Neutrino-nucleus reaction cross sections

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To draw physics conclusions from the SNO data, it is crucially important to have reliable estimates of the neutrino-deuteron reaction cross sections. I describe here the latest calculations of these cross sections and assess their reliability.

1. Introduction

Neutrino-nucleus reactions play crucial roles in various astrophysical phenomena and also in terrestrial experiments that use nuclear targets to detect neutrinos. One of the extremely important neutrino-nucleus reactions in neutrino physics today is the neutrino-deuteron reaction. This importance derives from highly consequential experiments that are under way at the Sudbury Neutrino Observatory (SNO) [1]. One of the primary goals of SNO is to study the solar neutrinos by monitoring reactions occurring in heavy water:

$$\nu_e + d \;\rightarrow\; e^- + p + p, \tag{1}$$
$$\nu_x + d \;\rightarrow\; \nu_x + p + n \quad (x = e,\, \mu,\, \text{or } \tau), \tag{2}$$
$$\bar{\nu}_e + d \;\rightarrow\; e^+ + n + n, \tag{3}$$
$$\bar{\nu}_x + d \;\rightarrow\; \bar{\nu}_x + p + n \quad (x = e,\, \mu,\, \text{or } \tau)\,. \tag{4}$$

The unique feature of SNO is its ability to register the charged-current (CC) and neutral-current (NC) reactions separately but simultaneously. Since the NC reaction measures the total flux of the solar neutrinos (regardless of their flavors), SNO experiments offer valuable information about the nature of possible neutrino oscillations. SNO is also capable of monitoring astrophysical neutrinos the energy of which extends well beyond the solar-neutrino energy regime, a prominent example being supernova neutrinos. Obviously, in interpreting experimental results to be obtained at SNO, the accurate knowledge of the ν-d reaction cross sections is a prerequisite.

Much effort has been invested in estimating the ν-d cross sections [2–9]. The latest developments, which I wish to describe here, come from two different fronts. One front is what may be called the phenomenological Lagrangian approach (PhLA) and the other front is the effective field theory (EFT) approach — both PhLA and EFT will be explained below. I wish to present an argument that, by combining these two approaches, we can have much more reliable estimates of the ν-d cross sections than before.

A highly successful method for describing nuclear responses to electroweak probes is to consider one-body impulse approximation terms and two-body exchange-current terms acting on nonrelativistic nuclear wave functions, with the exchange currents derived from a one-boson exchange model. In a modern realization of this approach [10,11], the vertices characterizing relevant Feynman diagrams are determined, as much as possible, with the use of the low-energy theorems and current algebra. This type of formalism may be called the phenomenological Lagrangian approach (PhLA). This formalism has been used extensively for electromagnetic processes in two-nucleon systems [12,13]. The reported good agreement between theory and experiment strongly suggests the basic soundness of PhLA. This method has also been applied to two-nucleon weak-interaction processes such as muon capture on the deuteron [4], the pp-fusion reaction [14], and the ν-d reactions [3–7]. The most elaborate PhLA calculations have been carried out very

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recently by Nakamura, Sato, Gudkov and myself (NSGK) [7], and I shall describe below the salient features of the NSGK calculation.

Meanwhile, a new approach based on effective field theory (EFT) has been scoring great success in describing low-energy electroweak processes in the two-nucleon systems [17–20]. In particular, the rate of thermal neutron radiative capture on the proton ($n+p \to d+\gamma$) has been calculated in chiral perturbation theory (χPT) and the result is found to be in perfect agreement with the data [17]. Butler and Chen [8] and Butler, Chen and Kong [9] have recently made elaborate studies of ν-d cross sections for the solar neutrino energies with the use of EFT. I will discuss how nicely the results of PhLA and EFT complement each other to yield significantly improved estimates of the ν-d cross sections.

2. NSGK calculation

The goal of our work described in NSGK [7] is to assess and improve the reliability of the PhLA calculation of the ν-d reaction cross sections, by carefully examining the dependence of the results on various input and approximations that go into calculations. The main points of improvements in NSGK over the previous estimates are: (1) Use of the "modern" NN potentials; (2) Use of the more accurate nucleon weak-interaction form factors; (3) Monitoring the strength of the $\pi N\Delta$ vertex that governs by far the dominant exchange-current contribution, with the use of empirical data for the related processes.

