Relic neutrino asymmetry generation from $\nu_\alpha \leftrightarrow \nu_s$ oscillations

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Active–sterile neutrino oscillations provide a mechanism by which large differences in the neutrino and antineutrino number densities can be created in the early universe. The quantum kinetic equations are employed in the study of these neutrino asymmetries, which, when solved analytically in the adiabatic limit, generate physically transparent evolution equations that are very useful for the understanding of the nature of the asymmetry growth.

1. Introduction

Active–sterile neutrino oscillations in the early universe can generate large differences in the number densities of relic neutrinos and antineutrinos [1–3]. For a wide range of small vacuum mixing angles θ_0 and squared mass differences Δm^2 (provided that $\Delta m^2 < 0$, meaning roughly that $m_{\nu_\alpha} > m_{\nu_s}$, where $\alpha = e$, μ or τ), this is achieved in the temperature range $T \sim 1 \to \mathcal{O}(10)$ MeV (before neutrino decoupling at $T \simeq 1$ MeV) in a CP asymmetric background. The final values of these so-called lepton number asymmetries,

$$L_{\nu_\alpha} \equiv \frac{n_{\nu_\alpha} - n_{\overline{\nu}_\alpha}}{n_\gamma}, \qquad (1)$$

where n_ψ is the number density of species ψ, are typically of the order 0.1, and have profound cosmological implications: (i) the suppression of ν_s production because of large matter effects [1,2], and (ii) a modification to primordial ^{4}He synthesis if a sufficiently large L_{ν_e} is created prior to or during big bang nucleosynthesis (BBN) [3]. For a review on asymmetry generation, see Ref. [4].

The study of asymmetry evolution is conducted in the density matrix formalism, and the quantum kinetic equations (QKEs) [5,6], which quantify the effects of (i) decohering collisions, (ii) matter-affected oscillations, (iii) expansion of the universe, and (iv) repopulation of empty states from the background, are the main tool of trade.

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2. Quantum kinetic equations

The properties of a $\nu_\alpha \leftrightarrow \nu_s$ system are encoded in the density matrices [5,6]

$$\rho = \frac{1}{2}(P_0 + \mathbf{P}\cdot\sigma), \qquad (2)$$

where $\mathbf{P} = P_x\hat{x} + P_y\hat{y} + P_z\hat{z}$. All quantities are understood to be functions of temperature T and momentum p. The diagonal entries represent, respectively, the ν_α and ν_s distribution functions:

$$N_{\nu_\alpha} = \frac{1}{2}(P_0 + P_z)N^0_{\rm eq},\; N_{\nu_s} = \frac{1}{2}(P_0 - P_z)N^0_{\rm eq}, \qquad (3)$$

in which the reference distribution function $N^0_{\rm eq}$ is of Fermi–Dirac (equilibrium) form,

$$N^\mu_{\rm eq} = \frac{1}{2\pi^2}\frac{p^2}{1+\exp\left(\frac{p-\mu}{T}\right)}, \qquad (4)$$

with chemical potential μ set to zero. The off-diagonal entries P_x and P_y are the coherences.

The variables P_0 and $\mathbf{P}$ advance in time according to the quantum kinetic equations (QKEs) [5]

$$\begin{aligned}\frac{\partial \mathbf{P}}{\partial t} &= \mathbf{V}\times\mathbf{P} - D(P_x\hat{x} + P_y\hat{y}) + R_\alpha\hat{z},\\ \frac{\partial P_0}{\partial t} &= R_\alpha \simeq \Gamma\left[\frac{N^\mu_{\rm eq}}{N^0_{\rm eq}} - \frac{1}{2}(P_0 + P_z)\right]. \end{aligned} \qquad (5)$$

Here, the matter potential vector $\mathbf{V} = \beta\hat{x} + \lambda\hat{z}$ governs the coherent matter-affected evolution of the ensemble, with [7,8]

$$\beta = \frac{\Delta m^2}{2p}\sin 2\theta_0,\; \lambda = \frac{\Delta m^2}{2p}(b - a - \cos 2\theta_0), \qquad (6)$$

and

$$a = -\frac{4\zeta(3)\sqrt{2}G_F L^{(\alpha)}T^3 p}{\pi^2\Delta m^2},$$
$$b = -\frac{4\zeta(3)\sqrt{2}G_F A_\alpha T^4 p^2}{\pi^2\Delta m^2 m_W^2}, \quad (7)$$

where G_F is the Fermi constant, m_W the W-boson mass, ζ the Riemann zeta function and $A_e \simeq 17$, $A_{\mu,tau} \simeq 4.9$. Also present in Eq. (7) is

$$L^{(\alpha)} = L_{\nu_\alpha} + L_{\nu_e} + L_{\nu_\mu} + L_{\nu_\tau} + \eta, \quad (8)$$

which contains a small term due to the cosmological baryon–antibaryon asymmetry η.

