-\mu|} \quad , \quad k_0 < k < k_2 \tag{3.12}$$

For later comparison with present observational data , it is convenient to replace all comoving frequencies k by the associated proper frequency $\omega = k/a(t)$, and to express the spectral distribution in terms of the final curvature scale $H_1 \equiv H(\eta_1)$, reached at the end of the inflationary phase, $H_1 \simeq (a_1\eta_1)^{-1} = \omega_1$. Since in our model η_1 is also the beginning of the radiation-dominated evolution for $a(t)$, it follows that H can be expressed in terms of the radiation energy density ρ_γ, as

$$H_1^2 \simeq \left(\frac{k_1}{a_1}\right)^2 \simeq G\rho_\gamma(\eta_1) \quad , \quad G\rho_\gamma(t) \simeq \left(\frac{k_1}{a_1}\right)^2 \left(\frac{a_1}{a(t)}\right)^4 \tag{3.13}$$

Note that we have used the Newton constant $G \simeq M_p^{-2}$ as the effective gravitational coupling during the post-inflationary cosmological evolution; the allowed deviations from this value turn out to be indeed negligible for our determination of the spectral behaviour (see Sec.5).

By using eq.(3.13), and by measuring $\rho(\omega)$ in units of critical energy density ρ_c, the spectral distribution (3.12) can be recast finally in the convenient form $(\Omega(\omega) \equiv \rho(\omega)/\rho_c)$

$$\Omega(\omega, t) \simeq G H_1^2 \Omega_\gamma(t) \left(\frac{\omega}{\omega_1}\right)^{4-2|\mu-\nu|} \quad , \quad \omega_2 < \omega < \omega_1$$

$$\Omega(\omega, t) \simeq G H_1^2 \Omega_\gamma(t) \left(\frac{\omega}{\omega_1}\right)^{4-2|\mu-\nu|} \left(\frac{\omega}{\omega_2}\right)^{\mp 2|\sigma-\mu|} \quad , \quad \omega_0 < \omega < \omega_2 \tag{3.14}$$

where $\Omega_\gamma(t) = \rho_\gamma(t)/\rho_c$ is the fraction of critical energy density present in the form of radiation, at the given observation time t.

This spectrum is parametrized by the scale H_1, and by the kinematical indices μ, ν, σ, which determine its frequency behaviour. It may be interesting to note that the high frequency part of the spectrum is decreasing, flat or increasing depending on whether $|\mu - \nu|$ is larger, equal or smaller than 2. For a primordial phase corresponding to isotropic inflation of $d = 3$ spatial dimensions, with frozen dilaton and internal radius ($\beta = \beta_1 = \gamma = \gamma_1 = 0$), eq.(3.4) gives, in particular, $|\mu - \nu| = 1 + \alpha$, so that the behaviour of the spectrum is the same as that of the curvature scale. For a de Sitter phase ($\alpha = 1$) one recovers indeed the well known flat spectrum[2,23] ($\Omega \simeq const.$), while for superinflation ($0 < \alpha < 1$) one obtain the growing spectrum recently discussed in Ref.22.

In the general case in which $d \neq 3$, and the additional contributions of a dilaton variation (as well as those of dimensional reduction) are included, however, the spectral behaviour may be flat or decreasing even if the curvature is growing. What is important to stress is that, in any case, all observational data and constraints on the present background of cosmic gravitational waves can be translated, thanks to eq.(3.13), into direct information on the curvature scale H_1 (marking the transition from the primordial inflationary phase to the standard decelerated scenario), and on the kinematics of the background evolution. This possibility will be discussed in Sec.5.

