Seesaw neutrino masses and mixing with extended democracy

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In the context of a minimal extension of the Standard Model with three extra heavy right-handed neutrinos, we propose a model for neutrino masses and mixing based on the hipothesis of a complete alignment of the lepton mass matrices in flavour space. Considering a uniform quasi-democratic structure for these matrices, we show that, in the presence of a highly hierarchical right-handed neutrino mass spectrum, the effective neutrino mass matrix, obtained through the seesaw mechanism, can reproduce all the solutions of the solar neutrino problem.

All the enthusiasm around the Super-Kamiokande evidence for atmospheric neutrino oscillations [1] has lead to a great activity in the search for models [2] which can reproduce the solar and atmospheric neutrino data [3].

In Ref. [4] we have proposed a simple model for lepton masses and mixing which fulfils these requirements.

As a starting point, we consider the existence of three heavy right-handed neutrinos which, together with the seesaw mechanism, will generate small neutrino masses. Furthermore, we assume that the charged lepton, Dirac and right-handed neutrino mass matrices (M_l, M_D and M_R respectively) are aligned in flavour space in a such a way that they are all "democratic", i.e., they are proportional to a matrix whose elements are, in leading approximation, all equal to one. We will refer to this assumption as "extended democracy".

In the context of the Standard Model (SM) with three additional heavy right-handed neutrinos, the lepton mass Lagrangian has the form:

$$-\mathcal{L}_{mass} = \bar{l}_{iL}\,(M_l)_{ij}\,l_{jR} + \bar{\nu}_{iL}\,(M_D)_{ij}\,\nu_{jR} + \\ + \frac{1}{2}\,\nu_{iR}^T\,C\,(M_R)_{ij}\,\nu_{jR} + h.c.\ . \tag{1}$$

Following the hints of some Grand Unified Theories (GUTs), we consider the mass spectrum of M_D similar to the one of the up-type quarks.

Since the matrices M_l, M_D and M_R are assumed to be, at leading order, proportional to the democratic matrix Δ, we write:

$$M_k = c_k\,[\,\Delta\,+\,P_k]\ ,\quad k = l, D, R\ , \tag{2}$$

$$\Delta_{ij} = 1\ \ ,\ \ P_k = \mathrm{diag}(0, a_k, b_k)\ , \tag{3}$$

with $|\,a_k|\,, |\,b_k| \ll 1$, so that all the matrices are close to the democratic limit. From Eq. (3) one can see that the breaking of the extended democracy is small and it has the same pattern for all the mass matrices. The effective neutrino mass matrix, $M_{\rm eff} = -{M_D}^T {M_R}^{-1} M_D$, is given by:

$$M_{\rm eff} = -c_{\rm eff}\,[\,\Delta + P_{\rm eff}\,]\,,\ P_{\rm eff} = \mathrm{diag}(0, x, y), \tag{4}$$

where $x = {a_D}^2/a_R$, $y = {b_D}^2/b_R$ and $c_{\rm eff} = {c_D}^2/c_R$. It is interesting to notice that this matrix has the same general form as the matrices M_k, i.e. the seesaw mechanism preserves our *Ansatz*. This is a remarkable feature of the scheme we propose in Eqs. (2) and (3).

Our next task is to constrain the parameters in the mass matrix $M_{\rm eff}$ so as to satisfy the experimental bounds on the values of the Δm^2's and mixing angles.

The hierarchical structure of the eigenvalues of M_l implies $|a_l|, |b_l| \ll 1$. Since the matrix Δ can

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be diagonalized as $F^T \Delta F = \mathrm{diag}(0,\,0,\,3)$ with

$$F = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ 0 & -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} \end{pmatrix} , \tag{5}$$

the matrix U_l that diagonalizes M_l can be written as $U_l = FW$, where, due to the hierarchy $|a_l| \ll |b_l| \ll 1$, the matrix W is close to the unit matrix. Let us consider that all the parameters a_k and b_k in Eqs. (2) and (3) are real [2]. It is instructive to analyse first the limit when the matrix W coincides with the unit matrix, which corresponds to neglecting m_e and m_μ. The parameters a_k, b_k and c_k are related to the masses of charged leptons, up-type quarks and heavy Majorana neutrinos through:

$$a_l \simeq 6\frac{m_e}{m_\tau} \quad , \quad a_D \simeq \frac{m_u}{m_t} \quad , \quad a_R \simeq 6\frac{M_1}{M_3} ,$$
$$b_l \simeq \frac{9}{2}\frac{m_\mu}{m_\tau} \quad , \quad b_D \simeq \frac{9}{2}\frac{m_c}{m_t} \quad , \quad b_R \simeq \frac{9}{2}\frac{M_2}{M_3} ,$$
$$|c_l| \simeq \frac{m_\tau}{3} \quad , \quad |c_D| \simeq \frac{m_t}{3} \quad , \quad |c_R| \simeq \frac{M_3}{3} , \tag{6}$$

