Quantum corrections of neutrino masses and mixings

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We discuss various aspects of the renormalisation of the neutrino mass operator. Due to quantum corrections, a given mixing at the GUT scale may be amplified or destroyed at low energies. The eigenvalues of the neutrino mass operator are also modified by quantum effects, which results in severe constraints on neutrino mass textures.

The Super-Kamiokande data [1] clearly indicate a ν_μ/ν_e ratio in the atmosphere that is significantly smaller than the Standard Model expectations. The most natural way to explain this deviation is by introducing ν_μ–ν_τ oscillations, with $\delta m^2_{\nu_\mu\nu_\tau} \approx (10^{-2}$ to $10^{-3})$ eV^2 and $\sin^2 2\theta_{\mu\tau} \geq 0.85$. Similarly, the solar neutrino deficit can be resolved through either vacuum or matter-enhanced (MSW) oscillations, requiring a mass splitting of the involved neutrinos in the range $\delta m^2_{\nu_e\nu_\alpha} \sim (0.5-1.1)\times 10^{-10}$ eV^2 and $\delta m^2_{\nu_e\nu_\alpha} \sim (0.3-20)\times 10^{-5}$ eV^2 respectively. It turns out that both deficits can be accommodated in minimal schemes with either three light neutrinos with hierarchical masses (of the order of the required mass differences) or with three almost-degenerate neutrinos with masses of ≈ 1 eV [2]. Naturally light neutrinos can be obtained by the see-saw mechanism, while the leptonic mixing matrix is given by $V_{MNS} = V_\ell V_\nu^\dagger$, where V_ℓ diagonalizes the charged-lepton mass matrix, while V_ν diagonalizes the light neutrino mass matrix, m_{eff}.

In the presence of neutrino masses, the running of the various couplings from the unification scale down to low energies is modified. For the neutrino sector, the Dirac neutrino Yukawa coupling, λ_N, runs until the scale M_N. Subsequently it decouples and the quantity that runs is the effective neutrino operator m_{eff}.

In order to understand the renormalization effects due to a non-zero λ_N between M_{GUT} and M_N, we can start with the small-$\tan\beta$ regime of a supersymmetric theory, where only the top and the Dirac neutrino Yukawa couplings contribute in a relevant way. In a diagonal basis, the renormalisation group equations of the Yukawa couplings take the following form:

$$
\begin{aligned}
16\pi^2 \frac{d}{dt}\lambda_t &= \left(6\lambda_t^2 + \lambda_N^2 - G_U\right)\lambda_t \\
16\pi^2 \frac{d}{dt}\lambda_N &= \left(4\lambda_N^2 + 3\lambda_t^2 - G_N\right)\lambda_N \\
16\pi^2 \frac{d}{dt}\lambda_b &= \left(\lambda_t^2 - G_D\right)\lambda_b \\
16\pi^2 \frac{d}{dt}\lambda_\tau &= \left(\lambda_N^2 - G_E\right)\lambda_\tau
\end{aligned} \tag{1}
$$

where $\lambda_\alpha : \alpha = t, b, \tau, N$, represent the third-generation Dirac Yukawa couplings for the up and down quarks, charged lepton and neutrinos, respectively, and the $G_\alpha \equiv \sum_{i=1}^{3} c_\alpha^i g_i(t)^2$ are functions that depend on the gauge couplings, with the coefficients c_α^i given in [3].

Below the right-handed Majorana mass scale, where m_{eff} is formed, λ_N decouples from the renormalisation group equations. However, the effective neutrino mass operator will be a running quantity. For a generic $\tan\beta$, one finds that

$$
\frac{1}{m_{eff}^{ij}}\frac{d}{dt}m_{eff}^{ij} = \frac{1}{8\pi^2}\left(-c_i g_i^2 + 3\lambda_t^2 + \frac{1}{2}(\lambda_i^2 + \lambda_j^2)\right)
$$

where i, j are lepton flavour indices, already indicating that large Yukawa terms, which lower the effective couplings, have a larger effect on m_{eff}^{33} than on the other entries. Finally, the neutrino mixing angle relevant to the atmospheric neutrino deficit, θ_{23}, is also a running quantity, given by the following formula [4]:

$$16\pi^2 \frac{d}{dt}\sin^2 2\theta_{23} = 2\sin^2 2\theta_{23}(1-2\sin^2\theta_{23}) (\lambda_\tau^2 - \lambda_\mu^2)\ \frac{m_{eff}^{33}+m_{eff}^{22}}{m_{eff}^{33}-m_{eff}^{22}} \quad (2)$$

where the initial conditions for the running from M_N down to low energies are determined by the running of couplings between M_{GUT} and M_N.

Already this equation indicates that the mixing angle may significantly change from the GUT scale to low energies (i) if λ_τ is large, and (ii) if the diagonal entries of m_{eff} are close in magnitude. Because of the running of the τ Yukawa coupling being larger than those for the other flavours of charged leptons, m_{eff}^{33} decreases more rapidly than m_{eff}^{22}. Then, if we start with $m_{eff}^{22} < m_{eff}^{33}$, at a later time we may obtain $m_{eff}^{22} = m_{eff}^{33}$ and the mixing angle becomes *maximal*. Subsequently, the running of λ_τ results in $m_{eff,0}^{33} < m_{eff,0}^{22}$, and thus in a decrease of the mixing.