We conclude this section with an estimate of the transition frequencies ω_1 and ω_2. At the present time t_0, the minimal proper frequency ω_0 is determined by the to-day value of the Hubble radius, i.e. $\omega_0 = H_0 \sim 10^{-18} Hertz$. The frequency ω_2, corresponding to the matter-radiation transition, can be easily related to ω_0 by noting that $a(t) \sim t^{\frac{2}{3}}$ during the matter-dominated regime, so that

$$\frac{\omega_2}{\omega_0} = \frac{k_2}{k_0} \simeq \frac{H_2 a_2}{H_0 a_0} \simeq \left(\frac{t_0}{t_2}\right)^{\frac{1}{3}} = \left(\frac{a_0}{a_2}\right)^{\frac{1}{2}} \tag{3.15}$$

On the other hand, the radiation temperature evolves adiabatically ($aT = const.$), so that the ratio (3.15) can be expressed in terms of the temperature T_2 at the transition time,

$$\frac{\omega_2}{\omega_0} \simeq \left(\frac{T_2}{T_0}\right)^{\frac{1}{2}} \sim 10^2 \tag{3.16}$$

where $T_0 \sim 1^0 K$ is the present temperature of the radiation background.

In a similar way we can relate ω_0 to the maximal cutoff frequency ω_1, which depends on the final curvature scale H_1. We can put, in fact,

$$\frac{\omega_1}{\omega_0} = \frac{k_1}{k_0} \simeq \frac{H_1 a_1}{H_0 a_0} = \left(\frac{H_1 a_1}{H_2 a_2}\right) \left(\frac{H_2 a_2}{H_0 a_0}\right) \tag{3.17}$$

and we note that, during the radiation dominated evolution, $a \sim t^{\frac{1}{2}} \sim H^{-\frac{1}{2}}$. We have, moreover, $H_2 \sim 10^6 H_0$ and (in units of Planck mass) $H_0 \sim 10^{-61} M_p$; therefore

$$\frac{\omega_1}{\omega_0} \simeq 10^2 \left(\frac{H_1}{M_p}\right)^{\frac{1}{2}} \left(\frac{M_p}{H_0}\right)^{\frac{1}{2}} \left(\frac{H_0}{H_2}\right)^{\frac{1}{2}} \sim 10^{29} \left(\frac{H_1}{M_p}\right)^{\frac{1}{2}} \tag{3.18}$$

4.The squeezing parameter

Another phenomenological signature of the primordial cosmological transitions, encoded into the cosmic gravity-wave background, is the squeezing parameter which characterizes the quantum state of the gravitons produced from the vacuum[13]. This parameter is directly related to the Bogoliubov coefficients, and is thus sensible to all the various components of the production process, including a possible variation of the dilaton background, just like the spectral energy distribution.

The graviton production discussed in the previous section is based on the expansion of the gravitational perturbation in terms of $|in>$ and $|out>$ states, namely

$$\psi(k,\eta) = b\psi_{in} + b^\dagger \psi_{in}^*$$
$$\psi(k,\eta) = a\psi_{out} + a^\dagger \psi_{out}^* \tag{4.1}$$

for each mode k. The two sets of solutions are connected by a Bogoliubov transformation which, when expressed in terms of the "in" and "out" mode solutions, takes the form

$$\begin{pmatrix} \psi_{in} \\ \psi_{in}^* \end{pmatrix} = \begin{pmatrix} c_+ & c_- \\ c_-^* & c_+^* \end{pmatrix} \begin{pmatrix} \psi_{out} \\ \psi_{out}^* \end{pmatrix} \tag{4.2}$$

where $c_\pm$ are defined, according to eq,(3.4), as $c_\pm = B_\pm/C$. The equivalent relation among the corresponding annihilation and creation operators of the second-quantization formalism is then

$$\begin{pmatrix} a \\ a^\dagger \end{pmatrix} = \begin{pmatrix} c_+ & c_-^* \\ c_- & c_+^* \end{pmatrix} \begin{pmatrix} b \\ b^\dagger \end{pmatrix} \tag{4.2}$$

If the Bogoliubov transformation is parametrized by two real numbers, r and θ, in such a way that

$$c_+ = \cosh r \quad , \quad c_-^* = -e^{2i\theta} \sinh r \tag{4.4}$$

the transformation (4.3) can be rewritten as

$$a = S^\dagger b S \quad , \quad a^\dagger = S^\dagger b^\dagger S \tag{4.5}$$

where S is a unitary operator defined by

$$S = \exp\,[\frac{1}{2} z (b^\dagger)^2 - \frac{1}{2} z^* b^2] \qquad , \qquad z = r e^{2i\theta} \tag{4.6}$$

This is a so-called "squeezing" operator: when applied to the vacuum (or, more generally, to a coherent state), generates a state for which the quantum fluctuations of the operator $X \sim b + b^\dagger$ (or its canonical conjugate) can be arbitrarily squeezed for a suitable choice of r (see for instance Ref.(24)). In particular, $\Delta X \to 0$ for $r \to \infty$.

