

Neutrino Mixing, Dirac and Majorana Leptonic CP-Violation and Leptogenesis

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Conca Specchiulla

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There were spectacular developments regarding the “CHOOZ” angle θ_{13} in the last year or so.

Started in 2008 with weak indications for $\theta_{13} \sim 0.1$:

$$\sin^2 \theta_{13} = 0.016 \pm 0.010, \quad \sin \theta_{13} = (0.077 - 0.161), \quad 1\sigma$$

E. Lisi *et al.*, [arXiv:0806.2649](#)

A.B. Balantekin, D. Yilmaz, [arXiv:0804.3345](#);

Atmospheric ν data: $\cos \delta = -1$ favored over $\cos \delta = +1$

J. Escamilla *et al.*, [arXiv:0805.2924](#)

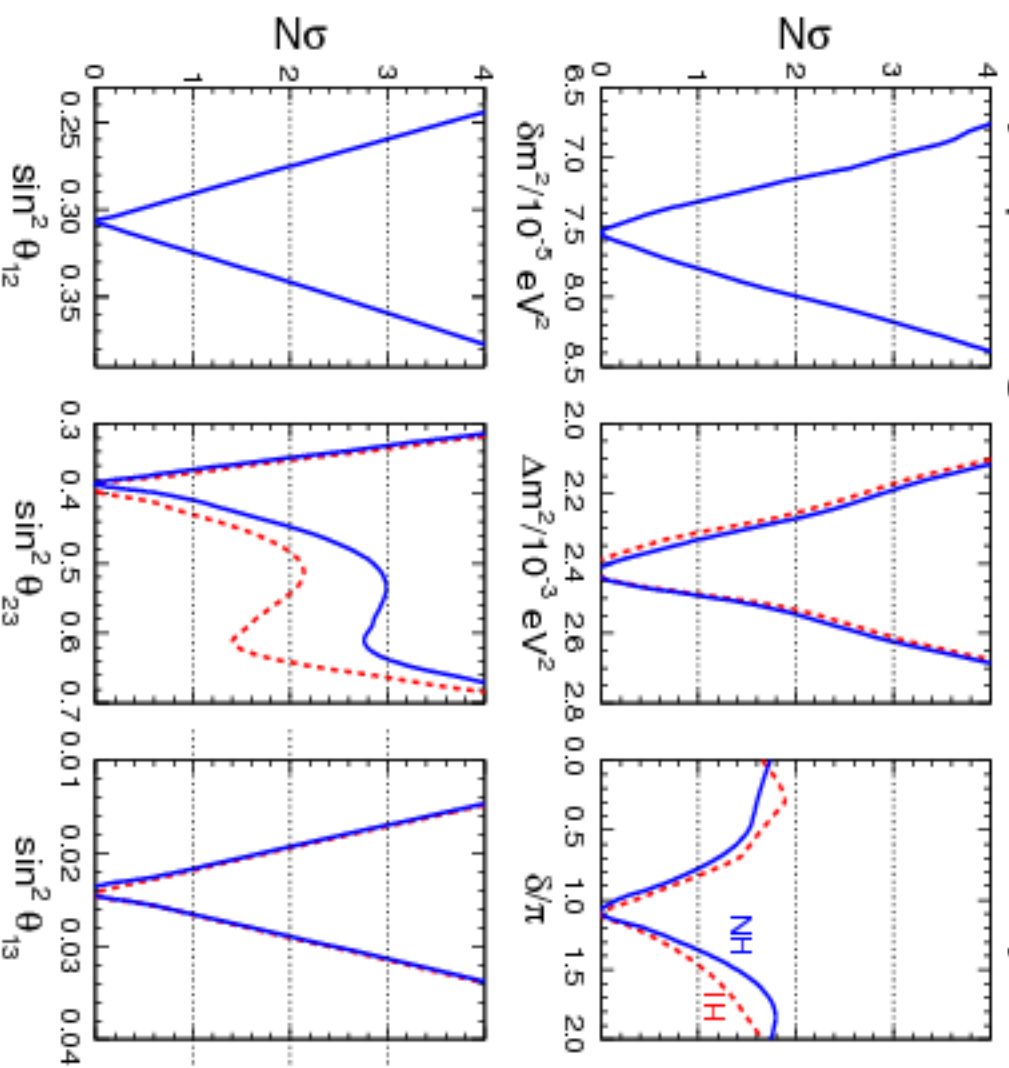


(c) 2000 ESHI

~ 420.0 mi / 675.8 km across

- June 14, 2011, T2K: 2.5σ evidence for $\theta_{13} \neq 0$.
- June 24, 2011, MINOS: 1.7σ evidence for $\theta_{13} \neq 0$.
- June 28, 2011, Fogli et al.: from global analysis more than 3σ evidence for $\theta_{13} \neq 0$; b.f.v.: $\sin^2\theta_{13} = 0.025$ (0.023).
- November 9, 2011, Double Chooz: $\theta_{13} \neq 0$ at 1.5σ .
- March 8, 2012, Daya Bay: 5.2σ evidence for $\theta_{13} \neq 0$, $\sin^2 2\theta_{13} = 0.092 \pm 0.016 \pm 0.005$.
- April 4, 2012, RENO: 4.9σ evidence for $\theta_{13} \neq 0$, $\sin^2 2\theta_{13} = 0.113 \pm 0.013 \pm 0.019$.
- Nu'2012 (June 4-9, 2012), T2K, Double Chooz: 3.2σ and 3.1σ evidence for $\theta_{13} \neq 0$.
- Nu'2012 (June 4-9, 2012), Daya Bay: $\sin^2 2\theta_{13} = 0.089 \pm 0.010 \pm 0.005$.
- June , 2012, Fogli et al., global analysis, b.f.v.: $\sin^2\theta_{13} = 0.0241$ (0.0244), NH (IH).

Synopsis of global 3ν oscillation analysis



Global data: $\cos \delta = -1$ favored over $\cos \delta = +1$;

best fit: $\sin \theta_{13} \cos \delta \cong -0.15$.

- $\Delta m_{\odot}^2 \equiv \Delta m_{21}^2 \cong 7.54 \times 10^{-5} \text{ eV}^2 > 0$, $\sin^2 \theta_{12} \cong 0.307$, $\cos 2\theta_{12} \gtrsim 0.28 \text{ (3}\sigma\text{)}$,
- $|\Delta m_{31(32)}^2| \cong 2.47 \text{ (2.46)} \times 10^{-3} \text{ eV}^2$, $\sin^2 \theta_{23} \cong 0.386 \text{ (0.392)}$, NH (IH),
- θ_{13} - the CHOOZ angle: $\sin^2 \theta_{13} = 0.0241 \text{ (0.0244)}$, NH (IH).
Fogli *et al.*, [arXiv:1205.5254v3](#)

Large $\sin^2 \theta_{13} \cong (0.01 - 0.02)$ - far-reaching implications for the program of research in neutrino physics:

- For the searches for CP violation in ν -oscillations;
- For the determination of the type of ν – mass spectrum (or of $\text{sgn}(\Delta m_{\text{atm}}^2)$) in neutrino oscillation experiments.
- For understanding the pattern of the neutrino mixing and its origins (symmetry, etc.?).
- For the predictions for the $(\beta\beta)_{0\nu}$ -decay effective Majorana mass in the case of NH light ν mass spectrum (possibility of a strong suppression).
- Important implications also for the “flavoured” leptogenesis scenario of generation of the baryon asymmetry of the Universe.

One of the next most important steps - determine the status of the CP symmetry in the lepton sector.

The reference scheme: 3- ν mixing

$$\nu_L = \sum_{j=1}^3 U_{lj} \nu_{jL} \quad l = e, \mu, \tau.$$

PMNS Matrix: Standard Parametrization

$$U = V P, \quad P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_{21}}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_{31}}{2}} \end{pmatrix},$$

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

- $s_{ij} \equiv \sin \theta_{ij}$, $c_{ij} \equiv \cos \theta_{ij}$, $\theta_{ij} = [0, \frac{\pi}{2}]$,
- δ - Dirac CP-violation phase, $\delta = [0, 2\pi]$,
- α_{21} , α_{31} - the two Majorana CP-violation phases.

S.M. Bilenky, J. Hosek, S.T.P., 1980

- $\Delta m_{21}^2 \equiv \Delta m_{21}^2 \cong 7.54 \times 10^{-5} \text{ eV}^2 > 0$, $\sin^2 \theta_{12} \cong 0.307$, $\cos 2\theta_{12} \gtrsim 0.28 \text{ (} 3\sigma \text{)}$,
- $|\Delta m_{31(32)}^2| \cong 2.47 \text{ (} 2.46 \text{)} \times 10^{-3} \text{ eV}^2$, $\sin^2 \theta_{23} \cong 0.386 \text{ (} 0.392 \text{)}$, NH (IH),
- θ_{13} - the CHOOZ angle: $\sin^2 \theta_{13} = 0.0241 \text{ (} 0.0244 \text{)}$, NH (IH).
Fogli *et al.*, arXiv:1205.5254v3

- Dirac phase δ : $\nu_l \leftrightarrow \nu_{l'}, \bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}, l \neq l'$; $A_{\text{CP}}^{(ll')} \propto J_{\text{CP}} \propto \sin \theta_{13} \sin \delta$:

P.I. Krastev, S.T.P., 1988

$$J_{\text{CP}} = \text{Im} \{ U_{e1} U_{\mu 2} U_{e2}^* U_{\mu 1}^* \} = \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos \theta_{13} \sin \delta$$

Current data: $|J_{\text{CP}}| \lesssim 0.045$ (can be relatively large!)