The decoherence function D is related to the total collision rate for ν_α, Γ, via $D = \Gamma/2 = k_\alpha G_F^2 T^4 p$, where $k_e \simeq 0.635$, $k_{\mu,\tau} \simeq 0.46$ [5], while the repopulation function R_α determines the rate at which a depleted momentum state in the distribution is refilled from the background plasma so as to restore thermal equilibrium.

A separate set of expressions, denoted with an overhead bar, parameterises the $\overline{\nu}_\alpha \leftrightarrow \overline{\nu}_s$ system.

3. Adiabatic limit approximation

By demanding $\alpha + s$ lepton number conservation, one may extract from the QKEs an exact evolution equation for the asymmetry, that is [9],

$$\frac{dL_{\nu_\alpha}}{dt} = \frac{1}{2n_\gamma}\int \beta(P_y - \overline{P}_y)N^0_{\rm eq}dp. \quad (9)$$

The role of the adiabatic limit approximation [9–11] is to supply approximate analytical solutions for P_y and $\overline{P}_y$ so as to render Eq. (9) a more physically transparent expression, through which the nature of the asymmetry growth may be revealed.

3.1. The Boltzmann limit

The first assumption is a small refilling function [9,11], i.e., $R_\alpha \simeq 0$, so that the QKEs reduce to

$$\frac{\partial \mathbf{P}}{\partial t} \simeq \mathcal{K}\mathbf{P} = \begin{pmatrix} -D & -\lambda & 0 \\ \lambda & -D & -\beta \\ 0 & \beta & 0 \end{pmatrix} \begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix}. \quad (10)$$

This may be solved by establishing an "instantaneous diagonal basis" via $\mathbf{Q} = \mathcal{U}\mathbf{P}$ and $\mathcal{K}_d \equiv \mathrm{diag}(k_1, k_2, k_3) = \mathcal{U}\mathcal{K}\mathcal{U}^{-1}$, with eigenvalues

$$k_{1,2} = -d \pm i\omega, \quad k_3 = -\frac{\beta^2 D}{d^2+\omega^2}, \quad (11)$$

where

$$d = D + \frac{k_3}{2}, \quad \omega^2 = \lambda^2 + \beta^2 + k_3 D + \frac{3}{4}k_3^2, \quad (12)$$

are interpreted as the *effective* damping factor and oscillation frequency respectively [10].

In the new basis, Eq. (10) is similar to

$$\frac{\partial \mathbf{Q}}{\partial t} = \mathcal{K}_d\mathbf{Q} - \mathcal{U}\frac{\partial \mathcal{U}}{\partial t}^{-1}\mathbf{Q}, \quad (13)$$

where the second term is explicitly dependent on the time derivatives of D, λ and β. The adiabatic limit is defined by $\mathcal{U}\frac{\partial \mathcal{U}}{\partial t}^{-1} \simeq 0$ [9], such that

$$P_\delta(t) \simeq \sum_{i,\epsilon} \mathcal{U}^{-1}_{\delta i}(t) e^{\int_0^t k_i(t')dt'}\mathcal{U}_{i\epsilon}(0)P_\epsilon(0), \quad (14)$$

where $\delta, \epsilon = x, y, z$, and $i = 1, 2, 3$, provides an approximate solution to Eq. (10).

Equation (14) simplifies further in the high temperature collision-dominated $D \gg |\beta|$ limit with the approximation

$$e^{\int_0^t k_{1,2}(t')dt'} \to 0, \quad (15)$$

since, in this limit, $d \simeq D$, $\omega \simeq \lambda$, and thus $|k_3| \ll |\mathrm{Re}(k_{1,2})|$ by Eqs. (11) and (12). Physically, frequent collisions lead to rapid and complete damping of the system's innate oscillations. The evolution of the ensemble is dictated instead by a slower rate k_3, which tends to equalise the ν_α and ν_s distribution functions [10].