The cosmic gravitons arising from the background transitions are thus produced in a squeezed state, with a parameter r which, according to eq (4.4), is given by

$$r = \ln\left(|c_-| + \sqrt{|c_-|^2+1}\right) \tag{4.7}$$

According to the model of background evolution considered in the previous section, and for $\omega > \omega_2$, the relic graviton background may be characterised, in general, by the following squeezing parameter

$$\begin{aligned} r(\omega) \simeq \ln|c_-| \simeq & -|\mu-\nu|\ln\left(\frac{\omega}{\omega_1}\right) \\ \simeq & |\mu-\nu|\left[25-\ln\left(\frac{\omega}{Hertz}\right)+\frac{1}{2}\ln\left(\frac{H_1}{M_p}\right)\right] \end{aligned} \tag{4.8}$$

(we have used eq.(3.7) for c_-, and the estimate (3.18) for ω_1).

The first term in eq.(4.8) is expected to be the dominant one, at least in the range of frequencies accessible, in a (hopefully) not too distant future, to a direct observation[3,4]. The second term takes into account the variation of r with frequency, and the third term provides a correction if the transition curvature scale is different from the Planck scale. A direct measurement of this parameter, at some definite value of frequency, would provide then significative information both on the curvature scale H_1, and on the background (dilaton included) dynamical evolution, through the $|\mu-\nu|$ dependence.

5.Phenomenological constraints on the graviton spectrum

The present energy distribution of a cosmic gravity-wave background is mainly constrained by three kind of direct observations[3,4]: the absence of fluctuations in the millisecond pulsar-timing data, the critical density value, and the isotropy of the cosmic microwave background radiation (CMBR). The first one applies on a narrow frequency interval around $\omega_p \sim 10^{-8} Hertz$, while the other two at all frequencies (the third one provides a bound which is frequency-dependent). Their relative importance, and the frequency at which they provide the must significant constraint, depend on the slope of the graviton energy spectrum $\Omega(\omega)$.

For a stochastic graviton background, the bound on the spectrum following from the CMBR isotropy constrains the wave amplitude $h(\omega)$, and scales like ω^{-2}. It provides then the most significative bound at the minimum frequency ω_0 (where it implies[3] $\Omega \leq 10^{-8}$), unless we have a spectrum which in its low frequency band ($\omega < \omega_2$) grows faster than ω^2. If the spectrum is growing at all frequencies, however, the most significant constraint, for the present values of experimental data, is provided in any case by the critical density bound $\Omega \lesssim 1$, applied to the highest frequency ω_1.

According to our three component model of background evolution, the spectrum may be increasing at low frequencies, and simultaneously flat or decreasing in the high frequency sector, only if (see eq.(3.14))

$$\mu > \nu \quad , \quad 2 \leq |\mu - \nu| < 2 + |\sigma - \mu| \tag{5.1}$$

Even in such a particular case, however, the growth of the low frequency sector cannot be significantly faster than ω^2, since, as we shall see later, $|\sigma - \mu|$ is not allowed to be notably larger than 1 by the present limits on the variation of the fundamentals constants. Therefore, the energy distribution of the graviton background can be significantly constrained by imposing on eq.(3.14) the three following bounds:

$$\Omega(\omega_1) < \Omega_c \quad , \quad \Omega(\omega_p) < \Omega_p \quad , \quad \Omega(\omega_0) < \Omega_i \tag{5.2}$$

where $\omega_p \sim 10^{-8} Hertz$, and Ω_c, Ω_p, Ω_i are the present value of the bounds on the energy density imposed, respectively, by critical density, pulsar timing data, and CMBR isotropy.

