Solar neutrino oscillations in the quasi-vacuum regime

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Motivated by recent experimental data, we study solar neutrino oscillations in the range $\delta m^2/E \in [10^{-10}, 10^{-7}]$ $\mathrm{eV}^2/\mathrm{MeV}$. In this range vacuum oscillations become increasingly affected by (solar and terrestrial) matter effects for increasing δm^2, smoothly reaching the MSW regime. A numerical study of matter effects in such "quasi-vacuum" regime is performed. The results are applied to the analysis of the recent solar neutrino phenomenology.

1. Introduction

Flavor oscillations either in vacuum [1] or in matter [2] provide a viable explanation to the solar neutrino problem [3]. The corresponding survival probability $P_{\nu_e\to\nu_e}$ is determined by the mass-mixing parameters δm^2 and ω, as well as by the electron density profile $N_e(x)$ along the ν path.

For solar neutrino oscillations three characteristic lengths can be identified: the astronomical unit $L = 1.496\times 10^8$ km, the oscillation wavelength in vacuum:

$$L_{\rm osc} = \frac{4\pi E}{\delta m^2} = 2.48\times 10^{-3}\frac{\delta m^2}{E}\ \mathrm{km}\ , \tag{1}$$

and the the refraction length in matter:

$$L_{\rm mat} = \frac{2\pi}{\sqrt{2}G_F N_e} = \frac{1.62\times 10^4}{N_e}\ \mathrm{km}\ , \tag{2}$$

where δm^2 is expressed in eV^2, the neutrino energy E in MeV and the electron density N_e in $\mathrm{mol/cm}^3$. Usually, two asymptotic regimes can be identified: 1) the "Mykheyev-Smirnov-Wolfenstein" (MSW) oscillation regime [2] ($\delta m^2/E \geq 10^{-7}$ $\mathrm{eV}^2/\mathrm{MeV}$) characterized by $L_{\rm osc}^{\rm MSW} \sim L_{\rm mat} \ll L$, and the "vacuum" (VAC) oscillation regime [4] ($\delta m^2/E \sim O(10^{-11})$ $\mathrm{eV}^2/\mathrm{MeV}$), characterized by $L_{\rm osc}^{\rm VAC} \sim L \gg L_{\rm mat}$. In the MSW regime, the many oscillation cycles in vacuum ($L_{\rm osc}^{\rm MSW} \ll L$) are responsible for complete decoherence of oscillations, and the survival probability depends on the detailed density profile in the Sun (and, during nighttime, in the Earth). Conversely, in the VAC regime the matter effects suppress oscillations in the Sun and in the Earth, so that (coherent) flavor oscillations take place only in vacuum, starting approximately from the Sun surface [5].

However, there is a transition regime [that might be called of "quasi-vacuum" (QV) oscillations], corresponding to the range $10^{-10} \leq \delta m^2/E \leq 10^{-7}$ $\mathrm{eV}^2/\mathrm{MeV}$, in which oscillations become increasingly affected by matter effects, and decreasingly decoherent, for increasing values of $\delta m^2/E$. In this regime, characterized by $L_{\rm mat} \leq L_{\rm osc}^{\rm QV} \leq L$, none of the approximations used in the MSW or in the vacuum regimes can be applied. In the past, QV oscillations have been considered only in a in few papers (see [6] and reference therein) since typical fits to solar ν rates allowed only marginal solutions in the range where QV effects are relevant. However, more recent analyses appear to extend the former ranges of the VAC solutions *upwards* and of the MSW solutions *downwards* in $\delta m^2/E$, making them eventually merge in the QV range (see, e.g., [7,8]).

The QVO regime hase been recently studied in detail by Friedland in [9]. Other numerical [6] and analytical [10] studies of QVO have also been performed. Here we discuss the results of [6] in the context of the most recent solar neutrino data.

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2. Calculation of the survival probability

From general quantum-mechanical arguments we can derive the probability that a ν_e produced in the interior of the Sun is detected as a ν_e on the Earth [6,11]:

$$\begin{aligned} P_{\nu_e\to\nu_e} &= P_\odot P_\oplus + (1-P_\odot)(1-P_\oplus) \\ &\quad + 2\sqrt{P_\odot(1-P_\odot)P_\oplus(1-P_\oplus)}\cos\xi \ , \end{aligned} \quad (3)$$

where $P_\odot$ and $P_\oplus$ are the transition probability $P_{\nu_e\leftrightarrow\nu_1}$ along the two partial paths inside the Sun (up to its surface) and inside the Earth (up to the detector), and the total phase ξ can be written as

$$\xi = \frac{\delta m^2}{2E}(L-R_\odot) + \xi_\odot + \xi_\oplus \ , \quad (4)$$

where $R_\odot$ is the solar radius and $\xi_\odot$ ($\xi_\oplus$) is the phase acquired in the Sun (Earth) matter.

