The Abdus Salam International Centre for Theoretical Physics

Bari Theory Xmas Workshop 2013

Myself and my research in 15 minutes:

Adversarial SAT: Climbing the Hierarchy of Complexity

arXiv: 1310.0967

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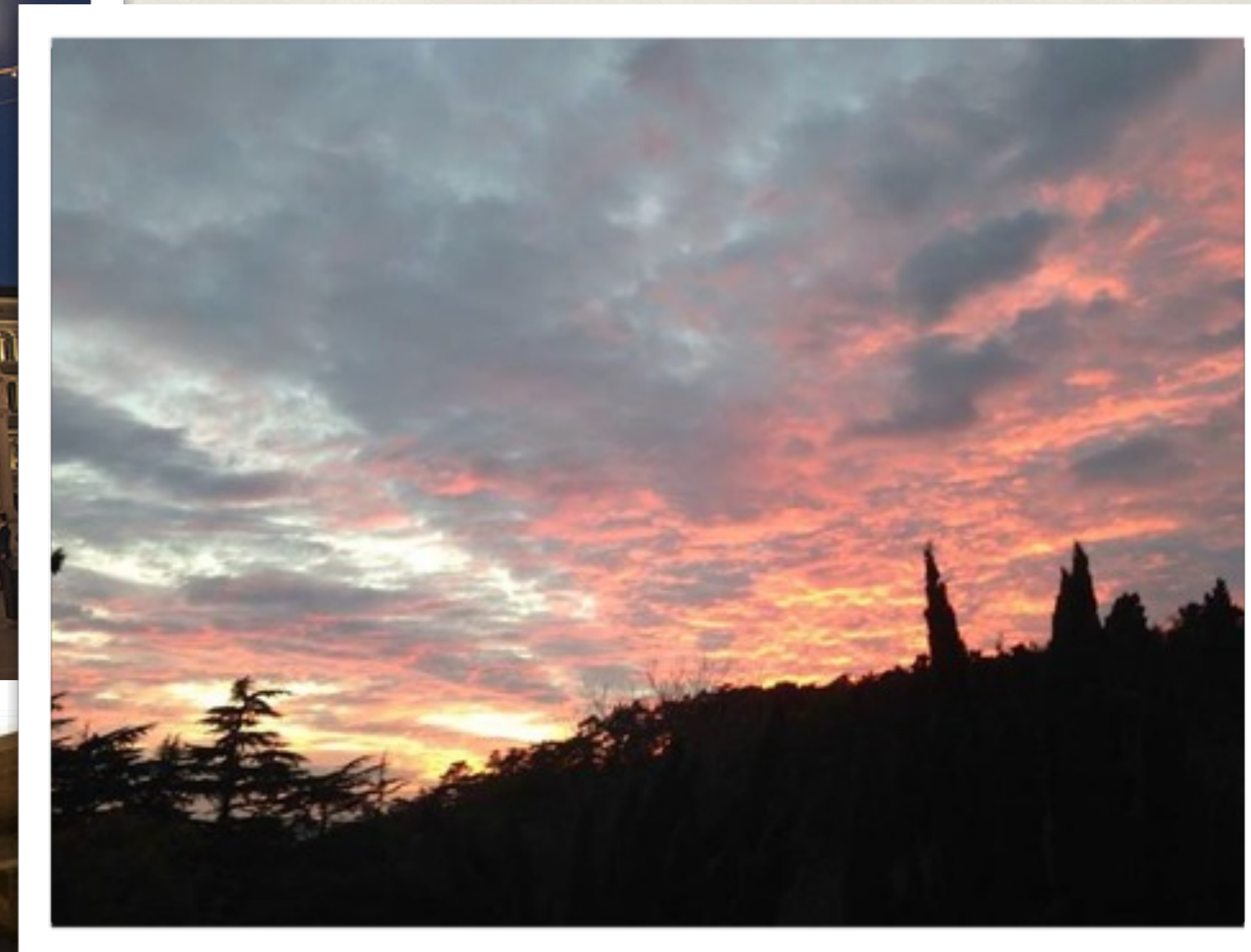


Research activity

Computational problems (AdSAT):

- Antonello Scardicchio (ICTP)
- Daniel Nagaj (University of Vienna, Slovak Academy of Sciences)

Intermediate Goods:

- Giacomo Livan (ICTP)
- Matteo Marsili (ICTP)

Evolution and Metabolic Networks:

- Areejit Samal (ICTP, Berkeley)
- Matteo Marsili (ICTP)

Outline

- What is Complexity?
- Not easy: K-SAT
- Even more difficult: AdSAT
- Random AdSAT
- Conclusions

Complexity Classes

- A complexity class is a set of functions that can be computed (for example by a Turing machine) within given resource bounds.