For our discussion of the constraints, it may be convenient to simplify the notations by defining the variables

$$x = |\mu - \nu| \quad , \quad y = |\sigma - \mu| \quad , \quad z = \log\left(\frac{H_1}{M_p}\right) \tag{5.3}$$

By using $\Omega_\gamma(t_0) \sim 10^{-4}$ for the present critical fraction of radiation energy density, and by inserting in eq.(5.2) the values of ω_0, ω_1, ω_2 determined in Sec.3, the three constraint equations in the parameter space, for our model of background evolution, can thus be written

$$z < 2 + \frac{1}{2}\log\Omega_c$$

$$z < \frac{1}{x}(80 + \log\Omega_p) - 38$$

$$z < \frac{1}{x}(120 \mp 4y + \log\Omega_i) - 58 \tag{5.4}$$

They follow, respectively, from the critical density, pulsars and isotropy bounds, and they define an allowed region in the (x, y, z) space which provides information on the past evolution of our Universe.

In order to discuss the extension of this region it should be noted, first of all, that the range of variation of the variable y,

$$y = |\sigma - \mu| = \frac{1}{2}|n_1(\beta_2 - \beta_1) + 2\gamma_2 - \gamma_1 - 2| \tag{5.5}$$

which parametrizes the time evolution of the dilaton and of the compactification radius during the matter and radiation dominated era (recall eq.(3.2)), is severely constrained by the present bounds on the variation of the fundamental constants.

Indeed, in a Brans-Dicke frame, and in a higher dimensional context with $n = D - 4$ dimensions lying in a compact internal space, with scale factor $b(t)$, the effective four dimensional Newton constant G_N evolves in time like $G_N \sim e^{\phi}/b^n$. We have then

$$\frac{\dot{G}_N}{G_N} = \dot{\phi} - n\frac{\dot{b}}{b} \tag{5.6}$$

During the matter dominated era the variation of the extra spatial dimensions is constrained by[25]

$$|\dot{b}/b| \leq 10^{-9} H_0 \tag{5.7}$$

and the variation of G_N by[26]

$$|\dot{G}_N/G_N| < 10^{-1} H_0 \tag{5.8}$$

where we have taken for H_0 the largest value allowed to-day, $H_0 \simeq 10^{-10} yr^{-1}$. These two bounds imply $|\dot{\phi}| < 10^{-1} H_0$. But, according to our parametrization (3.2), $\dot{\phi} = \gamma_2 H$ and $\dot{b}/b = -\beta_2 H/2$. It follows that

$$|\beta_2| \leq 10^{-9} \quad , \quad |\gamma_2| < 10^{-1} \tag{5.9}$$

Consider now the radiation-dominated era. During this phase, the best limits on $\dot{\phi}$ and $\dot{b}$ are obtained from the primordial nucleosynthesis. Denoting by b_{nucl}, G_{nucl}, and by b_0, G_0, the values of the radius of the internal space and of the Newton constant, at the epoch of nucleosynthesis and at the present epoch, respectively, one obtains that the change of b must be bounded by[25,27]

$$\frac{b_{nucl}}{b_0} = 1 + \epsilon \quad , \quad |\epsilon| < 10^{-2} \tag{5.10}$$

while the change of G is constrained by[28]

$$\frac{G_{nucl}}{G_0} = 1 + \epsilon \quad , \quad |\epsilon| < 3 \times 10^{-1} \tag{5.11}$$

Translated into limits on the time variation of b and ϕ, according to the parametrization (3.2), they imply

$$|\beta_1| < 10^{-3} \quad , \quad |\gamma_1| < 10^{-1} \tag{5.12}$$

The dilaton contribution is thus the dominant source of uncertainty in the value of the parameter y. Even taking into account the maximum allowed uncertainty, however, it follows from eqs.(5.9) and (5.12) that

$$0.9 \leq y \leq 1.1 \tag{5.13}$$

A first rough evaluation of the allowed region in the parameter space is thus obtained by fixing $y = 1$ in eqs.(5.4) (the allowed deviation of y from 1 is too small to be significant in view of our previous approximations).