Specifically, one has, from Eqs. (14) and (15),

$$P_y(t) \simeq \frac{\mathcal{U}^{-1}_{y3}(t)}{\mathcal{U}^{-1}_{z3}(t)}P_z(t) = \frac{k_3}{\beta}P_z(t), \quad (16)$$

which allows Eq. (9) to be approximated as

$$\frac{dL_{\nu_\alpha}}{dt} \simeq \frac{1}{2n_\gamma}\int [k_3(N_{\nu_\alpha} - N_{\nu_s}) - \overline{k}_3(N_{\overline{\nu}_\alpha} - N_{\overline{\nu}_s})]\,dp. \quad (17)$$

Equation (17) is essentially a classical Boltzmann rate equation that is dependent *only* on the instantaneous ν_α and ν_s distribution functions. The "rate constant" k_3 is of particular interest: A *matter- and collision-affected mixing angle* arises naturally from the QKEs [10]:

$$\sin^2 2\theta_{m,D} \equiv \frac{\beta^2}{(D+k_3)^2 + \lambda^2 + \beta^2}, \quad (18)$$

such that $k_3 = -\sin^2 2\theta_{m,D} \times D$; k_3 therefore reflects on the system's ability to mix and to collide.

After further algebraic manipulations, Eq. (17) shows that, for $\Delta m^2 < 0$, there exists a critical temperature above which $\frac{dL_{\nu_\alpha}}{dt} \propto -L^{(\alpha)}$, and below which $\frac{dL_{\nu_\alpha}}{dt} \propto +L^{(\alpha)}$. The former implies L_{ν_α} destruction, while the latter causes exponential growth [2]. These features have been confirmed by numerical integration of the exact QKEs.

3.2. Collision-affected MSW effect

Shortly after the initial explosive growth, the system enters a low temperature regime in which coherent Mikheyev–Smirnov–Wolfenstein (MSW) transitions [7] are the dominant asymmetry amplifier. Subsequent growth follows an approximate power law rate $L_{\nu_\alpha} \propto T^{-4}$, reaching a value of $\mathcal{O}(0.1)$ prior to neutrino decoupling [3]. In the following, however, I shall point out another interesting effect relevant for this epoch.

Together with the initial conditions $P_x(0) \simeq P_y(0) \simeq 0$, Eqs. (14) and (18) give, in the low temperature $|\beta| \gg D$ limit, the expression [12]

$$P_z(t) \simeq \cos 2\theta_{m,D}(t) \cos 2\theta_{m,D}(0) F_{\text{eff}} P_z(0), \quad (19)$$

where $F_{\text{eff}} = \exp \int_0^t k_3(t')dt'$ is generally analytically insoluble. However, supposing that the neutrino is created and "measured" well before and after resonance (plus other assumptions), one may write down an asymptotic solution:

$$F_{\text{eff}} = \exp \left[-\frac{\pi \beta D}{\left| \frac{d\lambda}{dt} \right|} \right]_{\text{res}}, \quad (20)$$

so that Eq. (19) becomes, in probability language,

$$\text{Prob}(\nu_\alpha \to \nu_s,\ t \gg t_{\text{res}}) \simeq \frac{1}{2} (1 + F_{\text{eff}}). \quad (21)$$

Physically, as the system crosses a resonance, collisions disrupt the coherent flavour conversion process and weaken the MSW mechanism, the extent of which is parameterised by an *efficiency factor* F_{eff}. In a collisionless environment, $F_{\text{eff}} = 1$, and Eq. (21) gives Prob $\simeq 1$ as expected.

The factor F_{eff} compares the interaction length $\ell_{\text{int}} = 1/D$ with the physical resonance width $\ell_{\text{res}} = |\beta / \frac{d\lambda}{dt}|$. If $\ell_{\text{res}} > \ell_{\text{int}}$, maximal mixing persists for a sufficiently long time during which substantial collision-induced equilibration may occur. An infinite resonance width means $F_{\text{eff}} \to 0$ and the system emerges from the resonance with equal ν_α and ν_s distribution functions.

4. Conclusion

Large relic neutrino asymmetries can be generated from active–sterile neutrino oscillations for a wide range of oscillation parameters. These asymmetries may then suppress sterile neutrino production and modify primordial ^{4}He synthesis. Approximate evolution equations obtained from the QKEs offer much analytical insight on the nature of the asymmetry growth.

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