- Majorana phases α_{21}, α_{31} :

- $\nu_l \leftrightarrow \nu_{l'}, \bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$ not sensitive;

S.M. Bilenky, J. Hosek, S.T.P., 1980;
P. Langacker, S.T.P., G. Steigman, S. Toshev, 1987

- $|\langle m \rangle|$ in $(\beta\beta)_{0\nu}$ -decay depends on α_{21}, α_{31} ;
- $\Gamma(\mu \rightarrow e + \gamma)$ etc. in SUSY theories depend on $\alpha_{21,31}$;
- BAU, leptogenesis scenario: $\delta, \alpha_{21,31}$!

- $\text{sgn}(\Delta m_{\text{atm}}^2) = \text{sgn}(\Delta m_{31}^2)$ not determined

$\Delta m_{\text{atm}}^2 \equiv \Delta m_{31}^2 > 0$, normal mass ordering
 $\Delta m_{\text{atm}}^2 \equiv \Delta m_{32}^2 < 0$, inverted mass ordering

Convention: $m_1 < m_2 < m_3$ - **NMO**, $m_3 < m_1 < m_2$ - **IMO**

$m_1 \ll m_2 < m_3$, **NH**,

$m_3 \ll m_1 < m_2$, **IH**,

$m_1 \cong m_2 \cong m_3$, $m_{1,2,3} \gg \Delta m_{\text{atm}}^2$, **QD**; $m_j \gtrsim 0.10 \text{ eV}$.

Dirac CP-Nonconservation: δ in U_{PMNS}

Observable manifestations in

$$\nu_l \leftrightarrow \nu_{l'} , \quad \bar{\nu}_l \leftrightarrow \bar{\nu}_{l'} , \quad l, l' = e, \mu, \tau$$

- not sensitive to Majorana CPVP α_{21} , α_{31}

CP-Invariance:

N. Cabibbo, 1978

S.M. Bilenky, J. Hosek, S.T.P., 1980;

$$P(\nu_l \rightarrow \nu_{l'}) = P(\bar{\nu}_l \rightarrow \bar{\nu}_{l'}) , \quad l \neq l' = e, \mu, \tau \quad \text{V. Barger, S. Pakvasa et al., 1980.}$$

CPT-invariance:

$$P(\nu_l \rightarrow \nu_{l'}) = P(\bar{\nu}_{l'} \rightarrow \bar{\nu}_l)$$

$$l = l' : \quad P(\nu_l \rightarrow \nu_l) = P(\bar{\nu}_l \rightarrow \bar{\nu}_l)$$

T-invariance:

$$P(\nu_l \rightarrow \nu_{l'}) = P(\nu_{l'} \rightarrow \nu_l), \quad l \neq l'$$

3ν —mixing:

$$A_{\text{CP}}^{(ll')} \equiv P(\nu_l \rightarrow \nu_{l'}) - P(\bar{\nu}_l \rightarrow \bar{\nu}_{l'}) , \quad l \neq l' = e, \mu, \tau$$

$$A_{\tau}^{(ll')} \equiv P(\nu_l \rightarrow \nu_{l'}) - P(\nu_{l'} \rightarrow \nu_l), \quad l \neq l'$$

$$A_{\text{T(CP)}}^{(e,\mu)} = A_{\text{T(CP)}}^{(\mu,\tau)} = -A_{\text{T(CP)}}^{(e,\tau)}$$

P.I. Krastev, S.T.P., 1988; V. Barger, S. Pakvasa *et al.*, 1980

In vacuum: $A_{\text{CP(T)}}^{(e\mu)} = J_{\text{CP}} F_{\text{osc}}^{\text{vac}}$

$$J_{CP} = \text{Im} \left\{ U_{e1} U_{\mu 2} U_{e2}^* U_{\mu 1}^* \right\} = \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos \theta_{13} \sin \delta$$

$$F_{\text{osc}}^{\text{vac}} = \sin\left(\frac{\Delta m_{21}^2}{2E}L\right) + \sin\left(\frac{\Delta m_{32}^2}{2E}L\right) + \sin\left(\frac{\Delta m_{13}^2}{2E}L\right)$$

In matter: Matter effects violate

P.I. Krastev, S.T.P., 1988

$$\text{CP : } P(\nu_l \rightarrow \nu_{l'}) \neq P(\bar{\nu}_l \rightarrow \bar{\nu}_{l'})$$

$$\text{CPT : } P(\nu_l \rightarrow \nu_{l'}) \neq P(\bar{\nu}_{l'} \rightarrow \bar{\nu}_l)$$

P. Langacker et al., 1987

Can conserve the T-invariance (Earth)

$$P(\nu_l \rightarrow \nu_{l'}) = P(\bar{\nu}_{l'} \rightarrow \bar{\nu}_l), \quad l \neq l'$$

In matter with constant density: $A_{\text{T}}^{(e\mu)} = J_{\text{CP}}^{\text{mat}} F_{\text{osc}}^{\text{mat}}$

$$R_{CP} \text{ does not depend on } \theta_{23} \text{ and } \delta; \quad |R_{CP}| \lesssim 2.5 \quad J_{\text{CP}}^{\text{mat}} = J_{\text{CP}}^{\text{vac}} R_{CP}$$

P.I. Krastev, S.T.P., 1988

P. Harrison, S. Scott, 2000

Rephasing Invariants Associated with CPVP

Dirac phase δ :

$$J_{CP} = \text{Im} \left\{ U_{e1} U_{\mu 2} U_{e2}^* U_{\mu 1}^* \right\} .$$

C. Jarlskog, 1985 (for quarks)

CP-, T- violation effects in neutrino oscillations

P. Krastev, S.T.P., 1988

Majorana phases α_{21} , α_{31} :

$$S_1 = \text{Im} \{ U_{e1} U_{e3}^* \} , \quad S_2 = \text{Im} \{ U_{e2} U_{e3}^* \} \quad (\text{not unique}); \quad \text{or} \\ S'_1 = \text{Im} \{ U_{\tau 1} U_{\tau 2}^* \} , \quad S'_2 = \text{Im} \{ U_{\tau 2} U_{\tau 3}^* \}$$

J.F. Nieves and P. Pal, 1987, 2001

G.C. Branco et al., 1986

J.A. Aguilar-Saavedra and G.C. Branco, 2000

CP-violation: both $\text{Im} \{ U_{e1} U_{e3}^* \} \neq 0$ and $\text{Re} \{ U_{e1} U_{e3}^* \} \neq 0$.

S_1 , S_2 appear in $|\langle m \rangle|$ in $(\beta\beta)_{0\nu}$ -decay.

In general, J_{CP} , S_1 and S_2 are independent.

Absolute Neutrino Mass Measurements

The Troitzk and Mainz ^3H β -decay experiments

$$m_{\nu_e} < 2.3 \text{ eV} \quad (95\% \text{ C.L.})$$

It is expected that the following sensitivity will be reached:

$$\text{KATRIN:} \quad m_{\nu_e} \sim 0.2 \text{ eV}$$

Cosmological and astrophysical data, the CMB data of the WMAP experiment, combined with supernovae data and data on galaxy clustering imply (depending on the model complexity and the input data used):

$$\sum_j m_j \leq (0.3 - 1.3) \text{ eV} \quad (95\% \text{ C.L.})$$

K.N. Abazajian *et al.*, [arXiv:1103.5083](#)

The WMAP and future PLANCK experiments can be sensitive to

$$\sum_j m_j \cong 0.4 \text{ eV}$$

Data on weak lensing of galaxies by large scale structure, combined with data from the WMAP and PLANCK experiments may allow to determine

$$\sum_j m_j : \quad \delta \cong 0.04 \text{ eV.}$$

M_ν from the See-Saw Mechanism

P. Minkowski, 1977.
M. Gell-Mann, P. Ramond, R. Slansky, 1979;
T. Yanagida, 1979;
R. Mohapatra, G. Senjanovic, 1980.

- Explains the smallness of ν -masses.
- Through **leptogenesis theory** links the ν -mass generation to the generation of baryon asymmetry of the Universe Y_B .

S. Fukugita, T. Yanagida, 1986; GUT's: M. Yoshimura, 1978.

- In SUSY GUT's with see-saw mechanism of ν -mass generation, the LFV decays

$$\mu \rightarrow e + \gamma, \quad \tau \rightarrow \mu + \gamma, \quad \tau \rightarrow e + \gamma, \text{ etc.}$$

are predicted to take place with rates within the reach of present and future experiments.

F. Borzumati, A. Masiero, 1986.

- The ν_j are **Majorana particles**; $(\beta\beta)_{0\nu}$ -decay is allowed.