We have to insert, moreover, in eq.(5.4) the values of the bounds implied by the present experimental data. We shall put $\Omega_c = 1$ (in order to avoid that the produced gravitons over-close our present universe), $\Omega_p = 10^{-6}$ as implied (at the 99% confidence level) by recent results from pulsar timing[29], and $\Omega_i = 10^{-8}$, following from the constraint [30] $h < 10^{-5}$ on the gravity-wave amplitude. With this data, the constraint eqs.(5.4) become

$$\begin{gathered} z < 2 \\ z < \frac{74}{x} - 38 \\ z < \frac{1}{x}(112 \mp 4) - 58 \end{gathered} \tag{5.14}$$

We recall that the negative (positive) sign in the last equation corresponds to $\mu < \nu$ ($\mu > \nu$). It should be mentioned, moreover, that in the context of a more stringent analysis, the first bound $z < 2$ could be replaced by $z < 0$, following from the fact that early nucleosynthesis seems to imply[31] , at high frequency, $\Omega < 10^{-4}$ for the energy density distribution of massless particles. This would correspond to a maximum scale $H_1 < M_p$ instead of $10^2 M_p$. This conclusion is, however, model dependent, and in this paper we prefer to rely on constraints following directly from observations.

The allowed region of the (x, z) plane delimited by eqs.(5.14) is illustrated in **Fig.1**. Because of the uncertainty of the experimental data, which has not been completely taken into account in our discussion, and because of the approximations made, this figure is expected to give only a qualitative picture of the phenomenological scenario. Nevertheless, we can draw from our analysis the following general conclusions.

1) There is a maximum allowed value for the curvature scale H_1 at the epoch of the transition from the phase of accelerated expansion, dilaton growth and dimensional reduction, to the decelerated radiation-driven evolution,i.e. $H_1 \leq 10^2 M_p$.

2) Models characterised by a sufficiently high scale, $H_1 \geq 10^{-2} M_p$, are constrained by pulsar timing if $\mu \geq \nu$, and by CMBR isotropy if $\mu \leq \nu$.

3) For any given scale H_1 lower than the maximum one there is a limiting slope of the spectrum, below which that scale is forbidden. Within our approximations, the limiting slope for a scale H_1 is fixed by

$$x < \frac{108}{58 + \log\left(\frac{H_1}{M_p}\right)} \tag{5.15}$$

if $\mu < \nu$, and

$$x < \frac{74}{38 + \log\left(\frac{H_1}{M_p}\right)} \tag{5.16}$$

if $\mu > \nu$. In the first case (which corresponds to all the physical models considered in the next section), the maximum scale $10^2 M_p$ is allowed for $x \lesssim 1.8$, while the Planck scale can be reached for $x \lesssim 54/29 \simeq 1.86$. A four dimensional inflationary background, with frozen dilaton and radius of the internal dimensions ($\gamma = \beta = 0$, $d = 3$), corresponds in particular to $\mu - \nu = -\alpha - 1 < 0$, and the Planck scale is thus reached for $\alpha \lesssim 25/29$, in agreement with the results of a previous analysis[22].

4) Finally, models corresponding to a spectrum which is flat or decreasing at high frequencies (i.e. with $x \geq 2$), are characterized by a maximum allowed scale $H_1 \leq 10^{-4} M_p$. We thus recover the well known bound on the scale of a four dimensional de Sitter inflation[2,22], since in that case $x = |\alpha + 1| = 2$ and one obtains the usual flat spectrum.

6. String cosmology pre-big-bang and other higher-dimensional models

In the standard cosmological model the curvature is monotonically increasing as we go back in time, and blows up at the initial singularity. A possible classical alternative to the singularity would seem to be provided by an initial inflationary de Sitter phase, at constant curvature, which extends in time indefinitely toward the past. However, as discussed in a recent paper [32], eternal exponential expansion, with no beginning, is impossible in the context of the conventional inflationary scenario, so that a primordial phase of constant curvature does not help to solve the problem of the initial singularity. Moreover, according to the constraints reported in the previous section, the constant value of the curvature during the initial de Sitter phase should lie at least four orders of magnitude below the Planck scale; this may seem unnatural, if one believes that the growth of the curvature is stopped and that the primordial curvature becomes stable just because of quantum effects.