The interaction Hamiltonian for semileptonic weak processes is given by the product of the hadron current (J_λ) and the lepton current (L^λ) as

$$H_W^{cc} = \frac{G_F \cos\theta_C}{\sqrt{2}} \int d\mathbf{x}[J_\lambda^{cc}(\mathbf{x}) L^\lambda(\mathbf{x}) + \text{h.c.}] \quad (5)$$

for the CC process and

$$H_W^{nc} = \frac{G_F}{\sqrt{2}} \int d\mathbf{x}\ [J_\lambda^{nc}(\mathbf{x}) L^\lambda(\mathbf{x}) + \text{h.c.}] \quad (6)$$

for the NC process. Here G_F is the Fermi coupling constant, and θ_C is the Cabibbo angle. The lepton current $L^\lambda(\mathbf{x})$ is well known. The hadronic charged current has the form

$$J_\lambda^{cc} = V_\lambda^\pm + A_\lambda^\pm, \quad (7)$$

where V_λ and A_λ denote the vector and axial-vector currents, respectively. The superscript $+(-)$ denotes the isospin raising (lowering) operator for the $\bar{\nu}(\nu)$-reaction. Meanwhile, the hadronic neutral current is given by

$$J_\lambda^{nc} = (1-2\sin^2\theta_W)V_\lambda^3 + A_\lambda^3 - 2\sin^2\theta_W V_\lambda^s, \quad (8)$$

where θ_W is the Weinberg angle. V_λ^s is the isoscalar part of the vector current, and the superscript '3' denotes the third component of the isovector current. In the present case the hadron current consists of one-nucleon impulse approximation (IA) terms and two-body meson exchange current (MEX) terms.

The IA current is determined by the single-nucleon matrix elements of the hadronic currents, *i.e.*, the nucleon weak-interaction form factors. In a PhLA due to Ivanov and Truhlik [11], the MEX operators are derived in a hard pion approach, in which one explicitly constructs a phenomenological Lagrangian consistent with current algebra, PCAC, and the vector meson dominance. There is, however, substantial evidence by now that only a small subset of the possible diagrams gives essentially the same results as the full set, and it is practically sufficient to only consider the following types of MEX. For the space component of A_λ, we include the isobar currents that arise from one-pion and one-ρ-meson exchange diagrams. For the space component of V_λ, we include the pair, pionic, and isobar currents. We adopt the one-pion exchange model for the pair and pionic current and the one-pion and one-ρ-meson exchange model for the isobar current.

In PhLA, the nuclear transition matrix elements are obtained by sandwiching the one-body IA and two-body MEX currents between the initial and final nuclear wave functions which obey the Schrödinger equation that involves a phenomenological nucleon-nucleon potential. The earlier work [4] indicates that, so long as we use a realistic NN potential that reproduces with sufficient accuracy the scattering phase shifts and the deuteron properties, the numerical results for the

ν-d cross sections are not too sensitive to particular choices of NN potentials. NSGK have extended this stability check to *modern* potentials that became available after the work described in [4]. As representatives of the "state-of-the-art" NN potentials, NSGK have considered the Argonne-v18 potential (ANLV18) [22], the Reid93 potential [23], and the Nijmegen II potential (NIJ II) [23].

NSGK have calculated the total cross sections for the four reactions in Eq. (1)- Eq. (4) for incident neutrino energies up to 170 MeV. For the CC reaction in Eq. (1), they have also calculated the electron spectrum, electron angular distribution and the electron double differential cross section, $d^2\sigma/(dE_e d\Omega_e)$. For the NC reaction in Eq. (2), they have calculated the neutron energy spectrum and the neutron angular distribution as well. The numerical results reported in NSGK have been posted in tabular and graphical forms at the web site: http://nuc003.psc.sc.edu/~kubodera/ NU-D-NSGK.