where M_i (i=1,2,3) are the heavy right-handed neutrino masses, which are the only free parameters in our model. The effective mass matrix $\tilde{M}_{\rm eff}$, in the basis where the charged leptons have been diagonalized, is then obtained from Eq. (4) through the rotation by the matrix F: $\tilde{M}_{\rm eff} = F^T M_{\rm eff} F \equiv -c_{\rm eff}\, y\, \tilde{M}_0$, with $\tilde{M}_0$ given by:

$$\begin{pmatrix} \frac{\varepsilon}{2} & -\frac{\varepsilon}{2\sqrt{3}} & -\frac{\varepsilon}{\sqrt{6}} \\ -\frac{\varepsilon}{2\sqrt{3}} & \frac{2}{3}+\frac{\varepsilon}{6} & -\frac{2}{3\sqrt{2}}+\frac{\varepsilon}{3\sqrt{2}} \\ -\frac{\varepsilon}{\sqrt{6}} & -\frac{2}{3\sqrt{2}}+\frac{\varepsilon}{3\sqrt{2}} & \frac{1}{3}+\frac{\varepsilon}{3}+\delta \end{pmatrix} , \tag{7}$$

where $\varepsilon = x/y$ and $\delta = 3/y$. The condition $\Delta m^2_{12} \equiv \Delta m^2_\odot \ll \Delta m^2_{32} \equiv \Delta m^2_{atm}$ requires $|\varepsilon|, |\delta| \ll 1$. Then, the largest eigenvalue of the matrix $\tilde{M}_0$ is always close to one. This leads to:

$$M_2 \simeq \frac{3}{2}\frac{m_c^2}{\sqrt{\Delta m^2_{atm}}} \simeq 4 \times 10^{10}\ \mathrm{GeV}\,. \tag{8}$$

[2]This treatment can be extended to the most general case of complex mass matrices, such as the special ones based on the hipothesis of universal strength of Yukawa couplings (USY) [4].

Moreover, we find:

$$M_1 \simeq \frac{2}{\varepsilon}\frac{m_u^2}{\sqrt{\Delta m^2_{atm}}} \quad , \quad M_3 \simeq \frac{1}{\delta}\frac{m_t^2}{\sqrt{\Delta m^2_{atm}}}\,. \tag{9}$$

It is interesting to notice that the values of M_2 are always nearly the same, which stems from the fact that they are related to Δm^2_{atm} (*cf.* Eq. (8)) and practically independent from $\Delta m^2_\odot$ and θ_{12}.

In the limit $|\varepsilon| \ll |\delta| \ll 1$ (relevant for the SMA solution of the solar neutrino problem), the light neutrino masses are

$$\{m_1,\, m_2,\, m_3\} = -c_{\rm eff}\, y \left\{ \frac{\varepsilon}{2}, \frac{2}{3}\delta + \frac{\varepsilon}{2}, 1 + \frac{\delta}{3} \right\} .$$

The diagonalization of $\tilde{M}_{\rm eff}$ yields

$$\varepsilon \simeq \sin 2\theta_{12}\sqrt{\frac{\Delta m^2_{21}}{\Delta m^2_{32}}} \quad , \quad \delta \simeq \frac{3}{2}\sqrt{\frac{\Delta m^2_{21}}{\Delta m^2_{32}}} , \tag{10}$$

$$\sin^2 2\theta_{23} \simeq \frac{8}{9}\left(1 + \frac{2}{3}\delta\right) \quad , \quad \sin\theta_{13} = -\frac{\varepsilon\delta}{3\sqrt{2}}\,. \tag{11}$$

Similarly, we can derive the following expressions for the eigenvalues of $\tilde{M}_{\rm eff}$ in the case $|\delta| \lesssim |\varepsilon| \ll 1$ (relevant for the VO, LOW and LMA solutions of the solar neutrino problem):

$$\{m_1,\, m_2,\, m_3\} =$$
$$-c_{\rm eff} y \left\{ \frac{\delta}{3} - \frac{\delta^2}{9\varepsilon}, \varepsilon + \frac{\delta}{3} + \frac{\delta^2}{9\varepsilon}, 1 + \frac{\delta}{3} \right\} .$$

Combined with the results obtained from the diagonalization of $\tilde{M}_{\rm eff}$, they lead to:

$$\tan\theta_{12} = 1 - \frac{2}{3}\frac{\delta}{\varepsilon} + \frac{2}{9}\frac{\delta^2}{\varepsilon^2} \ , \ \varepsilon = \sqrt{\tan\theta_{12}\frac{\Delta m^2_{21}}{\Delta m^2_{32}}} ,$$

$$\delta = \frac{3}{2}\left(1 - \sqrt{2\tan\theta_{12} - 1}\right)\sqrt{\tan\theta_{12}\frac{\Delta m^2_{21}}{\Delta m^2_{32}}} , \tag{12}$$

which replace Eqs. (10), whereas Eqs. (11) remain valid in this case.