Equation (3) is a general formula that smoothly interpolates from the vacuum to MSW regime. In particular, in the vacuum limit $P_\odot \simeq c^2_\omega \simeq P_\oplus$ and $\xi_\odot,\ \xi_\oplus \simeq 0$, we obtain the standard vacuum formula. Conversely, in the MSW limit the oscillating term is averaged away and $P_\odot \simeq \sin^2\omega^0_m P_c + \cos^2\omega^0_m(1-P_c)$, where ω^0_m is the mixing angle at the production point and P_c is the crossing probability $P_{\nu^m_2\to\nu_1}$ (where ν^m_2 is the heavier eigenstate in matter), we obtain the standard Parke's formula [12].[2]

he evolution inside the Sun has been performed by direct numerical integration of the MSW equation from solar the center to the solar surface.[3] The electron density was taken from [13]. Figure 2 in [6] shows, in the mass-mixing plane, isolines of the difference $c^2_\omega - P_\odot$ (solid curves), which becomes zero in the vacuum oscillation limit of very small $\delta m^2/E$. The variable $\tan^2\omega$ has been chosen in order to chart the first two octants of the mixing angle range. In [10] a semianalytical approximation of $P_\odot(\simeq P_c)$ has been find by modifying the standard resonance prescription [14] with the Maximum Violation of Adiabaticity (MVA) prescription. This approximation gives $P_\odot$ with a percent accuracy.

[2]In all the QV range $\omega^0_m \simeq \pi/2$ so that $P_\odot \simeq P_c$.

[3]In [6] has been explicitly verified that the results are independent from the specific ν production point, which can be effectively taken at the Sun center.

The evolution inside the Earth is done by evolving analytically the MSW equations at any given nadir angle η, using the technique described in [15], which is based on a five-step biquadratic approximation of the density profile from the Preliminary Reference Earth Model (PREM) [16] and on a first-order perturbative expansion of the neutrino evolution operator. Figure 4 in [6] shows isolines of the quantity $c^2_\omega - P_\oplus$, which becomes zero in the vacuum oscillation limit of very small $\delta m^2/E$ for two representative values of the nadir angle.

Regarding the phases $\xi_\odot$ and $\xi_\oplus$ in Eq. (4), it has been shown in [6] that they are negligible for current fits. Only $\xi_\odot$ could have non-negligible effects in future high-statistics experiments. A semianalytic formula for the calculation of $\xi_\odot$ has been recently derived in [10].

3. Decoherence of the oscillating term

For increasing values of $\delta m^2/E$, the oscillating term in Eq. (3) become increasingly suppressed by energy smearing. The onset of the corresponding decoherence effect is best studied for almost monochromatic neutrinos, namely for the ^{7}Be and *pep* lines, having a width $\Delta E \sim O(\mathrm{keV})$ [17]. When L_{osc}/L become greater of $\Delta E/\langle E\rangle$ (where $\langle E\rangle$ is the average neutrino energy) the damping effect is important. This happen for $\delta m^2/E \geq 10^{-8}$ eV^2/MeV (and before for continuos spectra).

For narrow line spectra, the energy-averaged oscillating factor can be written as $\langle\cos\xi\rangle \simeq D(\delta m^2 L/2\langle E\rangle^2)\cos\xi$, where D is the modulus of the Fourier-transform of the spectrum [6]. Figure 8 in [10] shows the function D as function of $\delta m^2/2\langle E\rangle$ for the ^{7}Be and *pep* lines. As anticipated, damping factor is important for $\delta m^2/E \geq 2\times 10^{-8}$ eV^2/MeV. For higher values of $\delta m^2/E$, the oscillating term can be completely neglected.

The survival probability $P_{\nu_e\to\nu_e}$ has to be averaged in time, since it depends from t, both through $P_\oplus$ via η and through the phase ξ via the earth orbit eccentricity. While $P_\oplus(t)$ and $\cos\xi(t)$ can be be easily averaged separately [6,15], the calculation of the time average of the term $\sqrt{P_\oplus(1-P_\oplus)}\cos\xi$ is in principle tedious.

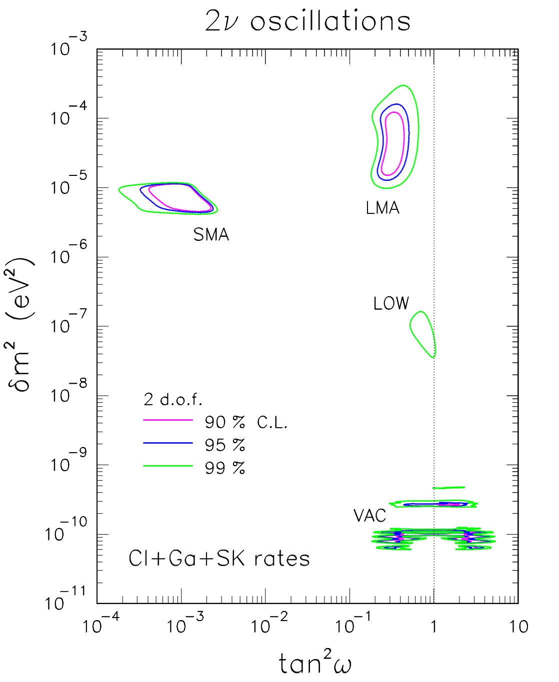

Figure 1. 2ν solutions to the solar neutrino problem at 90, 95 and 99% C.L. of CL+GA+K+SK rates.