- We focus on decision problems: functions whose output can be either true or false. Is $2^{57\,885\,161} - 1$ a prime number?

- Easy: a problem is in **P** if, in the worst case, it can be solved in a time scaling polynomially with the size of the problem:
  - given two vertices in a graph, does a path connecting them exist?
  - Königsberg bridges: does a path crossing all the bridges exactly once exist (Eulerian path)?

Complexity Classes

- Hard problems: a problem is in **NP** if, in the worst case, it is possible to verify in a time scaling polynomially with the size of the problem if a candidate solution is actually a solution:
  - integer factorization: given two integers *n* and *k*, does a divisor of *n* smaller than *k* exist?
- Harder problems: a problem is in **NPC** if it is in **NP** and if any other problem in **NP** can be polynomially reduced to it:
  - graph coloring: is there a way to color all the vertices of a graph such that there are not neighbors of the same color?
  - Super Mario Bros (?!?)
  - SAT…

Boolean K-Satisfiability

Given N boolean variables in the K-CNF form (here $K = 3$, $N = 7$, $M = 4$):

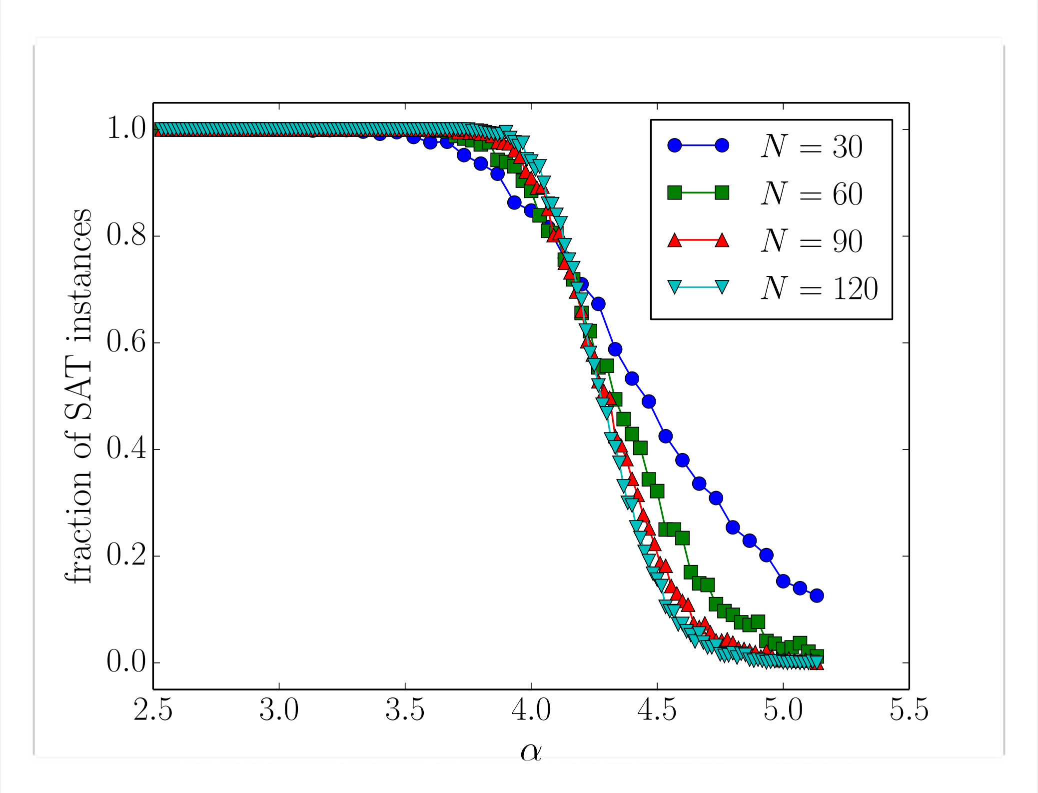
$$\phi(\vec{x}) = (x_1 \vee x_2 \vee \bar{x}_3) \wedge (x_4 \vee \bar{x}_2 \vee x_5) \wedge (\bar{x}_1 \vee x_6 \vee \bar{x}_4) \wedge (x_7 \vee \bar{x}_3 \vee x_2)$$

it can be either satisfiable or unsatisfiable:

$$\exists \vec{x} \ \text{ s.t. } \ \phi(\vec{x}) = 1 \ \Leftrightarrow \ \phi \in L_{\mathrm{SAT}}$$

$$\forall \vec{x} \ \ \phi(\vec{x}) = 0 \ \Leftrightarrow \ \phi \in L_{\mathrm{UNSAT}}$$

