Impact of Large Mixing Angle MSW Solution on Neutrino Mass Matrix

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We have discussed the quark-lepton mass matrices with the $U(1)$ and $O(3)$ flavor symmetry in SU(5), which lead to the large mixing angle MSW solution of solar neutrinos.

1. Introduction

Super-Kamiokande has almost confirmed the neutrino oscillation in atmospheric neutrinos, which favors the $\nu_\mu \to \nu_\tau$ process with $\sin^2 2\theta_{\rm atm} \geq 0.88$ and $\Delta m^2_{\rm atm} = (1.5 \sim 5) \times 10^{-3}{\rm eV}^2$ [1]. For the solar neutrinos [2], the 1117 days data in Super-Kamiokande favors the LMA-MSW solution [3], which is $\sin^2 2\theta_\odot = 0.65 \sim 0.97$ and $\Delta m^2_\odot = 10^{-5} \sim 10^{-4}{\rm eV}^2$. However, there are still allowed four solutions, the small mixing angle (SMA) MSW [4], the large mixing angle (LMA) MSW, the low Δm^2 (LOW) and the vacuum oscillation (VO) solutions.

There are a lot of ideas for the single large mixing angle in the neutrino mixing matrix (MNS matrix) [5]. However, it is not easy to get the nearly bi-maximal mixings with the LMA-MSW mass scale in the GUT models [6]. Therefore, it is important to search for the texture of the lepton mass matrix with the LMA-MSW solution.

In this paper, we study how to get the LMA-MSW solution with the help of the $U(1)$ and $O(3)$ flavor symmetries.

2. $U(1)$ flavor symmetry

Vissani [7] has already shown that the texture of the neutrino mass matrix with the $U(1)$ flavor symmetry leads to the LMA-MSW solution. Recently, Sato and Yanagida [8] have also studied this texture numerically and showed clearly that both the LMA-MSW and SMA-MSW solutions are obtained. However, since they have not discussed the quark mass matrices, simple relations among the quark-lepton mass matrices are not obtained. We discuss the quark-lepton mass matrix textures with the $U(1)$ flavor symmetry in the SU(5) GUT focusing on the solar neutrino solutions.

Let us consider the $U(1)$ flavor symmetry [9,10], in which fermions carry $U(1)$ charges, $U(1)$ is spontaneously broken by the VEV of the elctroweak singlet with $U(1)$ charge -1, and Yukawa couplings appear as effective operators through Froggatt-Nielsen mechanism [11].

We discuss the LMA-MSW solution in the SU(5) GUT. Taking the U(1) charges of $\mathbf{5}^*$ and $\mathbf{10}$ fermions for the (1st, 2nd, 3rd) family [12] in order to get the LMA-MSW solution:

$$\Psi_{\mathbf{5}^*} \sim (2,\ 0,\ 0)\ , \quad \Psi_{\mathbf{10}} \sim (3,\ 2,\ 0)\ , \qquad (1)$$

and putting $< S >/\Lambda = \lambda$, we obtain the quark mass matrices, which are consistent with the experimental values of the quark masses and the CKM mixing matrix, except for the value of the lightest u-quark mass.

The lepton mass matrices are given as follows:

$$M_E \sim \begin{pmatrix} \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} M_\nu \sim \begin{pmatrix} \lambda^4 & \lambda^2 & \lambda^2 \\ \lambda^2 & 1 & 1 \\ \lambda^2 & 1 & 1 \end{pmatrix} \qquad (2)$$

Maximal mixings of both (2-3) sectors lead to the nearly maximal mixing of the MNS matrix without cancellation if the relevant phase is chosen.

On the other hand, the left handed mixings between the first and the second (third) family should be carefully examined. The left-handed mixings in the charged lepton sector are given as $s^E_{12} \simeq s^E_{13} \simeq \lambda^2$ [13].

How large is the (1-2) family mixing in the neutrino sector? In order to answer this question, we discuss the following texture:

$$M_\nu \sim \begin{pmatrix} \epsilon_{11}^2 & \epsilon_{12} & \epsilon_{13} \\ \epsilon_{12} & 1 & 1 \\ \epsilon_{13} & 1 & 1 \end{pmatrix} \ , \tag{3}$$

where ϵ_{ij}'s are the same order except for factors of order one. At first, diagonalizing the (2-3) submatrix of this matrix by $s_{23}^\nu = 1/\sqrt{2}$, we get the mass matrix in the new basis:

$$\overline{M}_\nu \sim \begin{pmatrix} \overline{\epsilon}_{11}^2 & \overline{\epsilon}_{12} & \overline{\epsilon}_{13} \\ \overline{\epsilon}_{12} & \delta & 0 \\ \overline{\epsilon}_{13} & 0 & 1 \end{pmatrix} \ , \tag{4}$$

where $\overline{\epsilon}_{ij}$ are given in terms of ϵ_{ij}. The mass matrix in eq.(4) gives

$$\sin^2 2\theta_\odot = \frac{4\overline{\epsilon}_{12}^2}{(\delta - \overline{\epsilon}_{11}^2)^2 + 4\overline{\epsilon}_{12}^2} \ , \tag{5}$$

$$\Delta m_\odot^2 = (\delta + \overline{\epsilon}_{11}^2)\sqrt{(\delta - \overline{\epsilon}_{11}^2)^2 + 4\overline{\epsilon}_{12}^2}\ \Delta m_{\rm atm}^2,$$

where $\theta_{12} = \theta_\odot$ is taken. After eliminating the unknown parameter δ, we obtain

$$\frac{\Delta m_\odot^2}{\Delta m_{\rm atm}^2} = 4\frac{\overline{\epsilon}_{12}^2}{\sin^2 2\theta_\odot}(\cos 2\theta_\odot \pm \frac{\overline{\epsilon}_{11}^2}{\overline{\epsilon}_{12}}\sin 2\theta_\odot) \ . \tag{6}$$

Since ϵ_{12} and ϵ_{13} are complex in general, $\overline{\epsilon}_{12}$ is expected to be comparable to them.

