Non-Oscillation Searches of Neutrino Mass in the Age of Oscillations

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We focus on the implications of the oscillations for the shape of nuclear β-spectrum (=direct search for ν mass). This is of interest because of the existing bound, $m_{\nu_e} < 2.2$ eV, that could improve by one order of magnitude with future experiments. We stress important connections with the results of Liquid Scintillator Neutrino Detector (LSND), ν_e disappearance experiments, supernova (SN) neutrinos and neutrinoless double beta decay ($0\nu 2\beta$).

1. Massive Neutrinos and β-Spectrum

The nuclear β-spectrum with emission of N (undetected and not too heavy) massive ν's is the weighted sum of individual β-spectra [1,2]:

$$d\Gamma_{mass} = \sum_{j=1}^{N} |U_{ej}^2| \times d\Gamma(m_j^2), \quad m_j \le m_{j+1}$$

What is the "effective" mass $m_{\nu_e}^2$, for which $d\Gamma_{mass} \sim d\Gamma(m_{\nu_e}^2)$? Let us work out an answer. For $E_\nu \ll Q$, the behaviour of the β-spectrum is just due to phase space: $d\Gamma(m_j^2) \propto E_\nu (E_\nu^2 - m_j^2)^{1/2} dE_\nu$. If also the condition $E_\nu \gg m_j$ holds, we can approximate $d\Gamma(m_j^2) \propto [E_\nu^2 - m_j^2/2] dE_\nu$. In these hypotheses, $d\Gamma_{mass} \propto E_\nu^2 \sum_j |U_{ej}^2| - \sum_j |U_{ej}^2| m_j^2/2] dE_\nu$, so one is lead to define:

$$m_{\nu_e}^2 \equiv \frac{\sum_j |U_{ej}^2| \times m_j^2}{\sum_j |U_{ej}^2|} \quad (\text{or } \equiv \sum_j |U_{ej}^2| \times m_j^2)$$

The simpler formula [3] (that we keep for reference) holds if unitarity is assumed; or, in practice, if the normalization is not checked experimentally. A warning: In endpoint type experiments, the sums on j extend *just* on neutrino masses within the region of measurement (typically much narrower than the E_ν range, $E_\nu \le Q$).

Is is likely to measure something more than $m_{\nu_e}^2$? The answer is conditional; *No*, if the energy resolution δE_ν is larger than the level spacing; for this would mean averaging the β-spectrum $\langle d\Gamma_{mass}\rangle$, reducing it to $\langle d\Gamma(m_{\nu_e}^2)\rangle$ (far from endpoint the effective spectrum reproduces well the true one, as seen in fig. 1). *Maybe yes*, if one level (at least) stands out from the other ones. So the answer depends essentially on the detector characteristics. To be specific, if we assume that ν_H has mass in the $5-10$ eV region, $m_{\nu_e}^2$ could be not adequate to describe the existing tritium endpoint data [3,4] (unless the mixing of ν_H with ν_e is so little, that its effect is invisible). We assume that this does not happen; thus, a single parameter describes the modifications of the nuclear β-spectrum. However, we stress again that a single parameter suffices *if* the spectrum is "not resolved".

The parameter $m_{\nu_e}^2$ can be compared with $\mathcal{M}_{ee}^2 = |\sum_j |U_{ej}^2| \times m_j \times \exp(i\xi_j)|^2$ (the ee-entry of the Majorana neutrino mass matrix, squared) that leads to $0\nu 2\beta$ [5]. Note that:

- The presence of the Majorana phases ξ_j, that can produce cancellations in $\mathcal{M}_{ee}$;
- The different dependence on $|U_{ej}^2|$: Individual contributions scale as $\delta\mathcal{M}_{ee} = |U_{ej}^2| \times m_j$, while $\delta m_{\nu_e} = |U_{ej}| \times m_j$. Thence, the 1^{st} parameter is more severely suppressed than the 2^{nd} by $|U_{ej}^2|$.