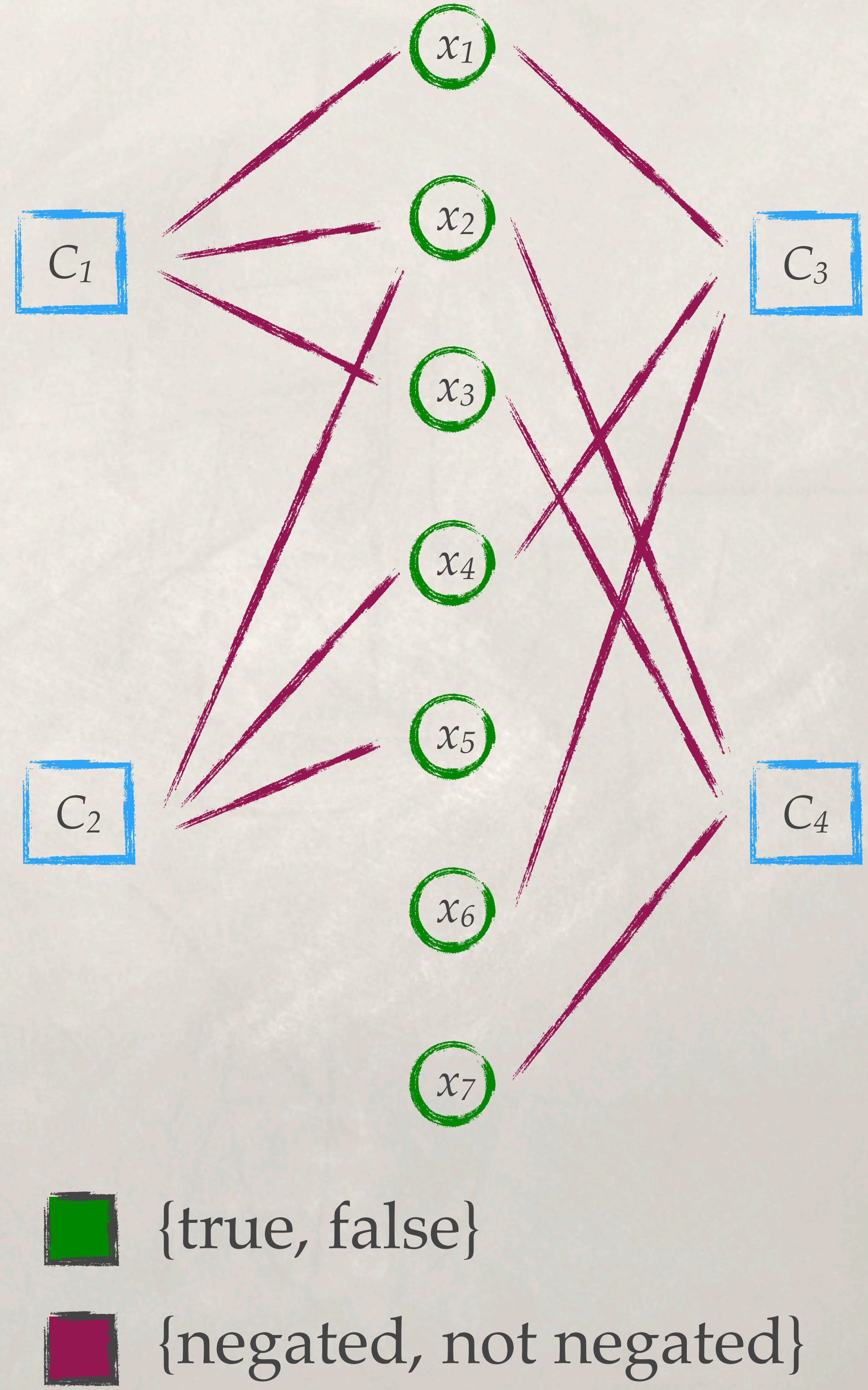
If we take random 3-CNF expressions:



sharp transition at $\alpha_c \simeq 4.27$

Adversarial Satisfiability

- Two players: the *positive* controls the *N* boolean variables, the *negative* controls the *KM* negations.
- The positive wins if, for all the possible configurations of negations, he is able to make the formula SAT.
- The negative wins if he finds a set of negations such that, no matter what the positive does, the formula is UNSAT.



