Bimaximal mixing in seesaw models

M. Jeżabek[a b *]

[a]Institute of Nuclear Physics, Kawiory 26a, PL-30055 Cracow, Poland

[b]Institute of Physics, University of Silesia, Uniwersytecka 4, PL-40007 Katowice, Poland

Neutrino masses and mixing are discussed in seesaw models assuming hierarchical structure of Dirac masses and predicting bimaximal mixing for three light neutrinos.

1. Introduction

Neutrino masses and oscillations belong to a very exclusive group of phenomena challenging the standard model of fundamental particles in a serious way. Thanks to the spectacular discovery by the Super-Kamiokande collaboration [1] we know that active neutrinos are massive. I call *active* these three neutrinos which interact electroweakly and couple to the Z^0 and W gauge bosons. In the following I assume that there are no light sterile neutrinos, i.e. spin $\frac{1}{2}$ fermions which do not interact electroweakly but participate in oscillations of the active neutrinos. In particular the deficit of atmospheric neutrinos [1] is explained as a result of $\nu_\mu \to \nu_\tau$ oscillations and not by $\nu_\mu \to \nu_s$ transitions, where ν_s denotes a sterile neutrino. In the following discussion only atmospheric and solar neutrino anomalies are considered whereas the signals of $\nu_\mu \to \nu_e$ oscillations reported by the LSND collaboration [2] are neglected. There are reasons both theoretical and experimental to accept such assumptions. On theoretical side it is very difficult to find a plausible mechanism leading to a similar range of masses for active and sterile neutrinos. The mass terms for the active neutrinos are generated by the non-zero vacuum expectation value of the standard model Higgs field Φ. Sterile neutrinos do not interact electroweakly. It is then quite difficult to understand why a sterile neutrino should have its mass in the same range as the active neutrinos. For this reason we cannot explain simultaneously and in a natural way both the anomalies in atmospheric and solar neutrinos and the LSND observations. On experimental side KARMEN does not confirm [3] LSND observations of neutrino oscillations.

A recent analysis of tritium decays by the Mainz group results in an impressive upper bound for the electron neutrino mass [4]: $m_{\nu_e} \leq 2.3\, eV$. Together with mass splittings which can be inferred from the data on atmospheric and solar neutrinos this bound implies that all three active neutrinos are lighter than about 2.3 eV. Therefore for three light neutrinos the mass of the heaviest active neutrino is many orders of magnitude smaller than the masses of the other fermions in the third generation of quarks and leptons: $m_{\nu_3}/m_{t,b,\tau} \sim 10^{-11} - 10^{-9}$. This remarkably small ratio suggests that the masses of the active neutrinos are related to physics beyond the standard model. In fact these masses can be described by an effective Lagrangian $\Delta\mathcal{L}$ bilinear in the lefthanded lepton doublets of electroweak SU_2 and quadratic in the Higgs field Φ. For three lepton flavours $(i, j = 1, 2, 3)$ all the couplings κ_{ij} in $\Delta\mathcal{L}$ have dimension - 1 in mass units and the effective Lagrangian describes a nonrenormalizable interaction which violates the lepton number. The smallness of the masses of the active neutrinos is related to the fact that the lepton number is violated at a large scale $M \sim 1/\kappa$. $\Delta\mathcal{L}$ can be understood as due to new superheavy particles whose propagators are proportional to

*Work supported in part by KBN grant 2P03B05418, EEC 5th Framework contract HPRN-CT-2000-00149 and by the M.Skłodowska-Curie Joint Fund II PAA/DOE-97-316.

$1/M$ at experimentally accessible 'low energies'. Thus these new particles are fermions and the simplest and the most economical assumption is to identify them with the righthanded partners of the active neutrinos. Then it is quite natural to assume that the operation of space-time reflection can be defined for fundamental fermions and each generation of quarks and leptons contains one righthanded neutrino which is a singlet of the standard model gauge group $SU_c(3) \times SU(2) \times U(1)$. As already stated the righthanded neutrinos are massive and their Majorana masses are of order M. For three generations of neutrinos

