Models of Neutrino Masses and Mixings

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I discuss models of neutrino masses that could lead to a large mixing angle for the atmospheric neutrino oscillations.

1. Introduction

Recent data from Superkamiokande [1] have provided a more solid experimental basis for neutrino oscillations as an explanation of the atmospheric neutrino anomaly. In addition, also the solar neutrino deficit, observed by several experiments [2], is probably an indication of a different sort of neutrino oscillations. Neutrino oscillations imply neutrino masses. The extreme smallness of neutrino masses in comparison with quark and charged lepton masses indicates a different nature of the former, presumably linked to lepton-number violation and the Majorana nature of neutrinos. Thus neutrino masses provide a window on the very large energy scale where lepton number is violated and on Grand Unified Theories (GUTs). Experimental facts on neutrino masses and mixings could give an important feedback on the problem of quark and charged lepton masses, as all these masses are possibly related in GUTs. In particular the observation of a nearly maximal mixing angle for $\nu_\mu \to \nu_\tau$ is particularly interesting. Perhaps also solar neutrinos may occur with large mixing angle. At present solar neutrino mixings can be either large or very small, depending on which particular solution will eventually be established by the data. Large mixings in the neutrino sector are very interesting because a first guess was in favour of small mixings, in analogy to what is observed for quarks. If confirmed, single or double maximal mixings can provide an important hint on the mechanisms that generate neutrino masses.

The experimental status of neutrino oscillations is still very preliminary. Thus, in order to be able to proceed, the theorist has to make a number of assumptions on how the data will finally look when the experimental situation will be completely clarified.

1.1. Three Light Neutrinos

Here we assume that only two distinct oscillation frequencies exist, the largest being associated with atmospheric neutrinos and the smallest with solar neutrinos. We assume that the hint of an additional frequency from the LSND experiment [3], not confirmed by the Karmen experiment [4] (but yet far from being completely excluded), will disappear. Thus we avoid the introduction of new sterile neutrino species and can deal with only the three known species of light neutrinos [1].

We interpret the atmospheric neutrino oscillations as nearly maximal $\nu_\mu \to \nu_\tau$ oscillations, in agreement with the Chooz results [6]. The solar-neutrino oscillations correspond to the disappearance of ν_e into nearly equal fractions of ν_μ and ν_τ. A priori we are open minded about which of the three most likely solutions for solar neutrino

[1]Can three light neutrinos accommodate solar, atmospheric and LSND oscillations? This would seem a priori possible if the solar Cl experiment were affected by a large unknown systematic error. In this case an energy–independent suppression of the solar neutrino flux by approximately a factor of two could reasonably describe the data, leaving the solar frequency undetermined in a vast range. Thus we might associate the two independent frequencies to LSND and atmospheric oscillations. It has been observed that such an attractive scenario is incompatible with the combined results of the Chooz and SuperKamiokande experiments, when also the atmospheric neutrino asymmetries are considered [5].

oscillations is adopted: the two MSW solutions with small (SA) or large (LA) mixing angle, or the vacuum oscillation solution (VO).

1.2. A Useful Parametrization

Maximal atmospheric neutrino mixing and the requirement that the electron neutrino does not participate in the atmospheric oscillations, as indicated by the Superkamiokande [1] and Chooz [6] data, lead directly to the following structure of the U_{fi} ($f = e,\mu,\tau$, i=1,2,3) real orthogonal mixing matrix, apart from sign convention redefinitions (here we are not interested in CP violation effects: all matrices are taken real)

$$U_{fi} = \begin{bmatrix} c & -s & 0 \\ s/\sqrt{2} & c/\sqrt{2} & -1/\sqrt{2} \\ s/\sqrt{2} & c/\sqrt{2} & +1/\sqrt{2} \end{bmatrix} \quad . \qquad (1)$$

This result is obtained by a simple generalization of the analysis of ref. [7] (also discussed in ref. [8]) to the case of arbitrary solar mixing angle ($s \equiv \sin\theta_{sun}$, $c \equiv \cos\theta_{sun}$): $c = s = 1/\sqrt{2}$ for maximal solar mixing (e.g. for vacuum oscillations $\sin^2 2\theta_{sun} \sim 0.75$) , while $\sin^2 2\theta_{sun} \sim 4s^2 \sim 5.5 \cdot 10^{-3}$ for the small angle MSW [9] solution. The vanishing of U_{e3} guarantees that ν_e does not participate in the atmospheric oscillations and the relation $|U_{\mu 3}| = |U_{\tau 3}| = 1/\sqrt{2}$ implies maximal mixing for atmospheric neutrinos. Note that we are assuming only two frequencies, given by

$$\Delta_{sun} \propto m_2^2 - m_1^2, \qquad \Delta_{atm} \propto m_3^2 - m_{1,2}^2 \qquad (2)$$

