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Constraints on RG flows and the Local Callan-Symanzyk equation

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a-theorem in 4D

- "a" theorem: There exists a function of the running coupling constants which decreases along the RG flow between the UV and IR conformal fixed points.
- Non-perturbative proof in 4D: put CFT in curved background $g_{\mu\nu}$. The Weyl symmetry is anomalous.
- Write effective action for dilaton (conformal mode of $g_{\mu\nu}$).Even in the flat-background the dilaton self-interaction does not vanish (proportional to "a" anomaly).
- Consider on-shell $2 \to 2$ dilaton scattering amplitude: $A(s) = \frac{\alpha(s)s^2}{f^4}$

$$\alpha(s \to \infty) - \alpha(s \to 0) = -8(a_{UV} - a_{IR})$$

- By using unitarity (optical theorem) and analiticity in the complex s-plane one proves $\alpha(s \to \infty) - \alpha(s \to 0) < 0$, i.e. $a_{UV} > a_{IR}$

Komargodski-Schwimmer '11, Luty-Polchinski-Rattazzi '12

The local Callan-Symanzyk equation

- Tool to constrain the RG-flow of QFTs: local Callan-Symanzik equation
- Generalize the standard Callan-Symanzik equation to account for local rescaling of the metric (Weyl transformations). For consistency, the coupling constants must also depend on space-time
- Relevant object: effective action for sources

$$\mathcal{W} = \mathcal{W}[\lambda^I, g_{\mu\nu}, A_\mu, \ldots] = \int \mathcal{D}[\phi] e^{i \int d^4x \left(\mathcal{L}_{CFT}[\phi] + \lambda^I O_I + \ldots\right)}$$

- The metric and couplings act as sources for operators, for instance:

$$\langle O_I(x) \ldots \rangle = \frac{\partial}{\partial \lambda^I(x)} \ldots \mathcal{W}$$

- We study the RG-flow around a fixed-point CFT by turning on (marginal) deformations parametrized by these couplings.

Jack,Osborn '90, Osborn '91

- Callan-Symanzyk equation includes a source-dependent anomaly $\mathcal{A}(\lambda^I, \dots)$

$$\begin{aligned} \Delta_\sigma \mathcal{W} &\equiv \int d^4x \left[\sigma 2 g^{\mu\nu} \frac{\partial}{\partial g^{\mu\nu}} - \sigma \beta^I \frac{\partial}{\partial \lambda^I} - \nabla_\mu \sigma S^A \frac{\partial}{\partial A^A_\mu(x)} + \dots \right] \mathcal{W} \\ &= \int d^4x \sigma(x) \mathcal{A}(\lambda^I, g_{\mu\nu}, \dots) \end{aligned}$$

- $\mathcal{A}(x)$ must respect Wess-Zumino consistency conditions for abelian symmetry:

$$\Delta_\sigma \mathcal{A}_{\sigma'} - \Delta_{\sigma'} \mathcal{A}_\sigma = 0$$

- Most of the anomalies can be made self-consistent by algebraic redefinitions.
- Only 2 or 3 interesting consistency conditions. For instance, "gradient equation":

$$\frac{\partial \tilde{a}}{\partial \lambda^I} = (\chi_{IJ} + F_{IJ})\beta^J$$

- It implies an "irreversibility" equation for $\tilde{a}$

$$\mu \frac{d\tilde{a}}{d\mu} = \chi_{IJ}\beta^I\beta^J > 0$$

- "Metric" in operator space χ_{IJ} positive-definite in unitary theories because related to 2-point function of operators.

Baume,Keren-Zur,Rattazzi,Vitale

$$\mu \frac{d\tilde{a}}{d\mu} = \chi_{IJ}\beta^I\beta^J$$

- $\tilde{a}$ coincides with "a" at the fixed points. "Strong" version of the a-theorem (in perturbation theory).
- This equation also implies SFT=CFT in unitary perturbative theories.
- The second consistency conditions related to the global symmetries of the theory has to be understood.

Thank you