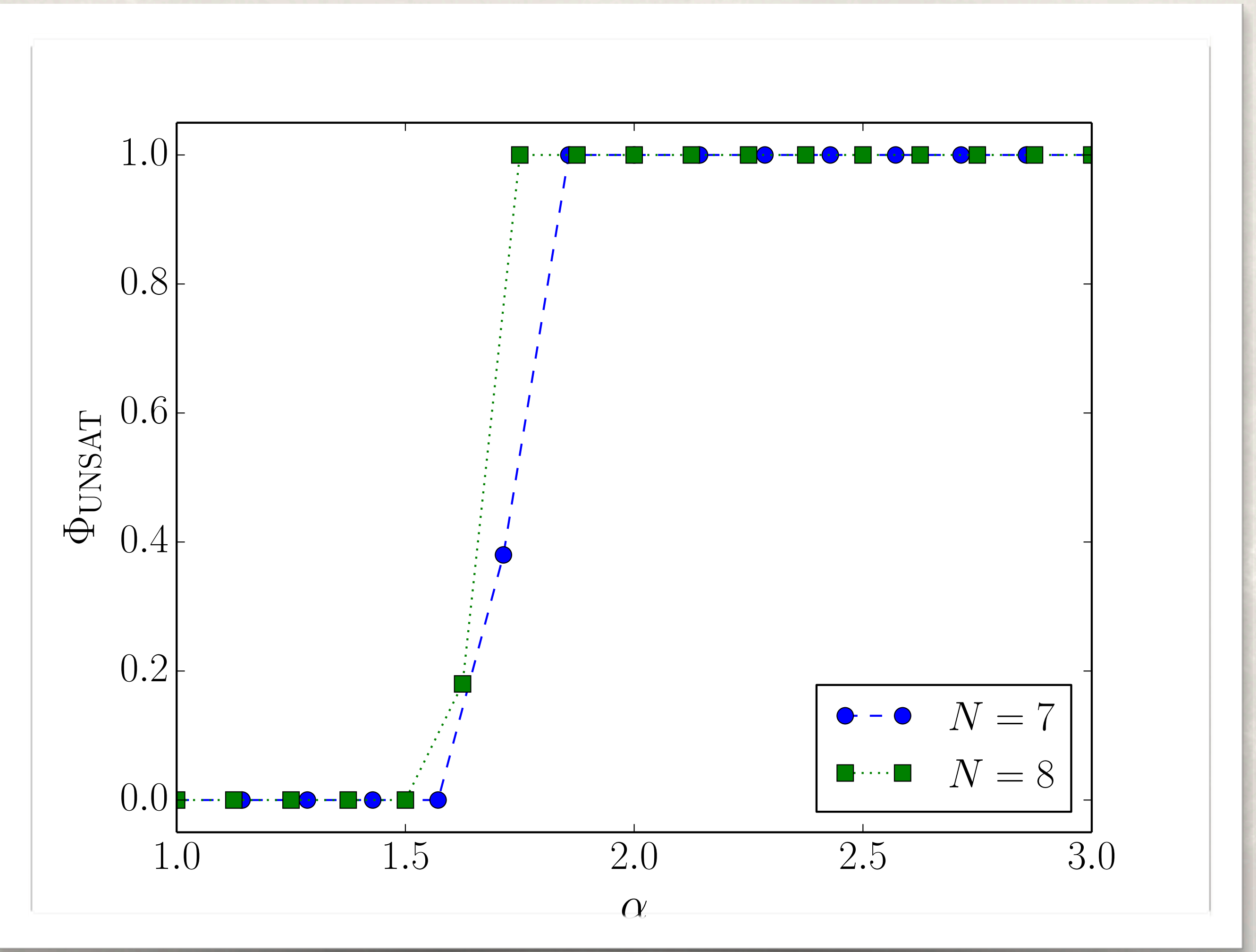
Adversarial Satisfiability

- Not in **NP**, al least not trivially: checking a configuration of negations is a *K*-SAT problem.
- The next step in the hierarchy of complexity: $\mathbf{\Sigma}_2^p$
- But for $K = 2$ we proved that AdSAT is in **P**!