In order to discuss the model dependence of our numerical results, let me introduce the term '*standard case*', which designates the result obtained in NSGK with both IA and MEX terms included and with the use of the ANLV18 potential. We first address comparison between the IA and MEX contributions. In Table 1, the column labeled "IA" gives the ratio of the total cross section obtained with the use of the IA terms alone to $\sigma_{\nu d}$(stnd), the result corresponding to our *standard case*.[2] We see that, at the low energies, the MEX contribution is about 5% of the IA contribution, and in the high energy region the relative importance of the MEX contribution is ~8%. It is reassuring that the ν-d reactions are dominated by the IA terms, which are under good theoretical control, for this means that we can concentrate on the task of assessing the accuracy of modest corrections due to MEX. Keeping this situation in mind, we examine the dependence of the NSGK results on input parameters. In Table 1, the third column gives the ratio of the $\sigma_{\nu d}$ reported in KN [6] to $\sigma_{\nu d}$(stnd), while the fourth column represents the ratio of $\sigma_{\nu d}$ obtained with the use of the NIJ II potential [23] to $\sigma_{\nu d}$(stnd). One can see that, in the solar neutrino energy region, the NSGK results agree with those of KN within ~1%. As the incident energy becomes higher, the NSGK results start to be somewhat larger than those of KN, the difference becoming ~6% towards the higher end of E_ν. This variance is well understood and it arises largely from the difference in the cutoff mass in the nucleon weak-form factor. Meanwhile, the dependence of $\sigma_{\nu d}$ on the nuclear potentials is found to be within 1% for all the reactions and for the entire energy region under study. The remaining model dependence to be checked is the structure and strength of the MEX operators, which will be discussed next.

[2] The IA results in NSGK are very similar to those of ref. [5], wherein only the IA term was considered.

Table 1
Model dependence of total cross sections. The second and third columns give, respectively, $\sigma_{\nu d}$(IA)/$\sigma_{\nu d}$(stnd) and $\sigma_{\nu d}$(KN)/$\sigma_{\nu d}$(stnd); here $\sigma_{\nu d}$(stnd) is the total cross section corresponding to our *standard case*. The fourth column gives the ratio of $\sigma_{\nu d}$ obtained with the use of the Nijmegen II potential to $\sigma_{\nu d}$(stnd). The last column shows the ratio of $\sigma_{\nu d}$ obtained with the Δ-current of ref. [14] to $\sigma_{\nu d}$(stnd).

$d(\nu,\nu)pn$				
E_ν[MeV]	IA	KN	NIJ	Δ(CRSW)
5	0.949	1.002	1.002	0.965
10	0.942	1.003	1.002	0.961
20	0.934	1.000	1.001	0.956
50	0.927	0.993	1.000	0.953
100	0.925	0.971	1.000	0.953
150	0.924	0.943	0.999	0.954
$d(\nu,e^-)pp$				
E_ν[MeV]	IA	KN	NIJ	Δ(CRSW)
5	0.952	1.019	1.003	0.968
10	0.945	1.008	1.002	0.964
20	0.937	1.002	1.001	0.959
50	0.928	0.990	1.000	0.956
100	0.924	0.968	0.999	0.956
150	0.922	0.941	0.999	0.956