See-Saw: Dirac ν -mass m_D + Majorana mass M_R for N_R

The See-Saw Lagrangian

$$\mathcal{L}^{\text{lep}}(x) = \mathcal{L}_{\text{CC}}(x) + \mathcal{L}_{\text{Y}}(x) + \mathcal{L}_{\text{M}}^{\text{N}}(x),$$

$$\mathcal{L}_{\text{CC}} = -\frac{g}{\sqrt{2}} \overline{l_L}(x) \gamma_{\alpha} \nu_{\text{LL}}(x) W^{\alpha\dagger}(x) + h.c.,$$

$$\mathcal{L}_{\text{Y}}(x) = \lambda_{i\ell} \overline{N_{iR}}(x) H^{\dagger}(x) \psi_{i\text{L}}(x) + Y_{\ell} H^c(x) \overline{l_R}(x) \psi_{i\text{L}}(x) + \text{h.c.},$$

$$\mathcal{L}_{\text{M}}^{\text{N}}(x) = -\frac{1}{2} M_i \overline{N_i}(x) N_i(x).$$

$\psi_{i\text{L}}$ - LH doublet, $\psi_{i\text{L}}^{\text{T}} = (\nu_{i\text{L}} \ l_{i\text{L}})$, l_R - RH singlet, H - Higgs doublet.

Basis: $M_R = (M_1, M_2, M_3)$; $D_N \equiv \text{diag}(M_1, M_2, M_3)$, $D_{\nu} \equiv \text{diag}(m_1, m_2, m_3)$.
 m_D generated by the Yukawa interaction:

$$-\mathcal{L}_{\text{Y}}^{\nu} = \lambda_{i\ell} \overline{N_{iR}} H^{\dagger}(x) \psi_{i\text{L}}(x), \quad v = 174 \text{ GeV}, \quad \nu \lambda = m_D - \text{complex}$$

For M_R - sufficiently large,

$$m_{\nu} \simeq v^2 \lambda^T M_R^{-1} \lambda = U_{\text{PMNS}}^* m_{\nu}^{\text{diag}} U_{\text{PMNS}}^{\dagger}.$$

$Y_{\nu} \equiv \lambda = \sqrt{D_N} R \sqrt{D_{\nu}} (U_{\text{PMNS}})^{\dagger} / v_u$, all at M_R ; R -complex, $R^T R = 1$.

In GUTs, $M_R < M_X$, $M_X \sim 10^{16}$ GeV;

J.A. Casas and A. Ibarra, 2001

in GUTs, e.g., $M_R = (10^9, 10^{12}, 10^{15})$ GeV, $m_D \sim 1$ GeV.

$Y_\nu \equiv \lambda = \sqrt{D_N} \ R \ \sqrt{D_\nu} \ (U_{\text{PMNS}})^\dagger / v_u$, all at M_R ;

R -complex, $R^T R = 1$.

J.A. Casas and A. Ibarra, 2001

$D_N \equiv \text{diag}(M_1, M_2, M_3)$, $D_\nu \equiv \text{diag}(m_1, m_2, m_3)$.

Models: R - CP conserving ($SU(5) \times T'$); CPV parameters in R determined by the CPV phases in U (class of A_4 models).

Texture zeros in Y_ν : CPV parameters in R and U - related.

In GUTs, $M_R < M_X$, $M_X \sim 10^{16}$ GeV;

in GUTs, e.g., $M_R = (10^9, 10^{12}, 10^{15})$ GeV, $m_D \sim 1$ GeV.

The CP-Invariance Constraints

Assume: $C(\bar{\nu}_j)^T = \nu_j$, $C(\bar{N}_k)^T = N_k$, $j, k = 1, 2, 3$.

The CP-symmetry transformation:

$$U_{\text{CP}} N_j(x) U_{\text{CP}}^\dagger = \eta_j^{NCP} \gamma_0 N_j(x'), \quad \eta_j^{NCP} = i\rho_j^N = \pm i, \\ U_{\text{CP}} \nu_k(x) U_{\text{CP}}^\dagger = \eta_k^{\nu CP} \gamma_0 \nu_k(x'), \quad \eta_k^{\nu CP} = i\rho_k^\nu = \pm i.$$

CP-invariance:

$$\lambda_{jl}^* = \lambda_{jl} (\eta_j^{NCP})^* \eta^l \eta^{H*}, \quad j = 1, 2, 3, \quad l = e, \mu, \tau,$$

Convenient choice: $\eta^I = i$, $\eta^H = 1$ ($\eta^W = 1$):

$$\lambda_{jl}^* = \lambda_{jl} \rho_j^N, \quad \rho_j^N = \pm 1,$$

$$U_{lj}^* = U_{lj} \rho_j^\nu, \quad \rho_j^\nu = \pm 1,$$

$$R_{jk}^* = R_{jk} \rho_j^N \rho_k^\nu, \quad j, k = 1, 2, 3, \quad l = e, \mu, \tau,$$

λ_{jl} , U_{lj} , R_{jk} - either real or purely imaginary.

Relevant quantity:

$$P_{jkm l} \equiv R_{jk} R_{jm} U_{lk}^* U_{lm}, \quad k \neq m,$$

$$CP: \quad P_{jkm l}^* = P_{jkm l} (\rho_j^N)^2 (\rho_k^\nu)^2 (\rho_m^\nu)^2 = P_{jkm l}, \quad \text{Im}(P_{jkm l}) = 0.$$

$$P_{jkm\ell} \equiv R_{jk} R_{jm} U_{lk}^* U_{lm}, \quad k \neq m,$$

$$\textcolor{red}{CP}: \quad P_{jkm\ell}^* = P_{jkm\ell} (\rho_j^N)^2 (\rho_k^\nu)^2 (\rho_m^\nu)^2 = P_{jkm\ell}, \quad \text{Im}(P_{jkm\ell}) = 0.$$

$$\text{Consider } \textcolor{blue}{NH} \ N_j, \text{ } \textcolor{blue}{NH} \ \nu_k: \quad P_{123\tau} = R_{12} R_{13} U_{\tau 2}^* U_{\tau 3}$$

$$\text{Suppose, } \textcolor{red}{CP}\text{-invariance holds at low } E: \quad \delta = 0, \quad \alpha_{21} = \pi, \quad \alpha_{31} = 0.$$

$$\text{Thus, } U_{\tau 2}^* U_{\tau 3} \text{ - purely imaginary.}$$

$$\text{Then } \textcolor{red}{\text{real}} \ R_{12} R_{13} \text{ corresponds to } \textcolor{blue}{CP}\text{-violation at "high" } E.$$

Baryon Asymmetry

$$Y_B = \frac{n_B - n_{\bar{B}}}{n_\gamma} = (6.1 \pm 0.3) \times 10^{-10}, \quad \text{CMB}$$

Sakharov conditions for a dynamical generation of $Y_B \neq 0$ in the Early Universe

- B number non-conservation.
- Violation of C and CP symmetries.
- Deviation from thermal equilibrium.

Leptogenesis

- The heavy Majorana neutrinos N_i are in equilibrium in the Early Universe as far as the processes which produce and destroy them are efficient.
- When $T < M_{N_1}$, N_1 drops out of equilibrium as it cannot be produced efficiently anymore.
- If $\Gamma(N_1 \rightarrow \Phi^- \ell^+) \neq \Gamma(N_1 \rightarrow \Phi^+ \ell^-)$, a lepton asymmetry will be generated.
- Wash-out processes, like $\Phi^+ + \ell^- \rightarrow N_1$, $\ell^- + \Phi^+ \rightarrow \Phi^- + \ell^+$, **etc.** tend to erase the asymmetry. Under the condition of non-equilibrium, they are less efficient than the direct processes in which the lepton asymmetry is created. The final result is a net (non-zero) lepton asymmetry.
- This lepton asymmetry is then converted into a baryon asymmetry by $(B + L)$ **violating but** $(B - L)$ **conserving** sphaleron processes which exist within the SM (at $T \gtrsim M_{\text{EWSB}}$).

In order to compute Y_B :

1. calculate the CP-asymmetry:

$$\varepsilon_1 = \frac{\Gamma(N_1 \rightarrow \Phi^- \ell^+) - \Gamma(N_1 \rightarrow \Phi^+ \ell^-)}{\Gamma(N_1 \rightarrow \Phi^- \ell^+) + \Gamma(N_1 \rightarrow \Phi^+ \ell^-)}$$

2. solve the Boltzmann equation to account for the wash-out of the asymmetry:

$$Y_L = \kappa \varepsilon$$

where $\kappa = \kappa(\tilde{m})$ is the “efficiency factor”, $\tilde{m}$ is the “the wash-out mass parameter” - determines the rate of wash-out processes;

3. the lepton asymmetry is converted into a baryon asymmetry:

$$Y_B = \frac{C_S}{g_*} \kappa \varepsilon$$

Baryon number violation in the SM

Sphaleron processes

SU(2) instantons lead to effective 12 fermion interactions:

$$O(B + L) = \prod_i q_{Li} q_{Li} q_{Li} l_{Li}$$

These would induce $\Delta B = \Delta L = 3$ processes:

$$u_L + d_L + c_L + s_L + t_L + b_L + \nu_{eL} + \nu_{\mu L} + \nu_{\tau L} \rightarrow \bar{d}_R + \bar{b}_R + \bar{s}_R$$

However, at $T = 0$ the probability of such processes is $\Gamma \sim e^{-4\pi/\alpha} \sim 10^{-165}$.