Since M_l is not exactly of the democratic form, the matrix W in $U_l = FW$ deviates slightly from the unit matrix due to the nonzero values of m_e and m_μ. Therefore, we obtain for $|\delta| \lesssim |\varepsilon| \ll 1$:

$$\frac{\sin^2 2\theta_{12}}{\sin^2 2\theta} = 1 - \frac{4}{3}\frac{m_e}{m_\mu}\frac{\cos 2\theta}{1 - \cos 2\theta}\left(1 + \frac{2}{3}\delta\right) , \tag{13}$$

Table 1
Results from exact numerical diagonalizations of $\tilde{M}_{\rm eff}$ corresponding to the four solutions of the solar neutrino problem. The corrections due to non-zero m_e and m_μ were included.

	LMA	SMA	LOW	VO
a_R	1.5×10^{-9}	8.5×10^{-8}	1.9×10^{-10}	4.1×10^{-9}
b_R	1.3×10^{-5}	2.2×10^{-5}	8.0×10^{-8}	3.9×10^{-8}
M_3 (GeV)	1.3×10^{16}	7.6×10^{15}	2.0×10^{18}	4.3×10^{18}
M_1 (GeV)	3.2×10^{6}	1.1×10^{8}	6.3×10^{7}	2.9×10^{9}
M_2 (GeV)	3.8×10^{10}	3.7×10^{10}	3.6×10^{10}	3.8×10^{10}
Δm^2_{12} (eV)^2	5.36×10^{-5}	7.25×10^{-6}	1.15×10^{-7}	1.02×10^{-10}
Δm^2_{23} (eV)^2	3.94×10^{-3}	4.15×10^{-3}	4.35×10^{-3}	3.96×10^{-3}
$\sin^2 2\theta_{12}$	0.95	5.1×10^{-3}	0.999	0.70
$\sin^2 2\theta_{23}$	0.95	0.96	0.94	0.94
U_{e3}	3.27×10^{-3}	2.27×10^{-3}	2.29×10^{-3}	2.29×10^{-3}

$$\sin^2 2\theta_{23} = \frac{8}{9}\left(1+\frac{2}{3}\delta\right)\left[1+\frac{m_\mu}{m_\tau}(1-3\delta)\right] , \quad (14)$$

$$\sin\theta_{13} = U_{e3} = -\frac{\varepsilon\delta}{3\sqrt{2}} - \frac{\sqrt{2}}{3}\frac{m_e}{m_\mu}\left(1-\frac{\delta}{3}\right) . \quad (15)$$

In the limit $m_e, m_\mu \to 0$ the corresponding expressions of Eqs. (11) and (12) are recovered.

When $|\varepsilon| \ll |\delta| \ll 1$, one has:

$$\sin^2 2\theta_{12} = 4\left[\frac{3}{4}\frac{\varepsilon}{\delta} - \frac{1}{3}\frac{m_e}{m_\mu}\left(1+\frac{2}{3}\delta\right)\right]^2 , \quad (16)$$

whereas $\sin^2 2\theta_{23}$ and U_{e3} are again given by Eqs. (14) and (15). Notice that the new contributions coming from nonzero m_e and m_μ tend to increase the values of $\sin^2 2\theta_{23}$, bringing it closer to the Super-Kamiokande best fit value. They also increase significantly the value of the mixing parameter $|U_{e3}|$.

We have performed exact numerical diagonalizations of the light neutrino effective mass matrix for each solution of the solar neutrino problem. All the results are within the allowed range of the experimental data for solar and atmospheric neutrinos (see table 1)[3]. Our analytical expressions also give very accurate results when compared with the exact ones.

A simple renormalization group analysis of our *Ansätze* shows that these solutions are stable against quantum corrections at one-loop level.

In conclusion, we have proposed a viable structure for the lepton mass matrices based on the extended democracy hipothesis, which, in the absence of right-handed neutrinos, would lead to small mixing in the leptonic sector, similar to what happens for quarks. The large mixing required by the atmospheric neutrino data appears naturally as a result of the seesaw mechanism combined with a strong hierarchy in the right-handed neutrino mass spectrum. By changing the values of the right-handed neutrino masses, we are able to reproduce all the possible solutions of the solar neutrino problem.

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