Let us consider the case of $\epsilon_{12} \simeq \epsilon_{13} \simeq \lambda^2$, which corresponds to the model in eq.(2). Taking $\overline{\epsilon}_{11} \simeq \overline{\epsilon}_{12} \simeq \lambda^2$, we show the allowed region on the $\sin^2\theta_\odot - \Delta m_\odot^2$ plane in Fig.1, where we take a typical value $\overline{\epsilon}_{11} = \overline{\epsilon}_{12} = 0.06$. The upper (lower) curve corresponds to the plus (minus) sign in the right hand side of eq.(6) with $\Delta m_{\rm atm}^2 = 5\ (1.5) \times 10^{-3}{\rm eV}^2$. In order to show the allowed region in the dark side [14], we use $\sin^2\theta_\odot$ as the horizontal axis. As seen in Fig.1, our mass matrices are consistent with only the LMA-MSW solution.

However, if the phase of ϵ_{12} is equal to the one of ϵ_{13}, $\overline{\epsilon}_{12}$ could be supressed. When we take $\overline{\epsilon}_{11} \simeq \lambda^2$ and $\overline{\epsilon}_{12} \ll \lambda^2$, we can get another allowed region as seen in Fig.1. We choose the typical values $\overline{\epsilon}_{11} = 0.06$ and $\overline{\epsilon}_{12} = 0.003$ in Fig.1. This case is consistent with both the SMA-MSW and the LOW solutions.

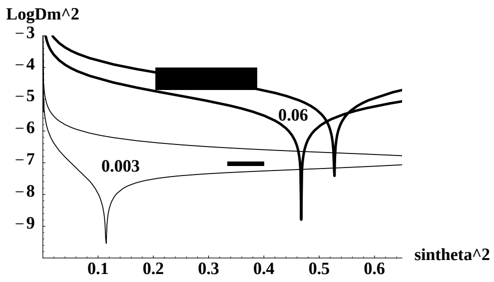

Figure 1. The allowed region on the $\sin^2\theta_\odot - \log_{10}(\Delta m_\odot^2/{\rm eV}^2)$ plane. The upper region corresponds to $\overline{\epsilon}_{12} = 0.06$ and the lower one to $\overline{\epsilon}_{12} = 0.003$. The black rectangle regions show the LMA-MSW, the SMA-MSW and the LOW solutions.

It is also important to discuss the magnitude of U_{e3}. Since U_{e3} is given approximately as $\max\{s_{12}^E,\ s_{13}^E,\ s_{13}^\nu\}$, its expected magnitude is λ^2, which is consistent with the CHOOZ data [15].

Since the parameter δ is give as

$$\delta \simeq 2\overline{\epsilon}_{12}\cot 2\theta_\odot + \overline{\epsilon}_{11}^2 \ , \tag{7}$$

the LMA-MSW solution corresponds to $\delta \simeq \lambda^2$. However, there is no principle to fix δ in advance within the framework of the $U(1)$ flavor symmetry. In other words, the lepton mass matrices in eq.(2) are consitent with the LMA-MSW, SMA-MSW and LOW solutions. Thus, these mass matrices cannot predict the solar neutrino solution because δ is the unknown parameter.

3. $O(3)$ flavor symmetry

There is a another aspect for the large mixings. If two or three families belong same multiplet, (for example, S_3 or $O(3)$ symmetry) the mass matrices in the symmetric limit are given as

$$M_E = \frac{c_E}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix},$$

$$M_\nu = c_\nu \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + b_\nu \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \qquad (8)$$

where $b_\nu \ll 1$ in $O(3)$ flavor symmetry [16]. The orthogonal matrix, which diagonalizes M_E

$$F = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \\ -1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \\ 0 & -2/\sqrt{6} & 1/\sqrt{3} \end{pmatrix}. \qquad (9)$$

Then, we get the mixing matrix

$$U_{MNS} \simeq F^T = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{6} & 1/\sqrt{6} & -2/\sqrt{6} \\ 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \end{pmatrix} \qquad (10)$$

which gives

$$\sin^2 2\theta_\odot = 1 \ , \qquad \sin^2 2\theta_{\rm atm} = \frac{8}{9}. \qquad (11)$$

Thus, one can get easily the nearly bi-maximal mixings in the non-abelian flavor symmetry.

4. Summary

We have discussed the lepton mass matrices with the $U(1)$ flavor symmetry. It is found that the solar neutrino solution depends on the parameters of the next-leading terms such as δ and $\overline{\epsilon}_{12}$. We also present $O(3)$ flavor symmetry, which leads to the bi-maximal mixings. In this case, the neutrino masses are nearly degenerated. Can we accept degenerated neutrino masses ? Fortunately, the neutrinoless $\beta\beta_{0\nu}$ experiments can test this case. Thus, the solar neutrino solution makes a big impact on the lepton mass matrix with the flavor symmetry. We expect that the LMA-MSW solution will be tested in KamLAND as well as SNO in the near future.

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