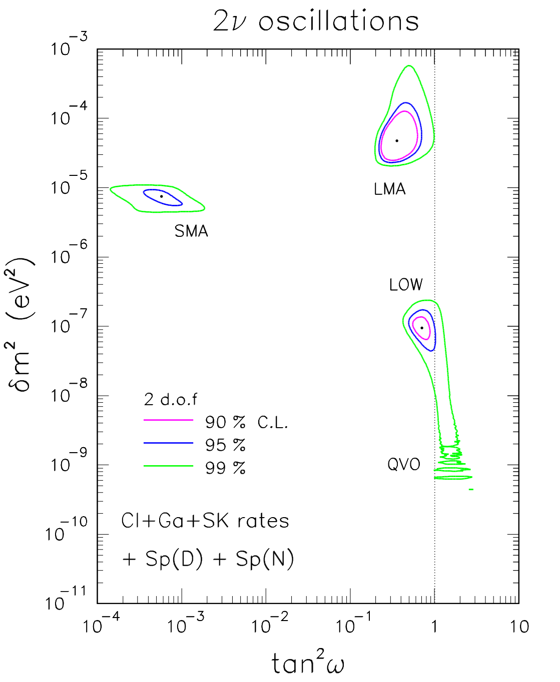

Figure 2. 2ν solutions to the solar neutrino problem at 90, 95 and 99% C.L. of all data.

Fortunately, the Earth effect is non negligible only when the oscillating term is doubly suppressed ($P_\odot \sim 0$ and $D \sim 0$). Therefore, in Eq. (3) one can safely take $P_\oplus \simeq c^2_\omega$ as far as the oscillatory term is non negligible.

4. 2ν solutions to the solar neutrino problem

In this Section we apply the previous results to the analysis of the more recent solar neutrino phenomenology. Usually, the analysis is performed in the first octant in ω and splitting the δm^2 range in the vacuum (few $\cdot\, 10^{-11} \leq \delta m^2 \leq$ few $\cdot\, 10^{-10}$ eV^2) and MSW ($\delta m^2 \geq 10^{-8}$ eV^2) subranges.

The lack of observation of a distorsion in the recoil electron spectrum in SuperKamiokande, together with a rate observed in SuperKamiokande and Gallium experiments compatible with about 1/2 suppression of the ν_e flux, strongly favor large mixing solutions. The LOW and VAC solutions tend to merge in the QV part of the plane, and acceptable solutions appear also for $\omega > \pi/4$ [7,8]. Consequently, the correct way to show the solutions is to use the entire mass-mixing plane without the artificial splitting in δm^2 and for $0 \leq \omega \leq \pi/2$.

Figure 1 shows the combined analysis of the total rate information coming from the SuperKamiokande (SK) [18], Homestake [19], SAGE [20], and GALLEX-GNO [21] experiments. Figure 2 shows the combination between rate information and day and night spectral information from SK [18]. The technical details of the analysis are explained in [22] and references therein. The value of $\chi^2_{\rm MIN}$ is 35.1 for the LMA, 37.8 for the LOW and 40.7 for the SMA solution, for 36 degrees of freedom.

The LMA and LOW solutions appear to be favored over the SMA solution in the global fit, although it is rather premature to think that the latter is ruled out. The vacuum solutions appear strongly disfavored by the nonobservation of SK spectral distorsions. From Fig. 2 we see that a solution to the solar neutrino problem emerges at 99% C.L. in the QV range. The matter effects in QV solutions break the symmetry $\omega \to \pi/2 - \omega$ valid in the vacuum range (and that can be well observed in the vacuum solutions in Fig. 1). From Fig. 2 it appears that large mixing angle solutions are favored. For this reason, several experimental tests have been proposed to check them [23,24].

5. Conclusions

We have studied solar neutrino oscillations in the hybrid regime $\delta m^2/E \in [10^{-10}, 10^{-7}]$ $\mathrm{eV}^2/\mathrm{MeV}$ ("quasi-vacuum" regime), in which the familiar approximations suitable in the vacuum and MSW regime are not valid. We have discussed a general formula that interpolates smoothly between vacuum and MSW regimes. The effects of energy smearing on the oscillating term and the Earth matter effect have been taken in account. The results have been applied to the analysis of the most recent solar neutrino data, and suitable solutions to the solar ν problem have been found in the QV range.

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