$$\Delta\mathcal{L} = -\frac{1}{2}\left(\bar{\nu}_L^c, \bar{\nu}_R^c\right)\begin{pmatrix} 0 & \mathbf{m_D} \\ \mathbf{m_D^T} & \mathbf{M_R} \end{pmatrix}\begin{pmatrix} \nu_L \\ \nu_R \end{pmatrix} + h.c. \quad (1)$$

where 3×3 matrices $\mathbf{m_D}$ and $\mathbf{M_R}$ describe Dirac and Majorana masses, respectively. For the sake of simplicity we neglect CP violating phases and assume that $\mathbf{m_D}$ and $\mathbf{M_R}$ are real. It is assumed that non-zero elements of $\mathbf{M_R}$ are of order of M and hence much larger than the matrix elements of $\mathbf{m_D}$. As a result of the famous seesaw mechanism [5] after diagonalization of the mass matrix (1) the particle spectrum is obtained which consists of three light and nearly lefthanded neutrinos and three nearly righthanded superheavy neutrinos whose masses are of order of the huge mass scale M. The masses of the active neutrinos can be calculated by considering Dirac masses as small perturbations. In the second order of perturbation theory the following 3×3 mass matrix is obtained

$$\mathcal{M}_\nu = -\mathbf{m_D}^{\mathrm{T}}\,\mathbf{M_R}^{-1}\,\mathbf{m_D} \quad (2)$$

with $\left(\mathbf{M_R}^{-1}\right)_{ij} \sim 1/M$. The masses of the active neutrinos are equal to absolute values of the eigenvalues of $\mathcal{M}_\nu$. Inversion of the matrix $\mathbf{M_R}$ is possible because the condition $\det \mathbf{M_R} \neq 0$ is assumed, which guarantees that all righthanded neutrinos are heavy.

2. Semi-hierarchical mass pattern

The Dirac masses of quarks and charged leptons exhibit a strongly ordered hierarchical structure $m_u \ll m_c \ll m_t$, $m_d \ll m_s \ll m_b$ and $m_e \ll m_\mu \ll m_\tau$. Following [6] we assume a similar hierarchical structure for the Dirac masses of neutrinos. However this hierarchical structure is drastically modified by huge Majorana masses of the righthanded neutrinos. As a result of the seesaw mechanism[5] three light neutrinos appear in the mass spectrum. Their Majorana masses can form patterns very different from the hierarchical one. In [6] a class of seesaw models was found corresponding to semi-hierarchical mass pattern ($m_1 \approx m_2 \ll m_3$). In these models the third mass eigenstate is much heavier than the other two whose masses are fairly close. Then other models leading to semi-hierarchical mass spectrum were proposed in [8]-[11]. In these models the mass matrix for active neutrinos has the following form

$$\mathcal{M}_\nu = \mu \begin{pmatrix} cr^2 & br & r \\ br & 1 & 1 \\ r & 1 & 1 \end{pmatrix} \quad (3)$$

with $r \ll 1$. Different models correspond to different values of b and c, e.g $b = c = 0$ in [6] and $c == -b = 1$ in [9]. In terms of the small parameter

$$r = \frac{2^{3/2}\left(\Delta m_\odot^2/2\Delta m_{atm}^2\right)^{1/3}}{(1-b)^{1/3}\left[(1+b)^2 - 4c\right]^{1/3}} \quad (4)$$

the eigenvalues of the active neutrino mass matrix (3) are given by $\lambda_i\sqrt{\Delta m_{atm}^2}$ where:

$$\begin{aligned} \lambda_{1,2} &= \pm(1-b)r/\sqrt{2} - [(1+b)^2 - 4c]r^2/8\ldots \\ \lambda_3 &= 2 + \mathcal{O}(r^2) \end{aligned} \quad (5)$$

3. MNS lepton mixing matrix

The structures of quark and lepton mixing matrices are also quite different. The Maki–Nakagawa–Sakata flavor mixing matrix U_{MNS} [12] relates the active neutrino mass and flavor eigenstates. For three light neutrinos it can be written as a product of two matrices $U_\pm$ describing transformations of the neutrinos ($I_3 = +1/2$) and the charged leptons ($I_3 = -1/2$)

$$U_{MNS} = U_-{}^{\dagger}\, U_+ \quad . \quad (6)$$

In [7] it has been assumed that the pattern of mixing angles in U_- describing the charged lepton sector resembles that in V_- for the down type quarks, i.e. there is an appreciable mixing in the electron–muon plane of the order of