Although, as mentioned, there is by now a substantial amount of evidence to support the basic soundness of PhLA, it is desirable to monitor its reliability here by simultaneously studying reactions that are closely related to the ν-d reactions. To this end, NSGK have used the $np \to \gamma d$ reaction and tritium β-decay. It turns out that the $\pi N\Delta$ vertex that features in the dominant exchange current for the ν-d reaction appears also in the $np \to \gamma d$ reaction, for which experimental cross sections are known for a wide range of the incident energy, from the thermal neutron energy up to the pion-production threshold. NSGK calculate both ν-d reaction and $np \to \gamma d$ cross sections in the same formalism and use the latter to gauge the reliability of their ν-d cross sections. With the use of the ANLV18 potential, NSGK obtain $\sigma(np \to \gamma d) = 335.1$ mb, with both the IA and MEX currents included. This is in good agreement with the experimental value, $\sigma(np \to \gamma d)^{exp} = 334.2 \pm 0.5$ mb [24]. With the IA contribution alone, the result would be $\sigma(np \to \gamma d)^{IA} = 304.5$ mb. The 10% contribution of MEX is due to the pion, pair and Δ-currents. Going beyond the thermal neutron energy regime, the $\sigma(np \to \gamma d)$ calculated in NSGK agree very well $\sigma(np \to \gamma d)^{exp}$ all the way up to $T_n \approx 100$ MeV. The fact that a PhLA calculation with no *ad hoc* adjustment of the input parameters is capable of reproducing $\sigma(np \to \gamma d)^{exp}$ for a very wide range of the incident energy gives us a reasonable degree of confidence in the basic idea of PhLA and the input parameters used.[3] Of course, strictly speaking, the electromagnetic and weak-interaction processes do not probe exactly the same sectors of PhLA, but the remarkable success with $\sigma(np \to \gamma d)$ gives, at least, partial justification of our PhLA as applied to weak-interaction reactions. Meanwhile, Carlson *et al.* [14], in estimating the solar *pp*-fusion cross section, used the tritium β-decay rate to fine-tune the $\pi N\Delta$ coupling constant that features in the Gamow-Teller exchange current. This method turns out to yield appreciably "quenched" Δ-excitation MEX effect in the *pp* fusion. It is therefore informative to examine the consequences of this second method for the ν-d reactions. The column labeled "Δ(CRSW)" in Table 1 shows the ratio of $\sigma_{\nu d}$ obtained with the use of the Δ-current employed in [14] to $\sigma_{\nu d}$(stnd). In the solar energy region this ratio is found to be 0.96 - 0.97, or the MEX contribution relative to the IA term is 2%, instead of 5% found in our *standard case*. At higher neutrino energies, the use of the Δ-current employed in [14] leads to a $\sim$4% MEX effect relative to the IA term, to be compared with the $\sim$8% effect found in our *standard case*. This change represents the largest uncertainty within the framework of PhLA.

As far as direct comparison with data is concerned, Tatara *et al.*'s estimate [4] of $\sigma(\nu_e + d \to e^- + p + p)$ averaged over the Michel spectrum of ν_e's agrees with the result of a stopped-pion-beam experiment [15] within large experimental errors (30%). Furthermore, the result of a Bugey reactor neutrino experiment [16] agrees, within 10% experimental errors, with the values of $\sigma(\bar{\nu}_e + d \to e^+ + n + n)$ and $\sigma(\bar{\nu}_e + d \to \bar{\nu}_e + p + n)$ given in KN [6]. These test also apply to the results of NSGK, since, at this level of precision, the cross sections reported in NSGK are equivalent to those of [4,6]. Furthermore, for $\mu^- d \to \nu_\mu nn$, the capture rate calculated in PhLA [4] agrees with the experimental value within the $\sim$10% experimental errors.

As emphasized by Bahcall *et al.* [21], one of the crucial quantities in neutrino oscillation studies at the SNO is the double ratio [NC]/[CC], where [NC] ([CC]) itself is the ratio of the observed neutrino absorption rate to the standard theoretical estimate for the NC (CC) reaction rate. This implies that the reliability of theoretical estimates for the ratio $R \equiv \sigma(NC)/\sigma(CC) \equiv \sigma(\nu d \to \nu np)/\sigma(\nu_e d \to e^- pp)$ is extremely important. We give in Table 2 the values of R resulting from the various models considered in NSGK. In the table we have chosen to normalize R by $R_{\rm stnd}$, the value corresponding to our *standard case*; $R_{\rm stnd}$ itself is shown in the second column of the table. We learn from Table 2 that all the models studied give essentially the same R; deviations from $R_{\rm stnd}$ are at most $\sim$1%. Thus, the largest source of model dependence in our work due to the Δ-

[3] Another similar success of PhLA is known in the $d(e,e')np$ reaction [13].

Table 2
Model dependence of $R \equiv \sigma(NC)/\sigma(CC) \equiv \sigma(\nu d \to \nu np)/\sigma(\nu_e d \to e^- pp)$. For representative values of E_ν [MeV], $R_{\rm stnd}$, the value of R for our *standard case* is given in the second column. The third, fourth and fifth columns give, respectively, R_a with a = IA, NIJ and Δ(CRSW), normalized by $R_{\rm stnd}$.