At finite T , the transitions proceed via thermal fluctuations with an unsuppressed probability: $\Gamma \sim \alpha T^4$.

Sphaleron processes are efficient for

$$T_{EW} \sim 100 \text{ GeV} < T < 10^{12} \text{ GeV}$$

Leptogenesis

$$Y_B = \frac{n_B - \bar{n}_B}{S} \sim 8.6 \times 10^{-11} \quad (n_\gamma: \sim 6.3 \times 10^{-10})$$

$$Y_B \cong -10^{-2} \quad \epsilon \kappa$$

W. Buchmüller, M. Plümacher, 1998;
W. Buchmüller, P. Di Bari, M. Plümacher, 2004

κ –efficiency factor; $\kappa \sim 10^{-1} - 10^{-3}$; $\epsilon \gtrsim 10^{-7}$.

ϵ : CP –, L – violating asymmetry generated in out of equilibrium N_{Rj} –decays in the early Universe,

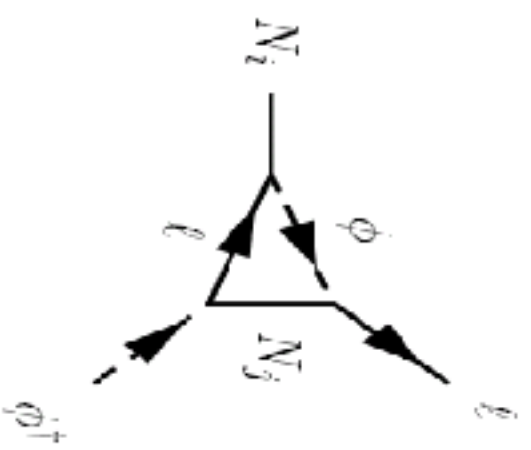
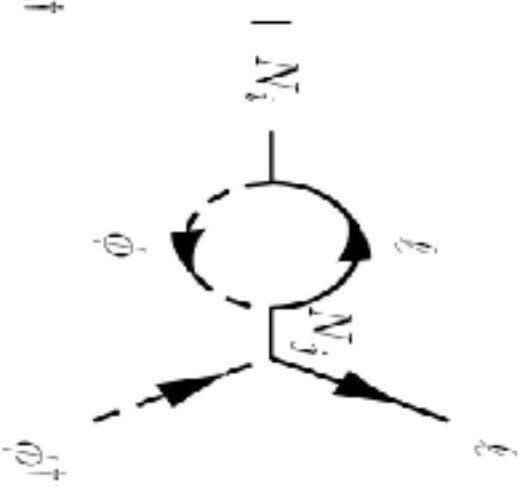
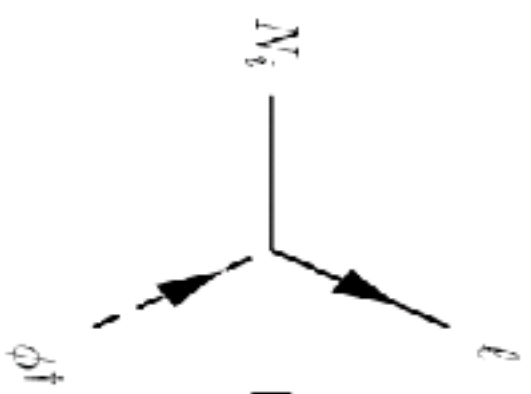
$$\epsilon_1 = \frac{\Gamma(N_1 \rightarrow \Phi^- \ell^+) - \Gamma(N_1 \rightarrow \Phi^+ \ell^-)}{\Gamma(N_1 \rightarrow \Phi^- \ell^+) + \Gamma(N_1 \rightarrow \Phi^+ \ell^-)}$$

M.A. Luty, 1992;
L. Covi, E. Roulet and F. Vissani, 1996;
M. Flanz *et al.*, 1996;
M. Plümacher, 1997;
A. Pilaftsis, 1997.

$\kappa = \kappa(\tilde{m})$, $\tilde{m}$ - determines the rate of wash-out processes:

$$\Phi^+ + \ell^- \rightarrow N_1, \quad \ell^- + \Phi^+ \rightarrow \Phi^- + \ell^+, \text{ etc.}$$

W. Buchmüller, P. Di Bari and M. Plümacher, 2002;
G. F. Giudice *et al.*, 2004



Low Energy Leptonic CPV and Leptogenesis

Assume: $M_1 \ll M_2 \ll M_3$

Individual asymmetries:

$$\epsilon_{1l} = -\frac{3M_1}{16\pi v^2} \frac{\text{Im}\left(\sum_{j,k} m_j^{1/2} m_k^{3/2} U_{lj}^* U_{lk} R_{1j} R_{1k}\right)}{\sum_j m_j |R_{1j}|^2}, \quad v = 174 \text{ GeV}$$

$$\widetilde{m}_l \equiv \frac{|\lambda_{1l}|^2 v^2}{M_1} = \left| \sum_k R_{1k} m_k^{1/2} U_{lk}^* \right|^2, \quad l = e, \mu, \tau.$$

The "one-flavor" approximation - $Y_{e,\mu,\tau}$ - "small":

Boltzmann eqn. for $n(N_1)$ and $\Delta L = \Delta(L_e + L_\mu + L_\tau)$.

$Y_l H^c(x) \bar{l}_R(x) \psi_{lL}$ - out of equilibrium at $T \sim M_1$.

One-flavor approximation: $M_1 \sim T > 10^{12} \text{ GeV}$

$$\epsilon_1 = \sum_l \epsilon_{1l} = -\frac{3M_1}{16\pi v^2} \frac{\text{Im}\left(\sum_{j,k} m_j^2 R_{1j}^2\right)}{\sum_k m_k |R_{1k}|^2},$$

$$\widetilde{m}_1 = \sum_l \widetilde{m}_l = \sum_k m_k |R_{1k}|^2.$$

Two-Flavour Regime

At $M_1 \sim T \sim 10^{12}$ GeV: Y_τ - in equilibrium, $Y_{e\mu}$ - not;

wash-out dynamics changes: τ_R^-, τ_L^+

$N_1 \rightarrow (\lambda_{1e} e_L^- + \lambda_{1\mu} \mu_L^- + \lambda_{1\tau} \tau_L^-) + \Phi^+$; $(\lambda_{1e} e_L^- + \lambda_{1\mu} \mu_L^- + \lambda_{1\tau} \tau_L^-) + \Phi^+ \rightarrow N_1$;

$\tau_L^- + \Phi^0 \rightarrow \tau_R^-, \quad \tau_L^- + \tau_L^+ \rightarrow N_1 + \nu_L$, etc.

$\epsilon_{1\tau}$ and $(\epsilon_{1e} + \epsilon_{1\mu}) \equiv \epsilon_2$ evolve independently.

Three-Flavour Regime

At $M_1 \sim T \sim 10^9$ GeV: Y_τ, Y_μ - in equilibrium, Y_e - not.

$\epsilon_{1\tau}, \epsilon_{1e}$ and $\epsilon_{1\mu}$ evolve independently.

Thus, at $M_1 \sim 10^9 - 10^{12}$ GeV: $L_\tau, \Delta L_\tau$ - distinguishable;

$L_e, L_\mu, \Delta L_e, \Delta L_\mu$ - individually not distinguishable;

$L_e + L_\mu, \Delta(L_e + L_\mu)$

Individual asymmetries:

Assume: $M_1 \ll M_2 \ll M_3$, $10^9 \lesssim M_1 (\sim T) \lesssim 10^{12}$ GeV,

$$\epsilon_{1l} = -\frac{3M_1}{16\pi v^2} \frac{\text{Im} \left(\sum_{j,k} m_j^{1/2} m_k^{3/2} U_{lj}^* U_{lk} R_{lj} R_{lk} \right)}{\sum_j m_j |R_{lj}|^2}$$

$$\widetilde{m}_l \equiv \frac{|\lambda_{1l}|^2 v^2}{M_1} = \left| \sum_k R_{lk} m_k^{1/2} U_{lk}^* \right|^2, \quad l = e, \mu, \tau.$$

The baryon asymmetry is

$$Y_B \simeq -\frac{12}{37g_*} \left(\epsilon_2 \eta \left(\frac{417}{589} \widetilde{m}_2 \right) + \epsilon_\tau \eta \left(\frac{390}{589} \widetilde{m}_\tau \right) \right),$$

$$\eta(\widetilde{m}_l) \simeq \left(\left(\frac{\widetilde{m}_l}{8.25 \times 10^{-3} \text{ eV}} \right)^{-1} + \left(\frac{0.2 \times 10^{-3} \text{ eV}}{\widetilde{m}_l} \right)^{-1.16} \right)^{-1}.$$

$$Y_B = -(12/37) (Y_2 + Y_\tau),$$

$$Y_2 = Y_{e+\mu}, \quad \epsilon_2 = \epsilon_{1e} + \epsilon_{1\mu}, \quad \widetilde{m}_2 = \widetilde{m}_{1e} + \widetilde{m}_{1\mu}$$