The effective light neutrino mass matrix is given by $m_\nu = U m_{diag} U^T$ with $m_{diag} = \mathrm{Diag}[m_1, m_2, m_3]$. For generic s one finds

$$m_\nu = \begin{bmatrix} 2\epsilon & \delta & \delta \\ \delta & \frac{m_3}{2} + \epsilon_2 & -\frac{m_3}{2} + \epsilon_2 \\ \delta & -\frac{m_3}{2} + \epsilon_2 & \frac{m_3}{2} + \epsilon_2 \end{bmatrix} \quad , \qquad (3)$$

with

$$\epsilon = (m_1 c^2 + m_2 s^2)/2 \ , \quad \delta = (m_1 - m_2) cs/\sqrt{2} \ ,(4)$$

$$\epsilon_2 = (m_1 s^2 + m_2 c^2)/2 \quad . \qquad (5)$$

We see that the existence of one maximal mixing and $U_{e3} = 0$ are the most important input that leads to the matrix form in eq. (3,5). The value of the solar neutrino mixing angle can be left free. While the simple parametrization of the matrix U in eq. (1) is quite useful to guide the search for a realistic pattern of neutrino mass matrices, it should not be taken too literally. In particular the data do not exclude a non-vanishing U_{e3} element. In most of the Superkamiokande allowed region the bound by Chooz [6] amounts to $|U_{e3}| < 0.2$. In the region not covered by Chooz $|U_{e3}|$ can even be larger [10,5]. Thus neglecting $|U_{e3}|$ with respect to s in eq. (1) is not really justified. Also note that in presence of a large hierarchy $|m_3| \gg |m_{1,2}|$ the effect of neglected parameters in eq. (3) can be enhanced by $m_3/m_{1,2}$ and produce seizable corrections. A non vanishing U_{e3} term can lead to different $(m_\nu)_{12}$ and $(m_\nu)_{13}$ terms. Similarly a deviation from maximal mixing $U_{\mu 3} \neq U_{\tau 3}$ distorts the ϵ_2 terms in the 23 sector of m_ν. Therefore, especially for a large hierarchy, there is more freedom in the small terms in order to construct a model that fits the data than it is apparent from eq. (3).

1.3. Hierarchical Spectrum

Since neutrino oscillations only measure differences of squared masses, the observed differences $(\Delta m^2)_{atm} = |m_3^2 - m_2^2| \gg (\Delta m^2)_{sun} = |m_2^2 - m_1^2|$ could correspond to A) hierarchical eigenvalues [11] $|m_3| \gg |m_{2,1}|$ or to partial or total near degeneracy: B) $|m_1| \sim |m_2| \gg |m_3|$ or C) $|m_1| \sim |m_2| \sim |m_3|$. The configurations B) and C) imply a very precise near degeneracy of squared masses. In model building such a degeneracy requires the adoption of flavour non-abelian symmetries, like those discussed by Joaquim and Tanimoto at this meeting [12]. For example, the case C) is the only one that could in principle accommodate neutrinos as hot dark matter together with solar and atmospheric neutrino oscillations. We think that it is not at all clear at the moment that a hot dark matter component is really needed [13] but this could be a reason in favour of the fully degenerate solution. Then the common mass should be around 1-3 eV. The solar frequency could be given by a small 1-2 splitting, while the atmospheric frequency could be given by a still small but much larger 1,2-3 splitting.

A strong constraint arises in this case from the non observation of neutrinoless double beta decay which requires that the ee entry of m_ν must obey $|(m_\nu)_{ee}| \leq 0.46$ eV [14]. As observed in ref. [15], this bound can only be satisfied if bimixing is realized (that is double maximal mixing, with solar neutrinos explained by the VO or MSW-LA solutions). But we would need a relative splitting $|\Delta m/m| \sim \Delta m^2_{atm}/2m^2 \sim 10^{-3} - 10^{-4}$ and a much smaller one for solar neutrinos explained by vacuum oscillations: $|\Delta m/m| \sim 10^{-10}$–$10^{-11}$. Such a tiny relative mass splitting, arranged at the large energy scale where lepton number is violated, can be easily upset by the renormalization group evolution down to the electroweak scale [16,17], unless a suitable flavour symmetry protects it during the running [18].