Moreover, we also observe that the neutrino masses will in fact vary non-trivially with the energy. It is convenient for the subsequent discussion to define the integrals

$$I_g = \exp[\frac{1}{8\pi^2}\int_{t_0}^{t}(-c_i g_i^2 dt)] \quad (3)$$

$$I_t = \exp[\frac{3}{8\pi^2}\int_{t_0}^{t}\lambda_t^2 dt] \quad (4)$$

$$I_i = \exp[\frac{1}{8\pi^2}\int_{t_0}^{t}\lambda_i^2 dt], \quad i = e,\mu,\tau \quad (5)$$

Then,

$$\frac{m_{eff}^{ij}}{m_{eff,0}^{ij}} = I_g \cdot I_t \cdot \sqrt{I_i}\cdot\sqrt{I_j} \quad (6)$$

where the initial conditions are denoted by $m_{eff,0}^{ij}$. As we have already mentioned, these conditions are defined at M_N, the scale where the neutrino Dirac coupling λ_N decouples from the renormalisation-group equations.

From (6), we see that the relative structure of m_{eff} is only modified by the charged-lepton Yukawa couplings. On the contrary, the top and gauge couplings give only an overall scaling factor. In order to quantify the renormalization effects on the physical neutrino masses, we can start with a texture which, at the GUT scale, leads to three *exactly degenerate* neutrinos, such as:

$$m_{eff} \propto \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{\sqrt{2}} & -\frac{1}{2} & \frac{1}{2} \end{pmatrix} \quad (7)$$

with scaled eigenvalues 1,-1,1 and calculate the respective values at low energies [5]. We take as illustrative initial conditions $\alpha_{GUT}^{-1} = 25.64$, $M_{GUT} = 1.1 \cdot 10^{16}$ GeV and $m_{SUSY} = 1$ TeV. We also choose λ_b/λ_τ such that an intermediate scale $M_N = 10^{13}$ GeV is consistent with the observed pattern of fermion masses. This way, we calculate the effects on the neutrino eigenvalues, mixings and mass differences, as shown in the last three columns of Table 1.

λ_τ	m_3	m_2	m_1
3.0	0.866	-0.952	0.997
1.2	0.903	-0.966	0.998
0.48	0.962	-0.987	0.9996
0.10	0.9478	-0.9993	0.99998
0.013	0.99998	-0.99999	1.00000

Table 1
The renormalized mass eigenvalues calculated for three exactly degenerate neutrinos for the unrenormalised texture, for $M_N = 10^{13}$ GeV.

We see that the renormalization effects on the neutrino-mass eigenvalues are significant and spoil the neutrino degeneracy. It is apparent from Table 1 that the breaking of the neutrino-mass degeneracy in this model is unacceptable even for small $\tan\beta$ [5]. However, we should add that ways to stabilise the neutrino textures have been proposed: For instance, one can have textures that owing to a symmetry are already non-degenerate at the high scales of the theory [6]. Moreover, it may be that the structure of the Dirac neutrino mass matrices stabilises the textures [7], and examples where this can happen have been found.

Moreover, there might be additional intrinsic instabilities on the neutrino mixing. To see this, let us look at the texture in 7. Since it has two exactly degenerate eigenvalues, there is arbitrariness in the choice of eigenvectors: the vectors corresponding to the two degenerate eigenvalues are not linearly independent, and can be rotated to different linear combinations, which still obey the orthogonality conditions. One example is

$$V_1' = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}, \quad V_2' = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}, \quad V_3' = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}$$

corresponding to bimaximal mixing: $\phi_1 = \frac{\pi}{4}$, $\phi_2 = 0$ and $\phi_3 = \frac{\pi}{4}$. However, this is an unstable solution. To see this, let us consider a perturbation ϵ from the texture in eq.(7):

$$m'_{eff} \propto \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}(1+\frac{\epsilon}{2}) \\ \frac{1}{\sqrt{2}} & \frac{1}{2} & -\frac{1}{2}(1+\frac{\epsilon}{2}) \\ \frac{1}{\sqrt{2}}(1+\frac{\epsilon}{2}) & -\frac{1}{2}(1+\frac{\epsilon}{2}) & \frac{1}{2}(1+\epsilon) \end{pmatrix}$$

Here, ϵ is a small quantity, which might arise from renormalisation group running or from some other higher-order effects. This perturbation lifts the degeneracy of the eigenvalues, which are now given by

$$1, \quad -1-\frac{\epsilon}{4}, \quad 1+\frac{3\epsilon}{4}$$

To this order, the eigenvectors are independent of ϵ and given by

$$V_1 = \begin{pmatrix} \frac{1}{\sqrt{3}} \\ \sqrt{\frac{2}{3}} \\ 0 \end{pmatrix}, \quad V_2 = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}, \quad V_3 = \begin{pmatrix} \frac{1}{\sqrt{6}} \\ -\frac{1}{2\sqrt{3}} \\ \frac{\sqrt{3}}{2} \end{pmatrix}$$

We see therefore that the stable solution is away from the bimaximal mixing.

To summarise, we discussed various aspects of the renormalisation effects of neutrino masses and interactions. Particularly in supersymmetric models with large $\tan\beta$, small mixing at the GUT scale may be amplified to maximal mixing at low energies, and vice versa. The eigenvalues of the neutrino mass operator are also modified by quantum corrections; given the very small mass differences required to address the solar and atmospheric neutrino deficits, several neutrino textures (especially those with degenerate neutrinos with an eV mass scale), can be constrained or, in certain models, even excluded.

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