$$|s| = \sqrt{m_e/m_\mu} \tag{7}$$

whereas the other mixing angles are much smaller. In the sector $I_3 = +1/2$ the situation is quite different. The structure of the matrix U_+ is not directly related to the Dirac mass matrix $\mathbf{m_D}$ for neutrinos. It is strongly affected by the structure of the Majorana mass matrix $\mathbf{M_R}$ for the righthanded neutrinos. As a consequence the structure of U_+ is different from the structure of the matrix V_+ for the up type quarks. Therefore the seesaw mechanism can also explain striking differences between the structures of the Cabibbo-Kobayashi-Maskawa mixing matrix for quarks and the Maki-Nakagawa-Sakata mixing matrix for leptons where maximal or nearly maximal mixing is preferred by the data on both atmospheric and solar neutrino anomalies. The mixing matrix in the $I_3 = +1/2$ sector

$$U_+ = U_{bm} + \mathcal{O}(r) \tag{8}$$

where

$$U_{bm} = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/2 & 1/2 & 1/\sqrt{2} \\ 1/2 & -1/2 & 1/\sqrt{2} \end{pmatrix} \tag{9}$$

corresponds to bimaximal mixing [13,14,6]. Neglecting small corrections $\mathcal{O}(r,s)$ the oscillations observed by the Super-Kamiokande Collaboration [1] can be understood as a result of transitions between ν_μ and ν_τ. An important parameter is the matrix element U_{e3}. For $b = c = 0$ [7]:

$$U_{e3} \approx s/\sqrt{2} + r/2^{3/2} \tag{10}$$

Other mass patterns for bimaximal mixing of three active neutrinos have been also considered in the literature [7,15]. These patterns lead to completely different predictions for the rates of neutrinoless double beta decays, see [7] for details.

The mass scale M for the righthanded neutrinos is 10^9–10^{10} GeV [6]. If CP violating phases are considered, see e.g.[9], it is possible that lepton number violating decays of these super-heavy particles generate a non-zero lepton number of the Universe. Then processes due to electroweak sphalerons which violate both baryon and lepton numbers but conserve B-L may explain the origin of the matter-antimatter asymmetry of the Universe [16].

REFERENCES

1. Fukuda Y. et al., Phys. Rev. Lett. 81 (1998) 1562.
2. Athanassopoulos C. et al., Phys. Rev. Lett. 81 (1998) 1774.
3. G.J.Drexlin in: Huitu, K. et al., High Energy Physics 99, Institute of Physics Publishing, Bristol and Philadelphia, 2000, p.891.
4. J.Bonn and Ch.Weinheimer, Acta Phys. Polon. B31 (2000) 1209.
5. M. Gell-Mann, P. Ramond and R. Slansky, in: Supergravity, North Holland, Amsterdam, 1979, p.315; T. Yanagida, in: Proc. of the Workshop on the Unified Theory and Baryon Number in the Universe, KEK report 79-18, 1979, p.95.
6. M. Jeżabek and Y. Sumino, Phys. Lett. B440 (1998) 327.
7. M. Jeżabek and Y. Sumino, Phys. Lett. B457 (1999) 139.
8. G. Altarelli and F. Feruglio, Phys. Lett. B439 (1998) 112.
9. B. Stech, Phys. Lett. B465 (1999) 219; Phys. Rev. D62 (2000) 3019.
10. Abud M. et al., Mod. Phys. Lett. A15 (2000) 15.
11. E.K. Akhmedov, G.C. Branco, M.N. Rebelo, Phys. Lett. B478 (2000) 215.
12. Z. Maki, M. Nakagawa and S. Sakata, Prog. Theor. Phys. 28 (1962) 870.
13. V. Barger, S. Pakvasa, T.J. Weiler and K. Whisnant, Phys. Lett. B437 (1998) 107.
14. A.J. Baltz, A.S. Goldhaber and M. Goldhaber, Phys. Rev. Lett. 81 (1998) 5730.
15. F. Feruglio, Acta Phys. Polonica B31 (2000) 1221 and references therein.
16. W. Buchmüller and T. Yanagida, Phys. Lett. B445 (1998) 399; W. Buchmüller and M. Plümacher, hep-ph/0007176.