What happens if we look at an ensemble of graphs? If there is a transition, the threshold would be an upper bound for the transition threshold of *K*-QSAT.

Random AdSAT

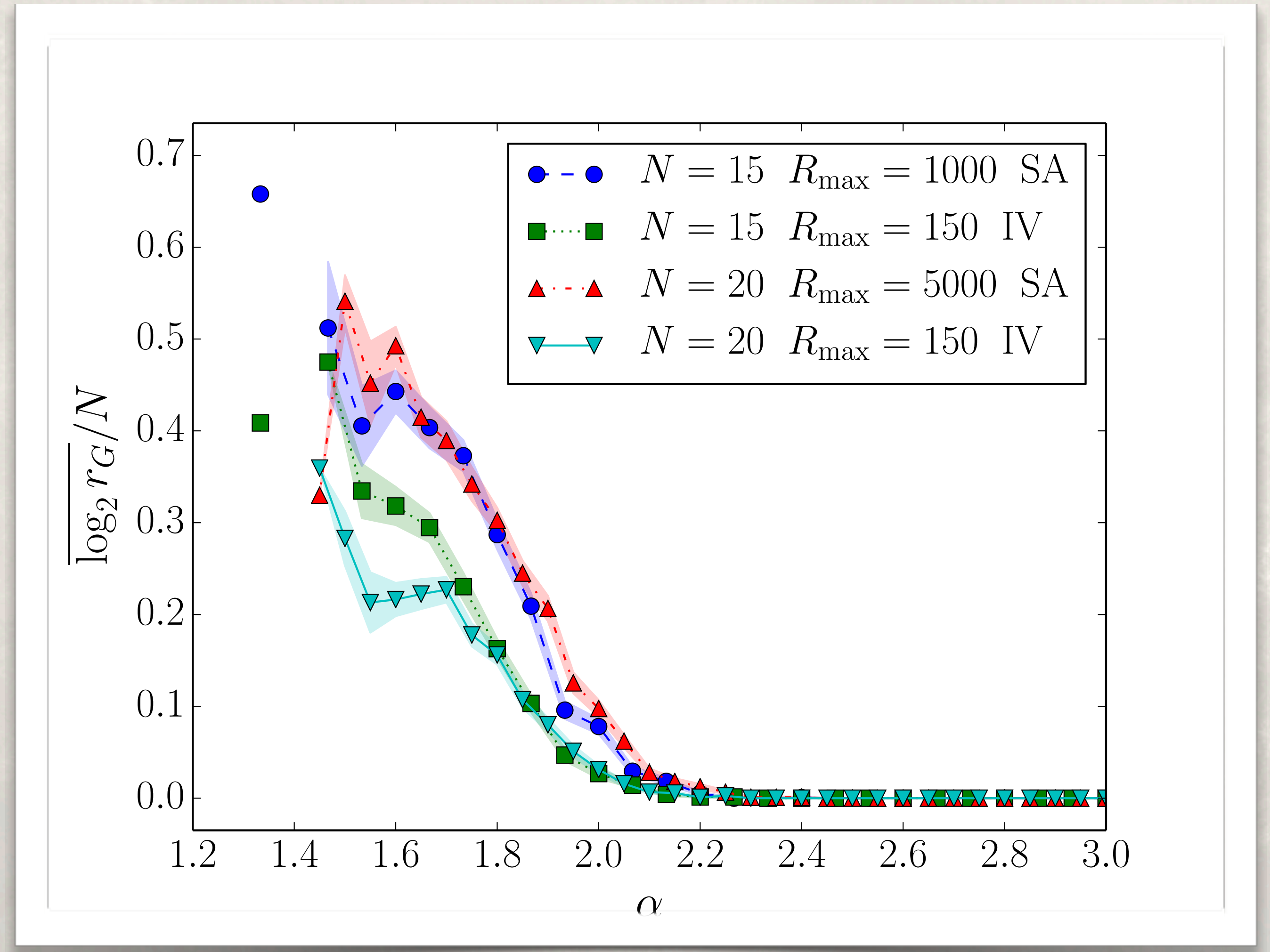
A complete algorithm is viable only for very small N: at $N = 7$ and $\alpha = 2$ we have to solve 2^{35} **NP** problems!



We must resort to a stochastic algorithm: simulated annealing using (the logarithm of) the number of solutions as a cost function, with restarts.

