Rudolf Peierls Centre for Theoretical Physics

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Studying complex fluids with lattice Boltzmann methods

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Numerical simulation of real fluids

CFD methods:

-construction of fluid equations(NS)

-discretisation of PDEs

-numerical integration

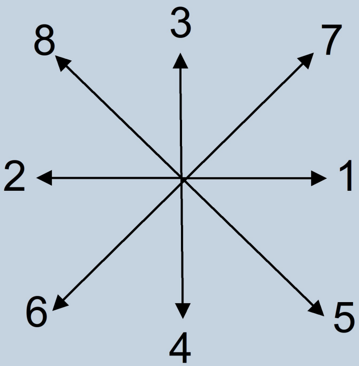
Lattice based methods:

-discrete kinetic theory (no further approximations)

-numerical integration

-conversion to fluid variables

The Lattice Boltzmann method



$f_i(\vec{x}, t)$ = particle density at site x at time t with i-th velocity

$v_i = (v_{0,} v_1 \ldots v_8)$ = set of discrete velocities

The evolution equation is the BGK equation:

$$f_i(\vec{x} + v_i \Delta t, t + \Delta t) - f_i(\vec{x}, t) = \frac{-1}{\tau}\left(f_i(\vec{x}, t) - f_i^{eq}(\vec{x}, t)\right)$$

The Lattice Boltzmann method-2

Real fluid quantities: $\rho(\vec{x},t)=\sum_i f_i(\vec{x},t)$

$$\rho\vec{u}=\sum_i f_i\vec{v}_i \qquad P_{(\alpha\beta)}=\sum_i f_i v_{(i\alpha)} v_{(i\beta)}$$

Equilibrium functions:

$$f_i^{eq}=a_i+b_i\frac{\vec{v}_i\cdot\vec{u}}{c_s^2}+c_i\frac{(\vec{v}_i\cdot\vec{u})^2}{c_s^4}$$

valid for low Mach number $u\ll c_s$

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Advantages

-easy implementation

-easy parallelization (local algorithm)

-easy treatment of boundary conditions

Complex fluids

E.g. binary fluids, Landau-type free energy:

$$F = \int \frac{a}{2}\varphi^2 + \frac{b}{4}\varphi^4 + \frac{\kappa}{2}|\nabla\varphi|^2 \, d^3x$$

In this case we need two distribution functions f and g

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Thank you!