1.4. See-saw Mechanism

For reasons of simplicity, we consider the simplest version of the see-saw mechanism with one Dirac, m_D, and one Majorana, M, mass matrix, related to the neutrino mass matrix m_ν, in the basis where the charged lepton mass matrix is diagonal, by

$$m_\nu = m_D^T M^{-1} m_D \quad . \tag{6}$$

As well known this is not the most general see-saw mechanism because we are not including the left-left Majorana mass block. It is unplausible that starting from hierarchical Dirac matrices we end up via the see-saw mechanism into a nearly perfect degeneracy of squared masses and the assumption of hierarchical Dirac masses and the see-saw mechanism naturally leads to a pattern of type A with $|m_3| \gg |m_2| \gg |m_1|$. Models with degenerate neutrinos (see, for example, refs. [19]) could be natural if the dominant contributions directly arise from non renormalizable operators which are apriori unrelated to other fermion masses, but we will not explore this possibility here.

2. Two Interesting Mechanisms

In general large mass splittings correspond to small mixings because normally only close-by states are strongly mixed. In a 2 by 2 matrix context, the requirement of large splitting and large mixing leads to a condition of vanishing determinant. For example the matrix

$$m \propto \begin{bmatrix} x^2 & x \\ x & 1 \end{bmatrix} \tag{7}$$

has eigenvalues 0 and $1 + x^2$ and for x of O(1) the mixing is large. Thus, in the limit of neglecting small mass terms of order $m_{1,2}$, the demands of large atmospheric neutrino mixing and dominance of m_3 translate into the condition that the subdeterminant 23 of the 3 by 3 mass matrix vanishes. The problem is to show that this vanishing can be arranged in a natural way without fine tuning.

Without loss of generality we can go to a basis where both the charged lepton Dirac mass matrix m_D^l and the Majorana matrix M for the right-handed neutrinos are diagonal. In fact, after diagonalization of the charged lepton Dirac mass matrix, we still have the freedom of a change of basis for the right-handed neutrino fields, in that the right-handed charged lepton and neutrino fields, as opposed to left-handed fields, are uncorrelated by the SU(2)×U(1) gauge symmetry. We can use this freedom to make the Majorana matrix diagonal: $M^{-1} = V^T d_M V$ with $d_M = \mathrm{Diag}[1/M_1, 1/M_2, 1/M_3]$. Then if we parametrize the matrix $V m_D$ by z_{ab} we have:

$$(m_\nu)_{ab} \equiv (m_D^T M^{-1} m_D)_{ab} = \sum_c \frac{z_{ca} z_{cb}}{M_c}. \tag{8}$$

From this expression we see that, while we can always arrange the twelve parameters z_{ab} and M_a to arbitrarily fix the six independent matrix elements of m_ν, the hierarchical case is special in that it can be approximately reproduced in two particularly simple ways, without relying on precise cancellations among different terms:

i) there are only two large entries in the z matrix, $|z_{c2}| \sim |z_{c3}|$, and the three eigenvalues M_a are of comparable magnitude (or, at least, with a less pronounced hierarchy than for the z matrix elements). Then, the subdeterminant 23 vanishes and one only needs the ratio $|z_{c2}/z_{c3}|$ to be close to 1. This possibility was discussed for instance in [20];

ii) one of the right-handed neutrinos is particularly light and, in first approximation, it is only coupled to μ and τ. Thus, $M_c \sim \eta$ (small) and $z_{c1} \sim 0$. In this case the 23 sub-determinant vanishes, and again one only needs the ratio $|z_{c2}/z_{c3}|$ to be close to 1. This possibility has been especially emphasized in refs. [21,22,5,23].

In a 2 by 2 matrix context (in the 23 sector), a typical example of mechanism i is given by a Dirac matrix m_D, defined by $\bar{R} m_D L$, which takes the approximate form:

$$m_D \propto \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & x & 1 \end{bmatrix} \quad . \tag{9}$$

This matrix has the property that for a generic Majorana matrix M one finds:

$$m_\nu = m_D^T M^{-1} m_D \propto \begin{bmatrix} 0 & 0 & 0 \\ 0 & x^2 & x \\ 0 & x & 1 \end{bmatrix} \quad . \tag{10}$$

The only condition on M^{-1} is that the 33 entry is non zero. It is important for the following discussion to observe that m_D given by eq. (9) under a change of basis transforms as $m_D \rightarrow V^\dagger m_D U$ where V and U rotate the right and left fields respectively. It is easy to check that in order to make m_D diagonal we need large left mixings. More precisely m_D is diagonalized by taking V=1 and U given by

$$U = \begin{bmatrix} c & -s & 0 \\ sc_\gamma & cc_\gamma & -s_\gamma \\ ss_\gamma & cs_\gamma & c_\gamma \end{bmatrix} \quad , \tag{11}$$

with

$$s_\gamma = -x/r \ , \quad c_\gamma = 1/r \ , \quad r = \sqrt{1+x^2} \quad . \tag{12}$$

The matrix U is directly the neutrino mixing matrix. The mixing angle for atmospheric neutrino oscillations is given by:

$$\sin^2 2\theta = 4 s_\gamma^2 c_\gamma^2 = \frac{4x^2}{(1+x^2)^2} \quad . \tag{13}$$

Thus the bound $\sin^2 2\theta > 0.8$ translates into $0.6 < |x| < 1.6$. As is clear, this mechanism is based on asymmetric Dirac matrices, with, in the case of the example, a large left-handed mixing already present in the Dirac matrix.