A different alternative has been recently suggested, on the grounds of string theory motivations[10,12,33], in which the singularity is avoided because the curvature grows up to a maximum (Planckian) value and then decreases back to zero. The standard radiation-dominated phase is then preceeded in time by a phase with "dual" dynamical behaviour (the curvature and the dilaton are growing, $\dot{H} > 0$, $\dot{\phi} > 0$, the evolution is accelerated, $\ddot{a} > 0$), called[12] "pre-big-bang". Particular examples of such a scenario are thus provided also by earlier models of superinflation and dynamical dimensional reduction, discussed in the context of Kaluza-Klein cosmology[34−36].

In this section we want to stress that if the initial configuration of our model of background evolution (i.e. for $\eta < -\eta_1$, see eq(3.11)) corresponds to a pre-big-bang scenario of this type, the consequent graviton spectrum is always growing fast enough to avoid the de Sitter bound $H < 10^{-4} M_p$ (i.e. $x < 2$), and to allow the Universe to inflate up to the maximal curvature scale, consistently with the bounds of the previous section.

Consider indeed the perfect-fluid dominated model of Refs.(34) and (35), which describes superinflation and dimensional decoupling, and belongs to the class parametrized by eq.(3.1) with $\gamma = 0$. One finds, for this model, $\mu - \nu = -\frac{1}{2}$, and $x = |\mu - \nu| = 0.5$. The model of Ref.(36) (based on the toroidal compactification of D=11 supergravity), corresponds to $\gamma = 0$, $\alpha = 0.26$ and $\beta = 0.22$, and gives $\mu - \nu = -0.49 < 0$. The model of string-driven inflation of Ref.(33)

has $\gamma = 0$, $n > 10$, and for $d = 3$ it gives $\mu - \nu = (4 - n)/3n < 0$. Finally, a typical pre-big-bang model[12], dual to the standard radiation phase, satisfies the Brans-Dicke equations (2.2) and (2.3) (with $\omega = 1$) for

$$\gamma = 2d \quad , \quad \alpha = \frac{2}{3 + d + n} = \beta \tag{6.1}$$

and implies

$$\mu - \nu = \frac{-2}{3 + d + n} < 0 \tag{6.2}$$

For all these models we have $\mu - \nu < 0$, and $|\mu - \nu| < 1.8$ (for any allowed number of internal dimensions), so that their final curvature scale is only constrained by the closure density bound.

We want to comment, finally, on the possibility that the CMBR anisotropy recently measured[37] by COBE be at least partially determined, at the quadrupole level, by a cosmic graviton background. It has been already pointed out[38], indeed, that a stochastic background of gravitational waves with flat spectrum, generated by a primordial de Sitter inflationary phase, could produce the entire observed signal, provided de Sitter inflation occurred at a vacuum energy scale $M_P v^{1/4} \simeq 1.5 \times 10^{16}$ GeV (at the 95% confidence level). This translates into a value of the Hubble constant $H = (8\pi M_P/3) v^{1/2} \sim 10^{-5} M_P$, which is not in conflict with the previously reported bound ($H_1 \lesssim 10^{-4}$ for $x = 2$, see **Fig.1**).

It should be noted, however, that a four-dimensional de Sitter inflation is not the only primordial phase which can be associated to a flat graviton spectrum. Indeed, in a more general higher-dimensional Brans-Dicke scenario, all the models with $|\mu - \nu| = 2$ provide a flat high frequency spectrum. Included in this class, in particular, are all the $(d + 1)$-dimensional models providing a phase with variable dilaton and isotropic superinflationary expansion, characterized in conformal time (according to eq.(3.1)) by the power $\alpha = 2/(d - 1 - \gamma)$.

It remains still open, moreover, the interesting possibility that the COBE anisotropy may be fitted a by non-flat graviton spectrum[39] with, in particular, $x < 2$, as predicted by the string pre-big-bang models. In this case we may expect, according to **Fig.1**, that the COBE data will select a higher transition scale H_1, and in such a case they could be interpreted, instead of a first direct evidence, via gravitational waves, for the GUT scenario[38], as evidence for the dilaton-driven string cosmology scenario. In order to discriminate between these two (exciting) possible interpretations, however, one should try to probe directly the energy density of the cosmic graviton background at some given frequency, for example through a gravity wave detector (such as LIGO[4]), or by means of astrophysical methods (such as timing measurements of millisecond pulsars[29]).