E_ν	$R_{\rm stnd}$	IA	NIJ	Δ(CRSW)
5	0.277	0.997	0.999	0.997
10	0.410	0.997	1.000	0.997
20	0.447	0.997	1.000	0.997
50	0.433	0.999	1.000	0.997
100	0.398	1.001	1.000	0.997
150	0.375	1.003	1.001	0.998

exchange current cancels out by taking the ratio between the NC and CC reactions.

3. Comparison with EFT calculations

In an EFT approach, one uses a general effective Lagrangian, $\mathcal{L}_{eff}$, that contains all possible terms compatible with given symmetries and a given order of expansion; the coefficient of each term in $\mathcal{L}_{eff}$ is called the low-energy coefficient (LEC). One then constructs nuclear transition operators and nuclear wave functions on the same footing, starting from $\mathcal{L}_{eff}$. The results of EFT calculations of the νd reactions [8,9] agree with those of PhLA in the following sense. Usually, some LEC's in EFT cannot be fixed by the symmetry requirements alone and hence need to be treated as parameters to be determined empirically. In [8,9], the coefficient L_{1A} of a four-nucleon axial-current counter term enters as an unknown parameter, although dimensional arguments suggest $|L_{1A}| \leq 6\,{\rm fm}^3$. According to [9], the cross sections obtained in EFT agree extremely well with those of the PhLA calculation for all the four reactions in Eqs.(1)-(4), by adjusting one parameter, L_{1A}. The optimal value of L_{1A} to reproduce the NSGK cross sections is $L_{1A} = 5.6\,{\rm fm}^3$ [9], a reasonable value as compared with the dimensional estimates. The fact that an *ab initio* calculation (modulo one free parameter) based on EFT is completely consistent with the results of PhLA provides further evidence for the basic reliability of PhLA. It is our view that, in calculating the low-energy ν-d cross sections, EFT and PhLA play complementary roles. EFT, being a general framework, is capable of giving model-independent results, *provided* all the LEC's in an effective Lagrangian $\mathcal{L}_{eff}$ are predetermined. At present, however, $\mathcal{L}_{eff}$ does contain an unknown LEC, L_{1A}.[4] Meanwhile, although PhLA is a model approach, its basic idea and the parameters contained in it have been tested using many observables. Thus, insofar as one accepts the validity of these tests, PhLA has predictive power. We should also mention that, beyond the solar neutrino energy regime, PhLA is at present the only available formalism for evaluating the ν-d cross sections. The EFT calculation in [8,9], by design, "integrates out" all the degrees of freedom but that of the heavy baryon. The nature of this so-called "nucleon-only" EFT limits its applicability to very low incident neutrino energies (typically the solar neutrino energies). On the other hand, there is no obvious conceptual obstacle in using PhLA in an energy regime significantly higher than that of the solar neutrinos. Therefore, once the reliability of PhLA is tested at low energies by comparison with experimental data or with the results of EFT, it is rather natural to use PhLA for higher energies as well.

4. Discussion and summary

We have examined the influence of changes in various inputs that feature in our PhLA. For the solar energy region, $E_\nu < 20$ MeV, the results are summarized as follows. We have confirmed that the ν-d cross sections are stable within 1% precision against any changes in the input that have been studied, except for somewhat higher sensitivity to the strength of the Δ-current. The MEX current in our *standard case* increases the cross sections by $\sim$5% from the IA values; we have used the $np \to \gamma d$ reaction to monitor the dominant part of our MEX current. If we use instead the Δ-excitation axial current renormal-

[4] In principle, however, it is possible to fix L_{1A} using a parity-violating electron scattering experiment [8,9].

ized by the tritium β-decay [14], the MEX current correction to the IA term turns out to be $\sim$2%, instead of 5% as in our *standard case*; see the column labeled Δ(CRSW) in Table 1. The difference between our *standard case* and Δ(CRSW) represents the range of uncertainty in the PhLA calculation of NSGK. We therefore consider it reasonable to use, as the best estimates of the low-energy νd cross sections, the values given by the *standard case* in NSGK and attach to them a possible *overall* reduction factor κ, with κ ranging from 0.96 to 1. In this language, the "1σ" uncertainty adopted by Bahcall *et al.* [21] corresponds to $\kappa = 0.95 - 1$, which represents the difference between the cross sections given in [5] and in [6].