If, for some reason, one prefers symmetric or nearly so matrices, then one can use mechanism ii. For example, one could want to preserve left–right symmetry at the GUT scale. Then, the observed smallness of left-handed mixings for quarks would also demand small right-handed mixings. So we now assume that m_D is nearly diagonal (always in the basis where m_D^l and M are diagonal) with all its off-diagonal terms proportional to some small parameter ϵ. Working in the subsector 23 and starting from

$$m_D \propto \begin{bmatrix} \epsilon^p & x\epsilon \\ x\epsilon & 1 \end{bmatrix}, \qquad M^{-1} \propto \begin{bmatrix} r_2 & 0 \\ 0 & 1 \end{bmatrix} \quad , \tag{14}$$

where x is of O(1) and $r_2 \equiv M_3/M_2$, we obtain:

$$m_\nu \propto \begin{bmatrix} \epsilon^{2p} r_2 + x^2\epsilon^2 & x\epsilon^{p+1} r_2 + x\epsilon \\ x\epsilon^{p+1} r_2 + x\epsilon & x^2\epsilon^2 r_2 + 1 \end{bmatrix} \quad . \tag{15}$$

For sufficiently small M_2 the terms in r_2 are dominant. For $p = 1, 2$, which we consider as typical cases, it is sufficient that $\epsilon^2 r_2 \gg 1$. Assuming that this condition is satisfied, consider first the case with $p = 2$. We have

$$m_\nu \propto x^2 \epsilon^2 r_2 \begin{bmatrix} \dfrac{\epsilon^2}{x^2} & \dfrac{\epsilon}{x} \\ \dfrac{\epsilon}{x} & 1 \end{bmatrix} \quad . \tag{16}$$

In this case the determinant is naturally vanishing (to the extent that the terms in r_2 are dominant), so that the mass eigenvalues are widely split. However, the mixing is nominally small: $\sin 2\theta$ is of $\mathrm{O}(2\epsilon/x)$. It could be numerically large enough if $1/x \sim$ 2–3 and ϵ is of the order of the Cabibbo angle $\epsilon \sim$ 0.20–0.25. This is what we call "stretching": the large neutrino mixing is explained in terms of a small parameter; this is not so small and can give a perhaps sufficient amount of mixing if enhanced by a possibly large coefficient. This minimalistic view was endorsed in refs. [24].

A more peculiar case is obtained for $p = 1$, which gives:

$$m_\nu \propto \epsilon^2 r_2 \begin{bmatrix} 1 & x \\ x & x^2 \end{bmatrix} \quad . \tag{17}$$

In this case the small parameter ϵ is completely factored out and for $x \sim 1$ the mixing is nearly maximal. The see-saw mechanism has created *large mixing from almost nothing* [25]: all relevant matrices entering the see-saw mechanism are nearly diagonal. Clearly, the crucial factorization of the small parameter ϵ^2 only arises for $p = 1$, that is the light Majorana eigenvalue is coupled to ν_μ and ν_τ with comparable strength. It is straightforward to extend the previous model to the 3 by 3 case [25]. In that case it is possible to reproduce both the SA and the LA MSW solutions. The required hierarchy among the matrix elements can be supported by a suitable Abelian flavour symmetry, which can be realized also at the level of an SU(5) grand unified theory. Moreover this hierarchy is not spoiled by the renormalization group evolution from the unification scale down to low energy. For an alternative discussion of large neutrino mixing in the framework of grand unified models, see [26].

3. Conclusions

If we start from three light neutrinos and the see-saw mechanism then a natural interpretation of the present data on neutrino oscillations is in terms of hierarchical light neutrino masses and asymmetric mass matrices (at least for d quarks and charged leptons). This has the advantage that no conspiracy is required between the Dirac and the Majorana sectors. There is also the peculiar possibility that large neutrino mixing is only produced by the see-saw mechanism starting from all nearly diagonal matrices. Although this possibility is certainly rather special, models of this sort can be constructed without an unrealistic amount of fine tuning. Both scenarios are well compatible with Abelian flavour symmetries and with grand unification ideas and the related phenomenology for quark and lepton masses.

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