Random Matrix Theory: Quantum Transport in chaotic cavities

Bari Theory Xmas Workshop 2013

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Introduction and Motivations

(My future work at Laboratoire de Physique Théorique et Modèles Statistiques Paris-Orsay with Pierpaolo Vivo)

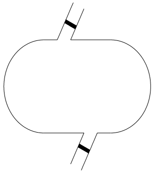
The Random Matrix Theory of Quantum Transport

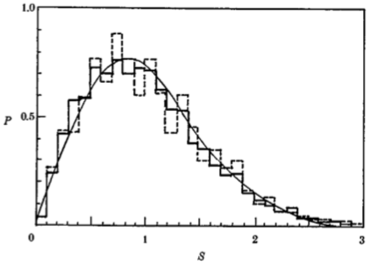
Statistics of transport of mesoscopic systems $\leftrightarrow$ statistics of random matrices.

Mesoscopic physics: quantum mechanical treatement + statistical approach

Fundamental problems which occur when a macroscopic object is miniaturized.

Chaotic cavities:

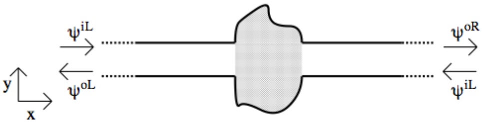




Quantum billiard: a billiard table which small ($\sim 1\mu m$), completely smooth and flat. Billiard balls shoot over this table, colliding elastically with the walls until they disappear into one of the pockets.The billiard balls are the electrons.

Scattering Matrix

Disordered region connected by ideal leads to two electron reservoirs.



The scattering matrix S relates the amplitudes of incoming and outgoing waves:

$$c^{\text{out}} = S c^{\text{in}} \ ,$$

$$S = \begin{pmatrix} r & t' \\ t & r' \end{pmatrix} .$$

Current conservation (unitarity): $SS^\dagger = I$.

Landauer formula: the scattering matrix directly determines the conductance

$$G = \lim_{\Delta V \downarrow 0} \frac{\bar{I}}{\Delta V} \qquad \boxed{G = G_0 \mathrm{Tr}\, tt^\dagger} \qquad G_0 = \frac{2e^2}{h} \simeq (13\, k\Omega)^{-1}$$

Random Matrix Theory of the Scattering Matrix

Idea: promote the scattering matrix $S = \begin{pmatrix} r & t' \\ t & r' \end{pmatrix}$ to a random matrix.

Maximum entropy argument $\rightarrow$ $\boxed{\Pr(S) = \text{constant}}$

Transmission matrix $T = tt^\dagger$ with eigenvalues $0 \leq T_i \leq 1$:

$$\boxed{P(T_1, \ldots, T_N) = \frac{1}{Z} \prod_{j<k} |T_j - T_k|^\beta \prod_i T_i^{\beta/2-1}}$$

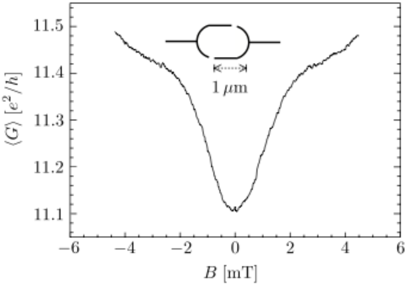
Dyson's threefold way: $\beta = 1, 2, 4$ ($S =$ orthogonal, unitary, symplectic) depending on the presence or absence of time-reversal and spin-rotation symmetry.
Experimental observables are linear statistics of the eigenvalues $A = \sum_i f(T_i)$:

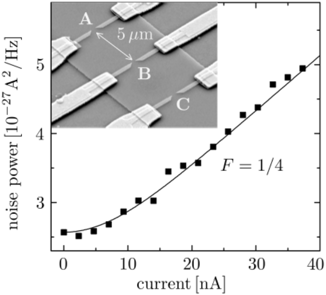
$$\begin{aligned} \text{Conductance} \quad G &= \sum_i T_i \\ \text{Shot noise} \quad P &= \sum_i T_i(1 - T_i) \\ \text{Fano factor} \quad F &= \frac{\text{Shot noise}}{\text{Conductance}} \end{aligned}$$

What is known

Weak localization and Universality Conductance Fluctuation

$$\boxed{\frac{\langle G\rangle}{G_0} = \frac{N}{2} + \frac{\beta-2}{4\beta} + O\left(N^{-1}\right)} \quad \text{(weak localization)}\,, \qquad \boxed{\mathrm{Var}\,\frac{G}{G_0} = \frac{1}{8\beta}} \quad \text{(UCF)}\,.$$





$$\boxed{\rho(T) = \frac{N}{\pi\sqrt{T(1-T)}}} \qquad \text{Transmission eigenvalue density } (N \gg 1)$$

$$\boxed{F \longrightarrow 1 - \frac{\int T^2\rho(T)\mathrm{d}T}{\int T\rho(T)\mathrm{d}T} = \frac{1}{4}} \qquad \text{(subpoissonian Fano factor)}$$

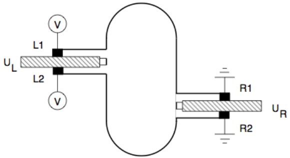
Possible directions

Non-ideal cavities: S is no longer uniformly distributed! $P(S) = \text{Poisson kernel.}$

$$P(S) \propto \left|\det(1 - S_0^\dagger S)\right|^{-(\beta N + 2 - \beta)}, \quad \text{where} \quad S_o = \langle S \rangle \ .$$

Weakly non-ideal leads (tunneling probabilities $\Gamma_i \lesssim 1$): pertubative methods in different regimes.

Entanglement: A quantum dot as an orbital entangler.



Entanglement in terms of transmission eigenvalues $\mathcal{E} = \mathcal{E}(T_1, T_2)$.

Optics: Absorbing random media, Grey-body radiation, Chaotic laser cavities.

References and Figures: M. V. Berry (Proc. R. Soc. **A414**, 1987), P. W. Brouwer (PhD Thesis, 1997), C. W. J. Beenakker et al. (Fundamental Problems of Mesoscopic Physics, 2004), P. Vivo et al. (PRL **101**, 2008), F. D. Cunden et al. (Eur. Phys. J. Plus **128**, 2013).

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