A. Abada et al., 2006; E. Nardi et al., 2006

A. Abada et al., 2006

Real (Purely Imaginary) R : $\varepsilon_{1l} \neq 0$, CPV from U

$$\varepsilon_{1e} + \varepsilon_{1\mu} + \varepsilon_{1\tau} = \varepsilon_2 + \varepsilon_{1\tau} = 0,$$

$$\begin{aligned} \varepsilon_{1\tau} &= -\frac{3M_1}{16\pi v^2} \frac{\text{Im}\left(\sum_{j,k} m_j^{1/2} m_k^{3/2} U_{\tau j}^* U_{\tau k} R_{1j} R_{1k}\right)}{\sum_j m_j |R_{1j}|^2} \\ &= -\frac{3M_1}{16\pi v^2} \frac{\sum_{j,k>j} m_j^{1/2} m_k^{1/2} (m_k - m_j) R_{1j} R_{1k} \text{Im}(U_{\tau j}^* U_{\tau k})}{\sum_j m_j |R_{1j}|^2}, \quad R_{1j} R_{1k} = \pm |R_{1j} R_{1k}|, \\ &= \mp \frac{3M_1}{16\pi v^2} \frac{\sum_{j,k>j} m_j^{1/2} m_k^{1/2} (m_k + m_j) |R_{1j} R_{1k}| \text{Re}(U_{\tau j}^* U_{\tau k})}{\sum_j m_j |R_{1j}|^2}, \quad R_{1j} R_{1k} = \pm i |R_{1j} R_{1k}| \end{aligned}$$

S. Pascoli, S.T.P., A. Riotto, 2006.

CP-Violation: $\text{Im}(U_{\tau j}^* U_{\tau k}) \neq 0$, $\text{Re}(U_{\tau j}^* U_{\tau k}) \neq 0$;

$$Y_B = -\frac{12}{37} \frac{\varepsilon_{1\tau}}{g_*} \left(\eta \left(\frac{390}{589} \widetilde{m_\tau} \right) - \eta \left(\frac{417}{589} \widetilde{m_2} \right) \right)$$

$$\textcolor{red}{m_1 \ll m_2 \ll m_3, \; M_1 \ll M_{2,3}; \; R_{12}R_{13} - \text{real}; \; m_1 \cong 0, \; R_{11} \cong 0 \; (N_3 \text{ decoupling})}$$

$$\epsilon_{1\tau} \; = \; - \frac{3M_1 \sqrt{\Delta m_{31}^2}}{16\pi v^2} \left(\frac{\Delta m_{\odot}^2}{\Delta m_{31}^2} \right)^{\frac{1}{4}} \frac{|R_{12}R_{13}|}{\left(\frac{\Delta m_{\odot}^2}{\Delta m_{31}^2} \right)^{\frac{1}{2}} |R_{12}|^2 + |R_{13}|^2}$$

$$\times \left(1 - \frac{\sqrt{\Delta m_{\odot}^2}}{\sqrt{\Delta m_{31}^2}} \right) \textcolor{red}{\mathrm{Im}\left(U_{\tau 2}^* U_{\tau 3} \right)}$$

$$\textcolor{red}{\mathrm{Im}\left(U_{\tau 2}^* U_{\tau 3} \right)} \; = \; -c_{13} \left[c_{23}s_{23}c_{12} \sin\left(\frac{\alpha_{32}}{2}\right) - c_{23}^2 s_{12}s_{13} \sin\left(\textcolor{red}{\delta} - \frac{\alpha_{32}}{2}\right) \right]$$

$$\alpha_{32} = \pi, \; \delta = 0: \; \mathrm{Re}(U_{\tau 2}^* U_{\tau 3}) = 0, \quad \textcolor{red}{\mathrm{CPV} \text{ due to } R}$$

$$\textcolor{violet}{S. Pascoli, S.T.P., A. Riotto, 2006.}$$

$$M_1 \ll M_2 \ll M_3, \quad m_1 \ll m_2 \ll m_3 \quad (\text{NH})$$

Dirac CP-violation

$$\alpha_{32} = 0 \ (2\pi), \quad \beta_{23} = \pi \ (0); \quad \beta_{23} \equiv \beta_{12} + \beta_{13} \equiv \arg(R_{12}R_{13}).$$

$$|R_{12}|^2 \cong 0.85, \quad |R_{13}|^2 = 1 - |R_{12}|^2 \cong 0.15 \text{ - maximise } |\epsilon_\tau| \text{ and } |Y_B|:$$

$$|Y_B| \cong 2.8 \times 10^{-13} |\sin \delta| \left(\frac{s_{13}}{0.2} \right) \left(\frac{M_1}{10^9 \text{ GeV}} \right).$$

$$|Y_B| \gtrsim 8 \times 10^{-11}, \quad M_1 \lesssim 5 \times 10^{11} \text{ GeV imply}$$

$$|\sin \theta_{13} \sin \delta| \gtrsim 0.11, \quad \sin \theta_{13} \gtrsim 0.11.$$

The lower limit corresponds to

$$|J_{\text{CP}}| \gtrsim 2.4 \times 10^{-2}$$

$$\text{FOR } \alpha_{32} = 0 \ (2\pi), \quad \beta_{23} = 0 \ (\pi):$$

$$|\sin \theta_{13} \sin \delta| \gtrsim 0.09, \quad \sin \theta_{13} \gtrsim 0.09; \quad |J_{\text{CP}}| \gtrsim 2.0 \times 10^{-2}$$

$$M_1 \ll M_2 \ll M_3, \quad m_1 \ll m_2 \ll m_3 \quad (\text{NH})$$

Majorana CP-violation

$$\delta = 0, \text{ real } R_{12}, R_{13} \quad (\beta_{23} = \pi \text{ (0)});$$

$$\alpha_{32} \cong \pi/2, \quad |R_{12}|^2 \cong 0.85, |R_{13}|^2 = 1 - |R_{12}|^2 \cong 0.15 \text{ - maximise } |\epsilon_\tau| \text{ and } |Y_B|:$$

$$|Y_B| \quad \cong 2 \times 10^{-12} \left(\frac{\sqrt{\Delta m_{31}^2}}{0.05 \text{ eV}} \right) \left(\frac{M_1}{10^9 \text{ GeV}} \right) .$$

$$\text{We get } |Y_B| \gtrsim 8 \times 10^{-11}, \quad \text{for } M_1 \gtrsim 3.6 \times 10^{10} \text{ GeV, or } |\sin \alpha_{32}/2| \gtrsim 0.15$$

$$M_1 \ll M_2 \ll M_3, \quad m_3 \ll m_1 < m_2 \text{ (IH)}$$

$m_3 \cong 0$, $R_{13} \cong 0$ (N_3 decoupling): impossible to reproduce Y_B^{obs} for real $R_{11} R_{12}$;

$|Y_B|$ suppressed by the additional factor $\Delta m_{\odot}^2/|\Delta m_A^2| \cong 0.03$.

Purely imaginary $R_{11} R_{12}$: no (additional) suppression

Dirac CP-violation

$$\alpha_{21} = \pi; \quad R_{11} R_{12} = i\kappa |R_{11} R_{12}|, \quad \kappa = 1;$$

$|R_{11}| \cong 1.07$, $|R_{12}|^2 = |R_{11}|^2 - 1$, $|R_{12}| \cong 0.38$ - maximise $|\epsilon_\tau|$ and $|Y_B|$:

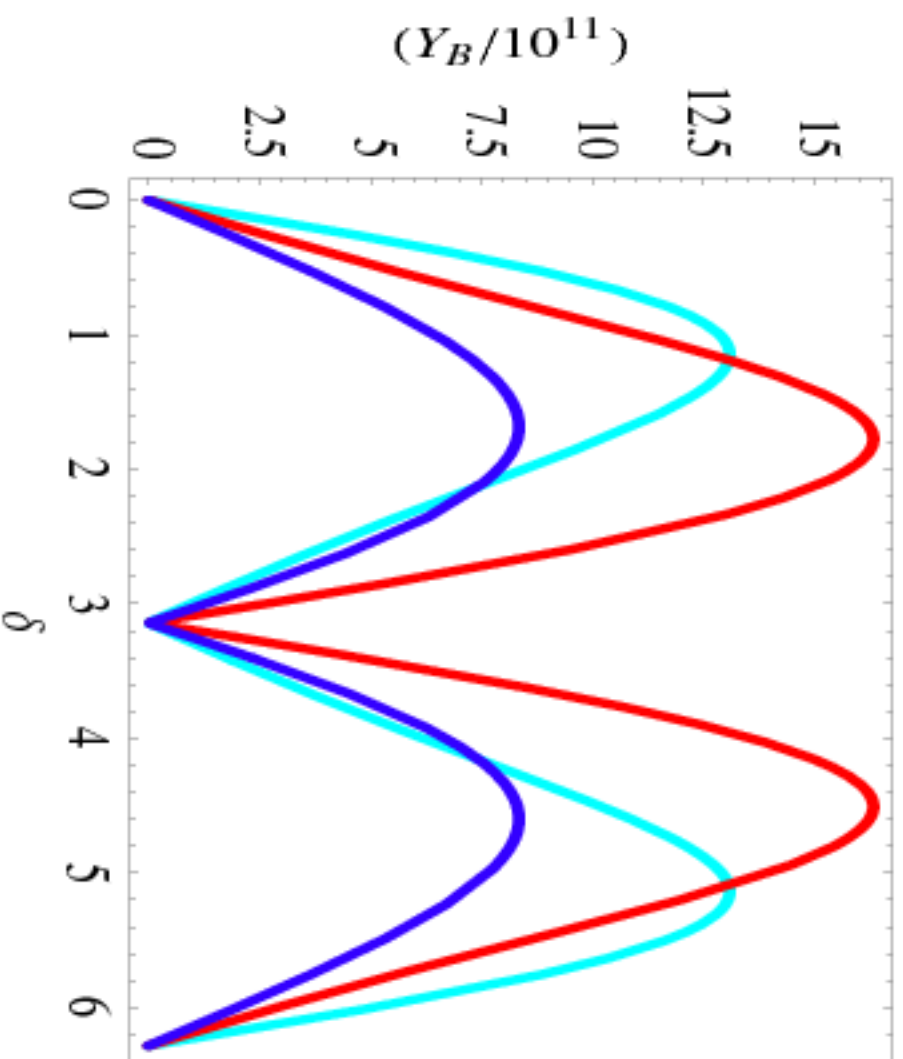
$$|Y_B| \cong 8.1 \times 10^{-12} |s_{13} \sin \delta| \left(\frac{M_1}{10^9 \text{ GeV}} \right).$$

$$|Y_B| \gtrsim 8 \times 10^{-11}, \quad M_1 \lesssim 5 \times 10^{11} \text{ GeV imply}$$

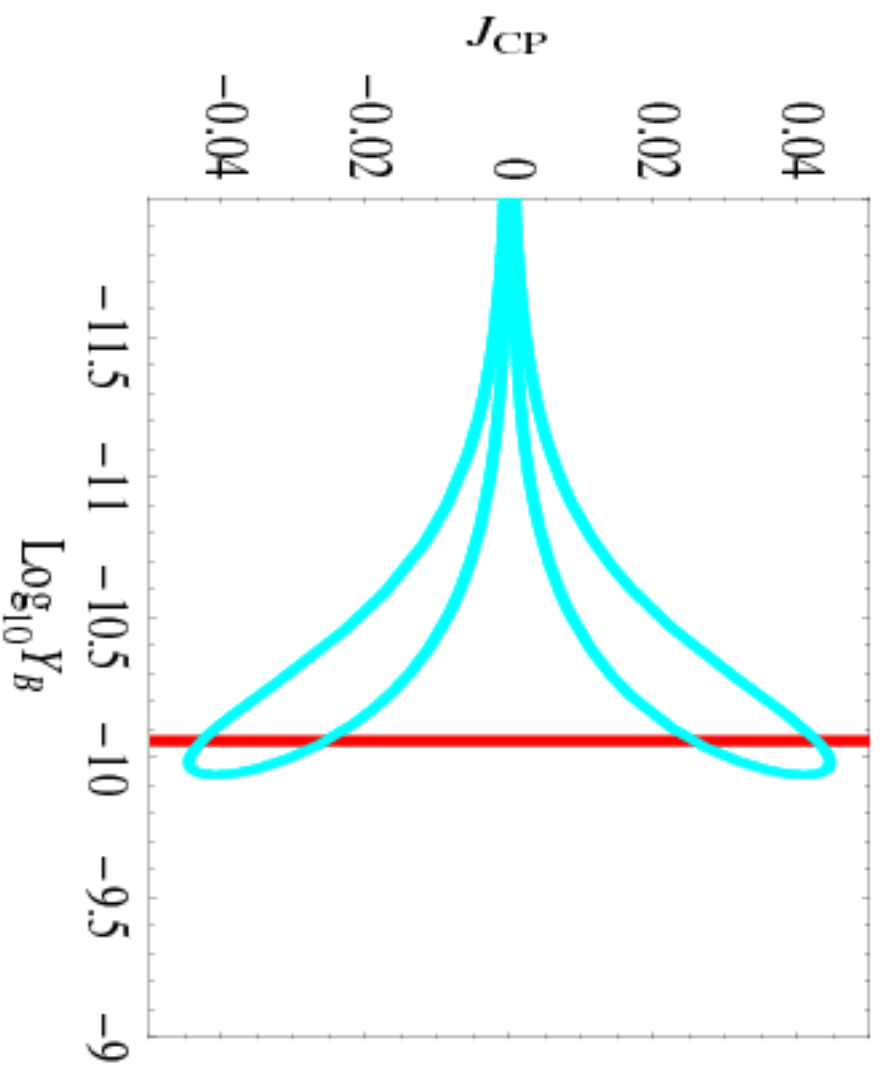
$$|\sin \theta_{13} \sin \delta| \gtrsim 0.02, \quad \sin \theta_{13} \gtrsim 0.02.$$

The lower limit corresponds to

$$|J_{CP}| \gtrsim 4.6 \times 10^{-3}$$



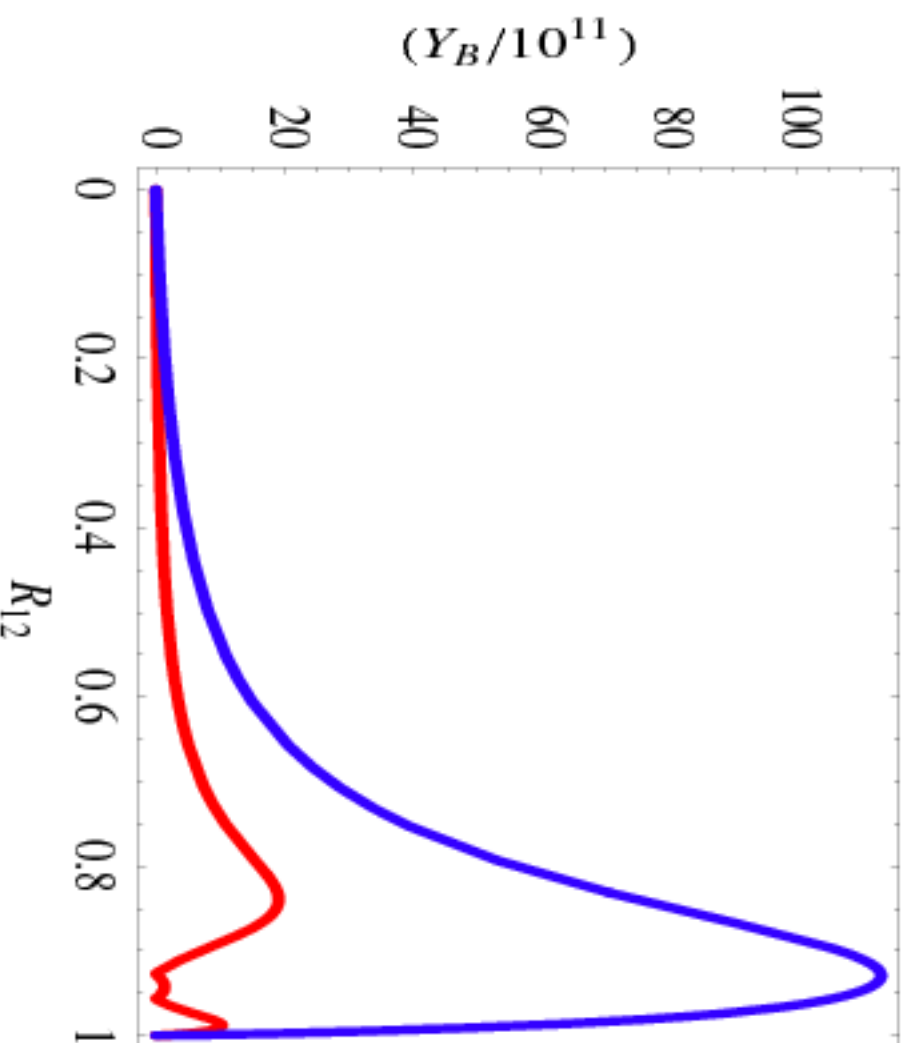
$M_1 \ll M_2 \ll M_3$, $m_1 \ll m_2 \ll m_3$; Dirac CP-violation, $\alpha_{32} = 0$; 2π ;
 real R_{12} , R_{13} , $|R_{12}|^2 + |R_{13}|^2 = 1$, $|R_{12}| = 0.86$, $|R_{13}| = 0.51$, $\text{sign}(R_{12}R_{13}) = +1$;
 i) $\alpha_{32} = 0$ ($\kappa' = +1$), $s_{13} = 0.2$ (red line) and $s_{13} = 0.1$ (dark blue line);
 ii) $\alpha_{32} = 2\pi$ ($\kappa' = -1$), $s_{13} = 0.2$ (light blue line);
 $M_1 = 5 \times 10^{11}$ GeV.