We have demonstrated that in the ratio $R \equiv \sigma(\mathrm{NC})/\sigma(\mathrm{CC})$ the model dependence is reduced down to the 1% level. In fact, this model independence is a rather natural consequence of the fact that, as first noted in [3] and further checked by NSGK, the contribution of the isoscalar NC current is extremely small. The transition operators for the NC and CC reactions are then simply related by isospin rotations, and the only non-trivial differences between the NC and CC cross sections come from isospin breaking in the two-nucleon wave functions and the phase spaces. These "external" features, however, should have little model dependence once an NN potential is adjusted to fit the two-nucleon data.

REFERENCES

1. The SNO Collaboration, Phys. Lett. **B194**, 321 (1987); nucl-ex/9910016; G. T. Ewan *et al.*, Sudbury Neutrino Observatory Proposal SNO-87-12, 1987.
2. S. D. Ellis and J. N. Bahcall, Nucl. Phys. **A114** (1968) 636.
3. J.N. Bahcall, K. Kubodera and S. Nozawa, Phys. Rev. D **38** (1988) 1030.
4. N. Tatara, Y. Kohyama and K. Kubodera, Phys. Rev. C **42** (1990) 1694.
5. S. Ying, W. C. Haxton and E. M. Henley, Phys. Rev. C **45** (1992) 1982.
6. K. Kubodera and S. Nozawa, Int. J. Mod. Phys. E **3** (1994) 101.
7. S. Nakamura, T. Sato, V. Gudkov and K.Kubodera, nucl-th/0009012, submitted to Phys. Rev. C.
8. M. Butler and J.-W. Chen, Nucl. Phys. **A675** (2000) 575.
9. M. Butler, J.-W. Chen and X. Kong, nucl-th/0008032.
10. M. Chemtob and M. Rho, Nucl. Phys. **A163** (1971) 1.
11. E. Ivanov and E. Truhlik, Nucl. Phys. **A316**, (1979) 451, 437.
12. D. O. Riska and G. E. Brown, Phys. Lett. **38B** (1972) 193.
13. See e.g., B. Frois and J.-F. Mathiot, Com. Part. Nucl. Phys. **18** (1981) 291, and references therein.
14. J. Carlson, D. O. Riska, R. Schiavilla and R. B. Wiringa, Phys. Rev. C **44** (1991) 619.
15. S. E. Willis *et al.*, Phys. Rev. Lett. **44** (1980) 522.
16. S. P. Riley *et al.*, Phys. Rev. C **59** (1999) 1780.
17. T.-S. Park, D.-P. Min and M. Rho, Phys. Rev. Lett. **74** (1995) 4153; Nucl. Phys. **A596** (1996) 515.
18. T.-S. Park, K. Kubodera, D.-P. Min and M. Rho, Phys. Rev. C **58** (1998) 637; Astrophys. J. **507** (1998) 443; Nucl. Phys. **A646** (1999) 83; Phys. Lett. **B472** (2000) 232.
19. D. B. Kaplan, M. J. Savage and M. B. Wise, Nucl. Phys. **B478** (1996) 629; Phys. Lett. **B424** (1998) 390.
20. For the latest extensive reviews, see, U. van Kolck, Prog. Part. Nucl. Phys. **43** (1999) 337; S.R. Beane et al., nucl-th/0008064.
21. J. N. Bahcall, P. I. Krastev and A. Yu. Smirnov, hep-ph/0002293.
22. R. B. Wiringa, V. G. J. Stoks and R. Schiavilla, Phys. Rev. C **51** (1995) 38.
23. V. G. J. Stoks *et al.*, Phys. Rev. C **49** (1994) 2950.
24. A. E. Cox *et al.*, Nucl. Phys. **74** (1965) 497.
25. T. S. Suzuki *et al.*, Astrophys. J. **439** (1995) 59.