7. Conclusions

In this paper we have considered a three-component model of cosmological evolution in which the standard radiation and matter-dominated expansion of the three-dimensional space is preceeded in time by a general d-dimensional phase of accelerated (i.e. inflationary) expansion. We have included, moreover, a possible variation of the effective gravitational coupling and of the compactification scale, parametrized, respectively by a logarithmic time dependence of the dilaton field, and by a power law evolution of the internal scale factor.

We have shown that the linearised equation for a metric fluctuation, obtained by perturbing the Brans-Dicke equations around this background, contains a coupling of the perturbation to the background metric and to the dilaton field ϕ. As a consequence, both the dimensional reduction process and the variation of G (via $\dot{\phi}$) contribute (besides inflation) to the process of the amplification of the gravitational perturbations (i.e. to the graviton production).

We have computed the spectral distribution $\Omega(\omega)$ of the energy density stored to-day in a cosmic graviton background (and the associated squeezing parameter $r(\omega)$), taking into account all possible contributions. The frequency behaviour of the spectrum turns out to be clearly related to the temporal behaviour of the background fields ($g_{\mu\nu}$ and ϕ); the observational constraints on $\Omega(\omega)$ provide then significative information both on the kinematics of the background evolution, and on the curvature scale H_1 characterizing the transition from the primordial inflationary phase (with variable dilaton), and the standard radiation-dominated phase.

We have shown, in particular, that for flat or decreasing spectra the transition scale cannot overcome a maximum value which lies, typically, four orders of magnitude below the Planck scale. For growing spectra, on the contrary, the allowed transition scale can be as high as the Planck one (and somewhat higher).

We have stressed, finally, that the contribution of the dilaton background to the cosmic production of gravitons may simulate the usual flat four-dimensional de Sitter spectrum, even if the inflationary evolution of the scale factor is not of the exponential type, and the curvature scale is growing, instead of constant, during the inflation. As a consequence, one could try to interpret the recently measured COBE anisotropy not only as evidence for de Sitter inflation at the GUT scale[38], but also (alternatively) as a possible evidence for a dilaton-driven string cosmology scenario[10,12].

Acknowledgments

We are very grateful to G.Veneziano for many useful discussions.

References

1. L.P.Grishchuk, Sov.Phys.JETP 40,409(1975);
   L.P.Grishchuk and A.Polnarev, in"General Relativity and Gravitation", ed. by A.Held (Plenum, New York, 1980) ol.2, p.393
2. V.A.Rubakov, M.V.Sazhin and A.V.Veryaskin, Phys.Lett.B115, 189(1992);
   R.Fabbri and M.D.Pollock, Phys.Lett.B125,445(1983);
   L.F.Abbott and M.B.Wise, Nucl.Phys.B244,541(1984)
3. L.P.Grishchuk, Sov.Phys.Usp.31,940(1988);
   Sov.Sci.Rev.E.Astrophys.Space Phys.7,267(1988).
4. K.S.Thorne, in"300Years of Gravitation", ed.by S.W.Hawking and W.Israel (Cambridge Univ.Press,Cambridge 1988) p.330
5. B.Allen, Phys.Rev.D37,2078(1988)
6. V.Shani, Phys.Rev.D42,453(1990)
7. L.P.Grishchuk and M.Solokhin, Phys.Rev.D43,2566(1991)
8. J.Garriga and E.Verdaguer, Phys.Rev.D39,1072(1991)
9. M.Demianski, in Proc. of the 9th Italian Conference on General Relativity and Gravitational Physics (Capri,1990), ed. by R.Cianci et al., (World Scientific, Singapore, 1991) p.19;
   L.Amendola, M.Litterio and F.Occhionero, Phys.Lett.B237,348(1990);
   M.Gasperini and M.Giovannini, Class.Quantum Grav.9,L137(1992)
10. G.Veneziano, Phys.Lett.B265,287(1991);

M.Gasperini, J.Maharana and G.Veneziano, Phys.Lett.B272,277(1991); M.Gasperini and G.Veneziano, Phys.Lett.B277,256(1992)