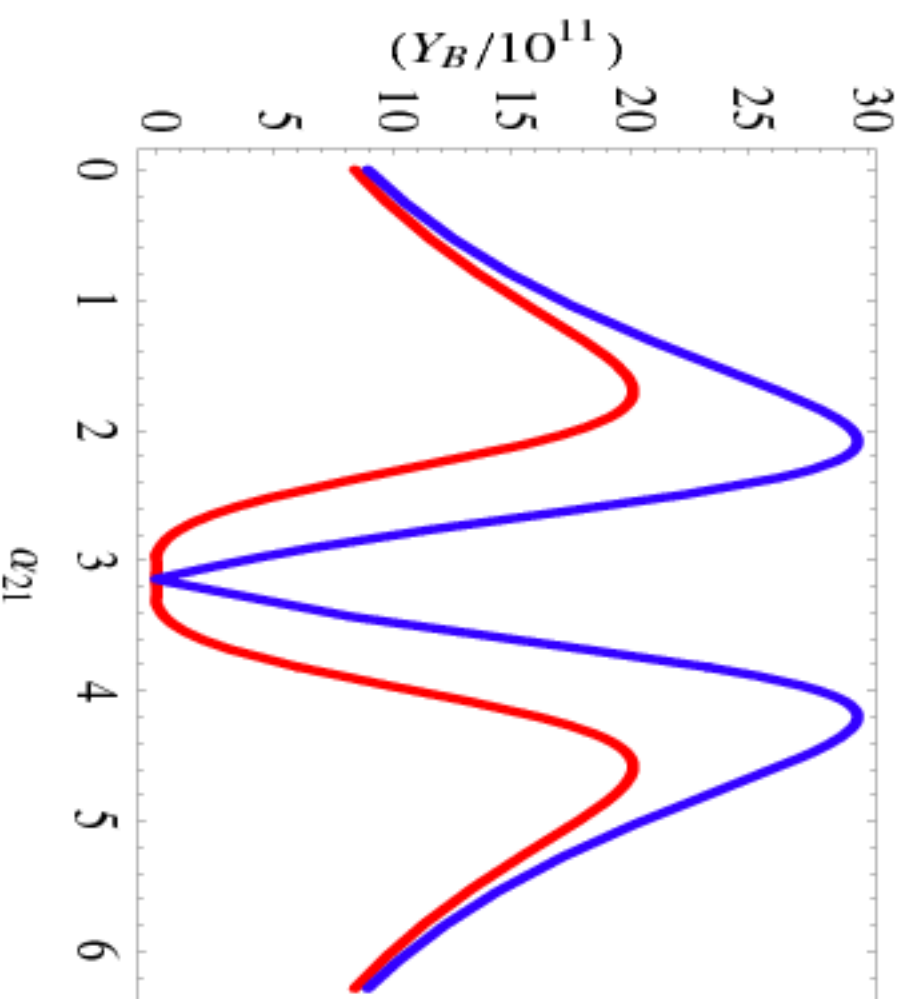
$M_1 \ll M_2 \ll M_3$, $m_1 \ll m_2 \ll m_3$; $M_1 = 5 \times 10^{11}$ GeV;
 Dirac CP-violation, $\alpha_{32} = 0$ (2π);

$|R_{12}| = 0.86$, $|R_{13}| = 0.51$, $\text{sign}(R_{12}R_{13}) = +1$ (-1) ($\beta_{23} = 0$ (π), $\kappa' = +1$);

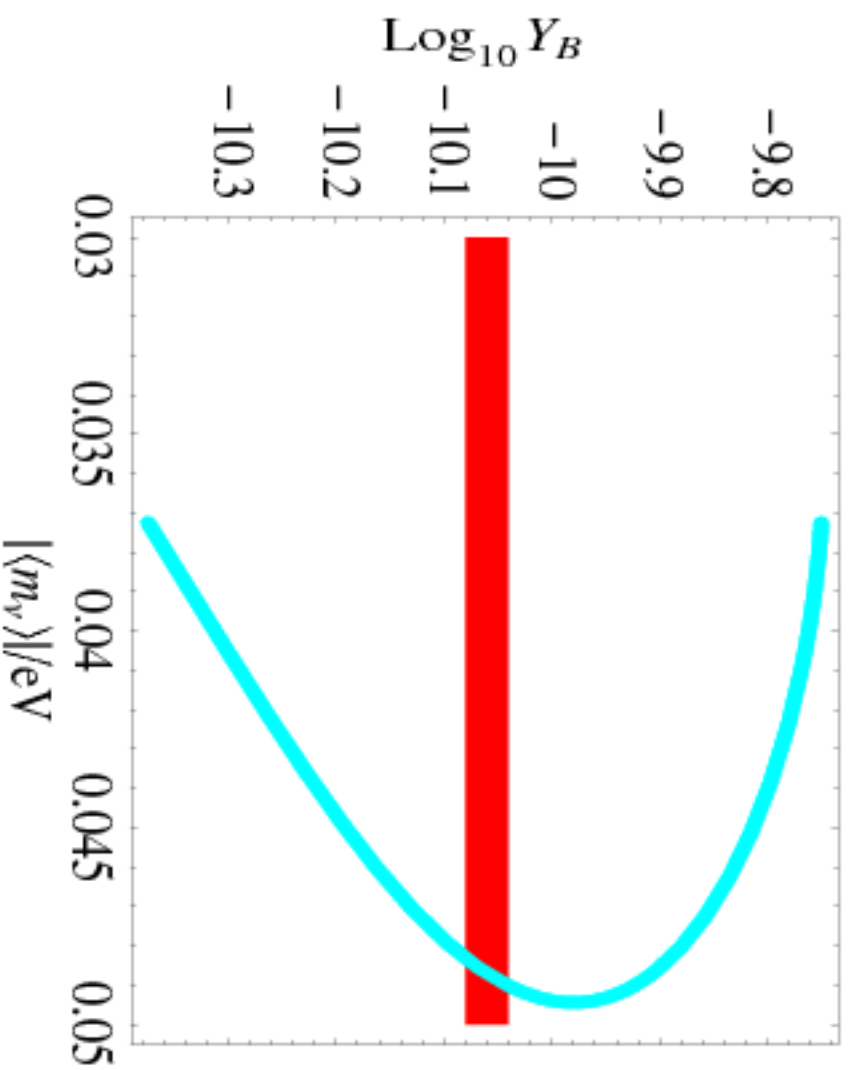
The red region denotes the 2σ allowed range of Y_B .



- $M_1 \ll M_2 \ll M_3$, $m_1 \ll m_2 \ll m_3$; $M_1 = 5 \times 10^{11}$ GeV;
real R_{12} , R_{13} , $\text{sign}(R_{12}R_{13}) = +1$, $R_{12}^2 + R_{13}^2 = 1$, $s_{13} = 0.20$;
a) Majorana CP-violation (blue line), $\delta = 0$ and $\alpha_{32} = \pi/2$ ($\kappa = +1$);
b) Dirac CP-violation (red line), $\delta = \pi/2$ and $\alpha_{32} = 0$ ($\kappa' = +1$);
 $\Delta m_{\odot}^2$, $\sin^2 \theta_{12}$, Δm_{31}^2 , $\sin^2 2\theta_{23}$ - fixed at their best fit values.



$M_1 \ll M_2 \ll M_3$, $m_3 \ll m_1 < m_2$; $M_1 = 2 \times 10^{11}$ GeV;
 Majorana CP-violation, $\delta = 0$;
 purely imaginary $R_{11}R_{12} = i\kappa|R_{11}R_{12}|$, $\kappa = -1$, $|R_{11}|^2 - |R_{12}|^2 = 1$, $|R_{11}| = 1.2$;
 $s_{13} = 0$ (blue line) and 0.2 (red line).



$M_1 \ll M_2 \ll M_3$, $m_3 \ll m_1 < m_2$; $M_1 = 2 \times 10^{11}$ GeV;
Majorana CP-violation, $\delta = 0$, $s_{13} = 0$;

purely imaginary $R_{11}R_{12} = i\kappa|R_{11}R_{12}|$, $\kappa = \pm 1$ $|R_{11}|^2 - |R_{12}|^2 = 1$, $|R_{11}| = 1.05$.