Random AdSAT

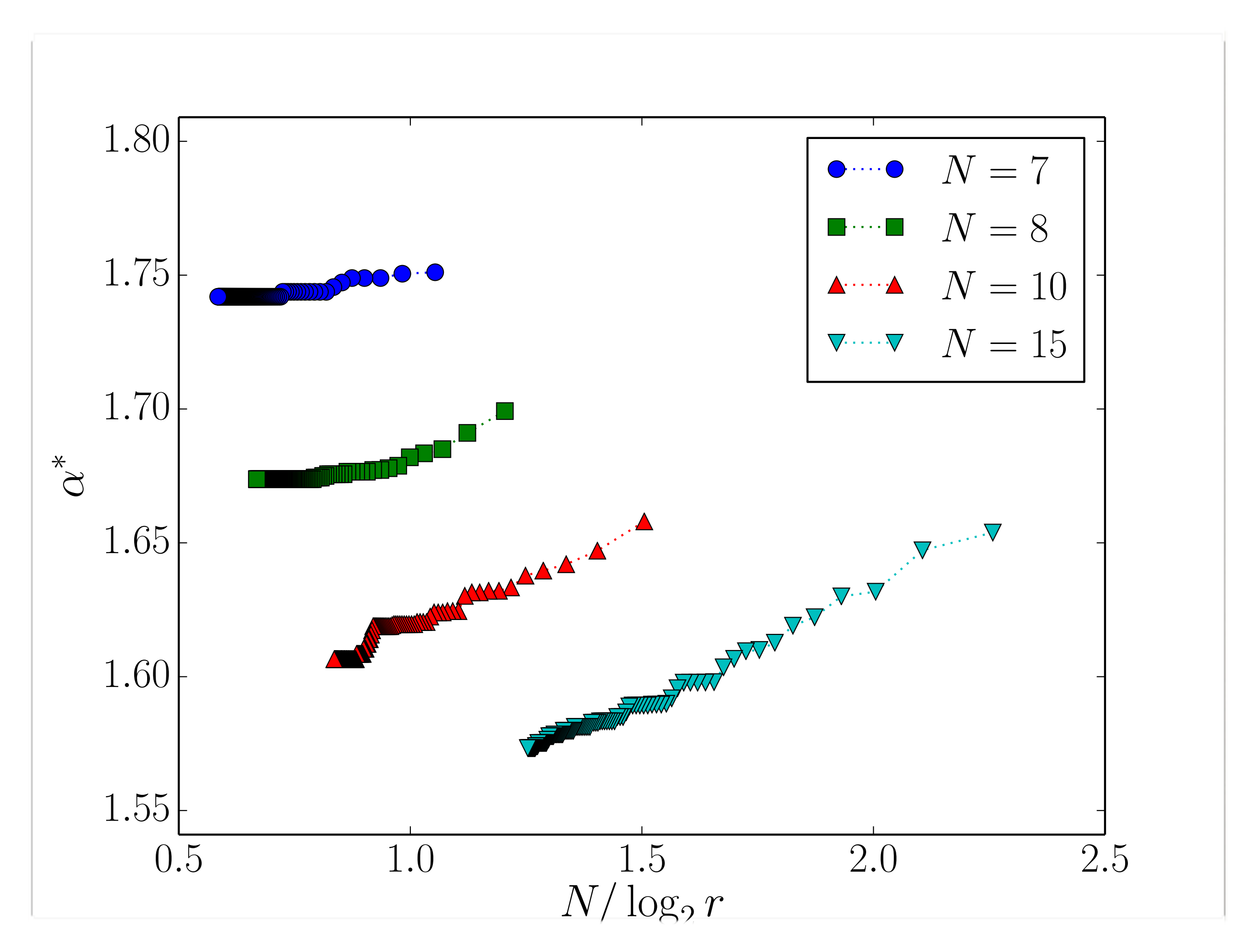
We augment an AdSAT graph so that we are deep in the UNSAT phase, and then gradually remove clauses.



Huge performance improvement, even more evident for larger N and smaller α.

Random AdSAT

Interpolating for the critical threshold we are able to set an upper bound: for $N = 15$, $\alpha_c < 1.6$!



We are also able to set for $N = 100$, $\alpha_c < 2.7$, in sharp contrast with previous results.

Conclusions

- For $K = 2$ AdSAT is **P**.
- Using an improved simulated annealing with restarts we are able to put upper bounds on the critical value of α.
- For $N = 15$, $\alpha_c < 1.6$, for $N = 100$, $\alpha_c < 2.7$.
- Can anything else be proved?