11. A.A.Tseytlin and C.Vafa, Nucl.Phys.B372,443(1992); A.A.Tseytlin, Class.Quantum Grav.9,979(1992)
12. M.Gasperini and G.Veneziano,"Pre-big-bang in string cosmology", CERN-TH. 6572/92 (July 1992)
13. L.P.Grishchuk and Y.V.Sidorov, Class.Quantum Grav.6,L161(1989); Phys.Rev.D42,3413(1990); L.P.Grishchuk, in Proc.of the Workshop"Squeezed states and uncertainty relations", University of Maryland, ed. by D.Han, Y.S.Kim and W.W.Zachary (Nasa Conference Pub.N.31353, 1992) p.329
14. E.M.Lifshitz, Zh.Eksp.Teor.Phys.16,587(1946); E.M.Lifshitz and I.M.Khalatnikov, Adv.of Phys.12,208(63)
15. L.H.Ford and L.Parker, Phys.Rev.D16,1601(1977)
16. K.A.Meissner and G.Veneziano, Phys.Lett.B267,33(1991); Mod.Phys.Lett.A6,3397(1991)
17. L.D.Landau and E.M.Lifshitz, Teoria dei Campi (Editori Riuniti, Roma, 1985) §115, p.486
18. B.L.Hu, Phys.Rev.D18,969(1978)
19. M.Abramowitz and I.A.Stegun, "Handbook of mathematical functions" (Dover, New York, 1972)
20. N.D.Birrel and P.C.W.Davies, "Quantum fields in curved space" (Cambridge Univ.Press, Cambridge, 1982)
21. E.Verdaguer, "Gravitational particle creation in the early Universe" UAB-FT-276(November 1991)
22. M.Gasperini and M.Giovannini, Phys.Lett.B282,36(1992)
23. A.A.Starobynski, JETP Lett.30,682(1979)
24. J.Grochmalicki and M.Lewnstein, Phys.Rep.208,189(1991)
25. J.D.Barrow, Phys.Rev.D.35,1805(1987)
26. R.W.Hellings, in Proc.of the 10th Course of the Int.School of Cosmology and Gravitation (Erice, 1987),ed. by V.De Sabbata and V.N.Melnikov (Kluwer Acad.Pub.,Dordrecht,1988) p.215
27. E.W.Kolb, M.J.Perry and T.P.Walker, Phys.Rev.D33,869(1986)
28. F.S.Accetta, L.M.Krauss and P.Romanelli, Phys.Lett.B248, 146(1990)
29. D.R.Stinebring et al., Phys.Rev.Lett.65,285(1990)

30. G.F.Smoot, in Proc.of the First Course of the Int. School of Astrophysics D.Chalonge, (Erice,1991) ed. by N.Sanchez (World Scientific, Singapore)
31. V.F.Schwartzaman, JETP Lett.9,184(1969)
32. A.Vilenkin,"Did the Universe have a beginning?", CALT-68-1772(1992)
33. M.Gasperini, N.Sanchez and G.Veneziano, Nucl.Phys.B364,365(1991)
34. R.B.Abbott, S.M.Barr and S.D.Ellis, Phys.Rev.D30,720(1984)
35. D.Shadev, Phys.Rev.D39,3155(1989); Phys.Rev.D30,2495(1984)
36. R.G.Morhouse and J.Nixon, Nucl.Phys.B261,172(1985)
37. G.F.Smoot et al.,"Structure with COBE DMR first Year Map", COBE Preprint (21 April 1992)
38. L.M.Krauss and M.White, "Grand Unification, gravitational waves, and the cosmic microwave background anisotropy", YCTP-P15-92 (April 1992)
39. T.Souradeep and V.Sanhi, "Density perturbations, gravity waves and the cosmic microwave background", IUCAA Preprint (July 1992);
F.Lucchin, S.Matarrese and S.Mollerach, "The gravitational wave contribution to CMB anisotropies and the amplitude of mass fluctuations from COBE results", Fermilab-Pub-92/185-A (July 1982);
M.Gasperini and M.Giovannini, in preparation

Figure caption

Fig.1

The maximum allowed value of the transition scale H_1 (in units of Planck mass), versus the parameters determining the kinematics of the background evolution. The allowed region with $H_1 \lesssim 10^2 M_p$ extends from $x = 1.8$ down to $x = 0$.