2. The Connection with Oscillations

Mainz [3] and Troitsk [4] Collaborations pushed the limit on m_{ν_e} down to 2.2 eV, and there are plans under discussion to reach the $200-400$ meV level. Being at the Neutrino Oscillation Workshop, it is natural to ask what we expect for $m_{\nu_e}^2$, if neutrino do oscillate. With little algebra:

$$\begin{aligned} m_{\nu_e}^2 &= \textstyle\sum_j |U_{ej}^2| \times m_j^2 = \\ &\textstyle\sum_{j>1} |U_{ej}^2| \times \Delta m_{j1}^2 + m_1^2 \equiv \delta m^2 + m_1^2 \end{aligned} \quad (1)$$

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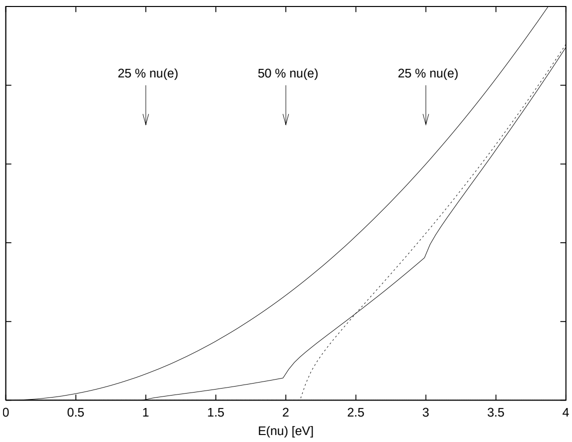

Figure 1. *Illustrative* endpoint spectrum, shown as function of E_ν, in arbitrary unities. Arrows show the position of the mass levels, with indicated values of $|U^2_{ej}|$. True spectrum (continuous line) is compared with the spectrum with effective mass $m^2_{\nu_e}$ (dotted). Massless spectrum is also shown for comparison. [A comment on two more *realistic* cases: 1) for a `SAL` 4-ν mass spectrum, an "LSND peak" at ~ 1 eV can be visually appreciated only after magnifying by some $100\times$ on $E_\nu <$ few eV; 2) for a `SA` 3-ν mass spectrum, to see "ATM or SOLAR peaks" requires further $1000\times$ zooming on $E_\nu < 100$ meV.]

we separate the part of $m^2_{\nu_e}$ that we can obtain from oscillations (namely, δm^2, which is ≥ 0) from the rest, irrelevant to oscillations (namely, m^2_1, the squared mass of the lightest neutrino).

To proceed, we will consider certain scenarios of oscillations and neutrino mass spectra. Motivated by existing indications of oscillation [6], we select 6 cases with 4-ν ($(N-1)!$ level permutations) and 6 cases with 3-ν (implies to discard one of the indications). The caption of Tab. 1 and two examples should suffice to clarify our terminology: 1) The spectrum `SAL` is the 4-ν spectrum, where the splitting Δm^2_{21} is related to solar neutrinos, Δm^2_{32} to atmospheric neutrinos, Δm^2_{43} to LSND. 2) The spectrum `SA` does not account for LSND, and entails 3-ν. We assume that a "sterile" neutrino plays a role, but only in the 4-ν cases. There are 3 main cases:

1. δm^2 LARGE. This happens for the 4-ν spectra `ALS`, `LSA`, `LAS` [7], and for the 3-ν spectra `AL` and `LS`, where $\delta m = 400 - 1400$ meV. Indeed, the ν_e state has to stand above the "LSND mass gap", because it must be involved in the solar doublet of levels, or (only for the `AL` case[1]) because we know that the atmospheric doublet is mostly $\nu_\mu - \nu_\tau$. This is the most appealing case for the future experiments that aim at finding an effect of massive neutrinos in β-decay spectra. However, all these 5 spectra might have troubles with SN1987A ν's [8]: In fact, the mixing of $\vartheta_{e\mu}$ that we need to explain LSND produces resonant MSW conversion[2] [9] of $\overline{\nu}_\mu$ into $\overline{\nu}_e$. This implies an average energy of the dominant class of events ($\overline{\nu}_e p \to e^+ n$) significantly larger than expected, that does not seem to be what data suggest.

2. δm^2 MEDIUM. There are 5 sub-cases: (1) The first two spectra are `SAL` and `ASL`. For them, $\delta m \sim (\Delta m^2_{lsnd})^{1/2} \times \vartheta_{ee} = 50 - 180$ meV. In fact, the mixing that lead to appearance in LSND is in these schemes [7] $\vartheta_{e\mu} \approx \vartheta_{ee} \times \vartheta_{\mu\mu}$, the product of those mixings that would lead to disappearance in Bugey [10], ϑ_{ee}, and in CDHS [11], $\vartheta_{\mu\mu}$. (The *final* LSND data do not contradict this, but the bound is almost saturated). (2) Then we have the spectrum `LA`. δm is tunable up to $120 - 180$ meV, since one can arrange the lighter state to be $\nu_1 \approx \nu_e + \vartheta_{ee}\nu_\tau + \vartheta_{e\mu}\nu_\mu$; the mixing $\vartheta_{e\mu}$ is fixed by LSND, while ϑ_{ee} (that is what matters for us) is only loosely constrained by Bugey. (3) Next case is `SL`; again, δm is tunable. If $\nu_3 = \nu_\mu + \vartheta_{e\mu}\nu_e + ...$, $\delta m = 20 - 50$ meV, if $\nu_3 = \nu_\tau + \vartheta_{ee}\nu_e + ...$, instead, $\delta m = 120 - 180$ meV. (4) The *naive* expectation for the `SLA` spectrum is $\delta m^2 = \Delta m^2_{lsnd} \times \vartheta^2_{e\mu}$, but in fact δm can be larger, if:

$$\nu_e \approx n + \vartheta_{e\mu} N + \vartheta_{ee} N_\perp, \quad \nu_\mu \approx N + \vartheta_{\mu\mu} n_\perp - \vartheta_{e\mu} n$$

this case shows that δm^2 can go up $\Delta m^2_{lsnd} \times \vartheta^2_{ee}$ [n is a linear combination of ν_1 and ν_2, N is a

[1] This case is special, also because it does not permit to arrange cancellations for $\mathcal{M}_{ee}$. In the other four cases, it is instead *possible* to arrange such a cancellation for $0\nu2\beta$; but the solar mixing has to be large.

[2] The electron density in the SN core is so large (*e.g.*, when compared with solar densities) that can lead to an MSW resonance for any of the Δm^2's, LSND and atmospheric included.

linear combination of ν_3 and ν_4, and the orthogonal states have obvious notations], still keeping the probability of appearance $P_{e\mu} \sim 4 \times \vartheta^2_{e\mu} \times \sin^2\varphi$, and the probability of disappearance $P_{ee} \sim 1 - 4 \times \vartheta^2_{ee} \times \sin^2\varphi$ (5) Last case is the spectrum AS, where the trivial identification $\delta m = (\Delta m^2_{atm})^{1/2} = 40-80$ meV holds. We remind the reader that this case is among the targets of next generation $0\nu2\beta$ experiments, indeed $\mathcal{M}^2_{ee} \geq \Delta m^2_{atm} \times (1-\sin^2 2\theta_{sol})$. SN1987A bounds can be avoided (at the price of a fine tuning of the relevant mixing).

3. δm^2 SMALL. This includes only the 3-ν spectrum SA, when $\delta m^2 = \Delta m^2_{sol} \times |U^2_{e2}| + \Delta m^2_{atm} \times |U^2_{e3}| = (2.5-20\text{ meV})^2$ (LMA has been assumed). If solar oscillations will be confirmed, but MiniBooNE should not support LSND findings, this case would be quite (most?) likely. For easy reference, the results of the discussion above are reported in one table:

Table 1
Summary of the expectations on δm (=lower bound on m_{ν_e} from oscillations, Eq. 1). The letters of the acronyms stand for L=Δm^2_{lsnd}, A=Δm^2_{atm}, S=Δm^2_{sol}; the order in which they are written (from left to right) indicates how the Δm^2's appear in the given neutrino mass spectrum (from lighter to heavier one).

δm [meV]	SPECTRUM
$400-1400$	ALS,LSA,LAS,AL,LS
$50-180$	SAL,ASL
$20-180$	SLA,LA,SL
$40-80$	AS
$2.5-20$	SA

3. Discussion

Oscillations lead to consider neutrino mass; this could be related with $\mu \to e\gamma$, proton decay, *etc.* But perhaps, the *most direct* connections are those with the $0\nu2\beta$ decay, and possible distortions of the β-decay spectra. In this view, we considered the parameter $m^2_{\nu_e}$ and discussed the expectations on that part of it, δm^2, related with oscillations (Eq. 1). We have outlined a troublesome connections of those schemes that predict the largest values of δm^2 with SN1987A neutrinos. Still, δm^2 could be relatively large (perhaps observable in future setups) if LSND indications are due to $\bar\nu_e$ appearance. This is strictly connected with $\bar\nu_e$ *disappearance*, since the implied mixing ϑ_{ee} is the parameter that matters for β-decay spectra. If LSND signal is not due to oscillations, $m^2_{\nu_e}$ and m^2_1 could be identified for practical purposes: Only "quasi degenerate" neutrinos could significantly modify β-spectra, unless the 50 meV level is attained (which at present seems quite difficult).

In conclusion, we stress again that the expectations for $m^2_{\nu_e}$ are closely related with oscillations: LSND; neutrinos from SN1987A and future type II SN's; existence of sub-dominant ν_e mixing... and, in many (but not all) cases, also with the rate of the neutrinoless double beta transition.

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