The Majorana phase α_{21} is varied in the interval $[-\pi/2, \pi/2]$.

$$M_1 \ll M_2 \ll M_3, \quad m_3 \ll m_1 < m_2 \text{ (IH)}$$

Majorana or Dirac CP-violation

$m_3 \neq 0$, $R_{13} \neq 0$, $R_{11}(R_{12}) = 0$: possible to reproduce Y_B^{obs} for real $R_{12(11)}R_{13} \neq 0$

Requires $m_3 \cong (10^{-5} - 10^{-2})$ eV; non-trivial dependence of $|Y_B|$ on m_3

Majorana CPV, $\delta = 0$ (π): requires $M_1 \gtrsim 3.5 \times 10^{10}$ GeV

Dirac CPV, $\alpha_{32(31)} = 0$: typically requires $M_1 \gtrsim 10^{11}$ GeV

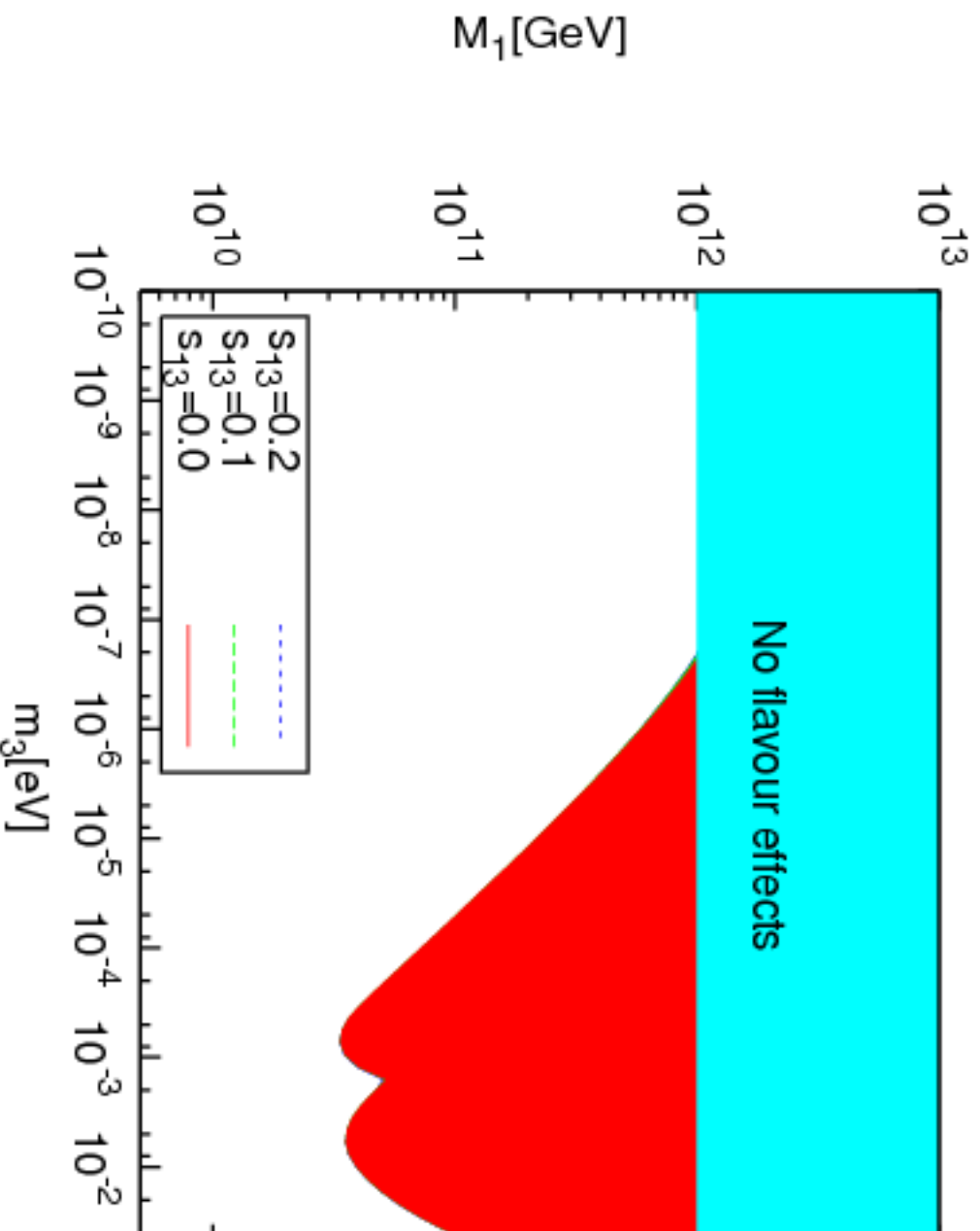
$$|Y_B| \gtrsim 8 \times 10^{-11}, \quad M_1 \lesssim 5 \times 10^{11} \text{ GeV imply}$$

$$|\sin \theta_{13} \sin \delta|, \quad \sin \theta_{13} \gtrsim (0.04 - 0.09).$$

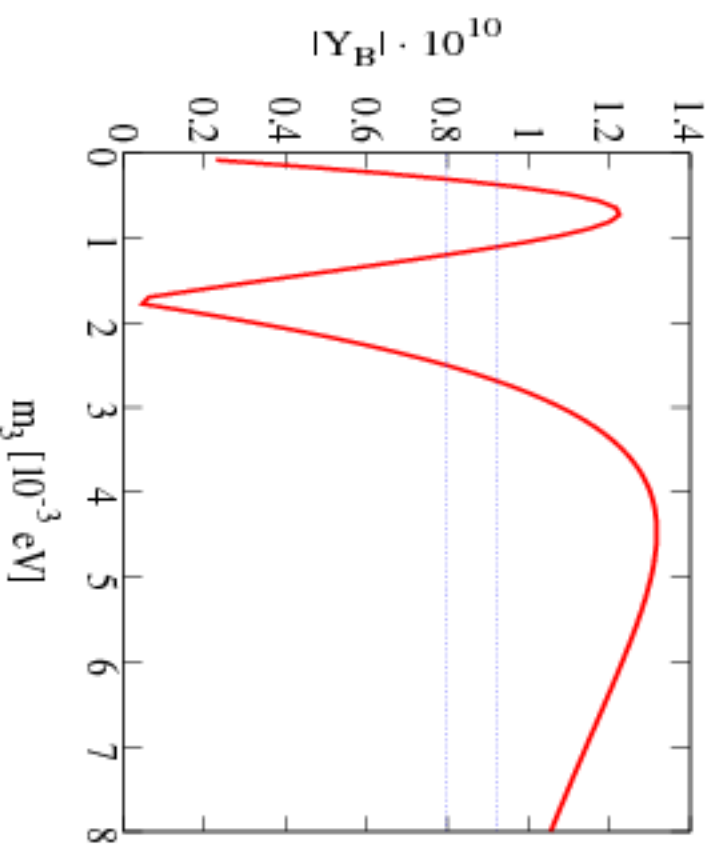
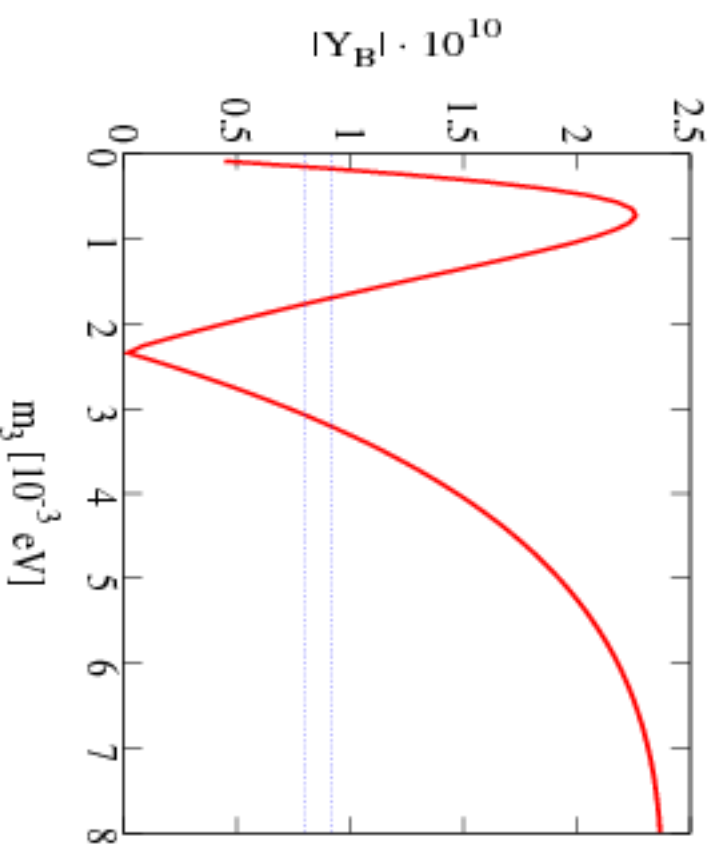
The lower limit corresponds to

$$|J_{CP}| \gtrsim (0.009 - 0.02)$$

NO (NH) spectrum, $m_1 < (\ll) m_2 < m_3$: similar dependence of $|Y_B|$ on m_1 if $R_{12} = 0$, $R_{11}R_{13} \neq 0$; non-trivial effects for $m_1 \cong (10^{-4} - 5 \times 10^{-2})$ eV.



$m_3 < m_1 < m_2$, $M_1 \ll M_2 \ll M_3$, real R_{1j} ; $M_1 = (10^9 - 10^{12})$ GeV, $s_{13} = 0.2; 0.1; 0$;
 R_{1j} varied within $|R_{13}|^2 + |R_{12}|^2 + |R_{13}|^2 = 1$; $\alpha_{21}, \alpha_{31}, \delta$ varied in $[0, 2\pi]$;
 $\min(M_1)$ for given m_3 : $|Y_B| = 8.6 \times 10^{-11}$; absolute minima of M_1 :
 $m_3 \cong 5.5 \times 10^{-4}$, 5.9×10^{-3} eV, $\alpha_{32} \cong \pi/2$, $M_1 = 3.4$ (3.5) $\times 10^{10}$ GeV.

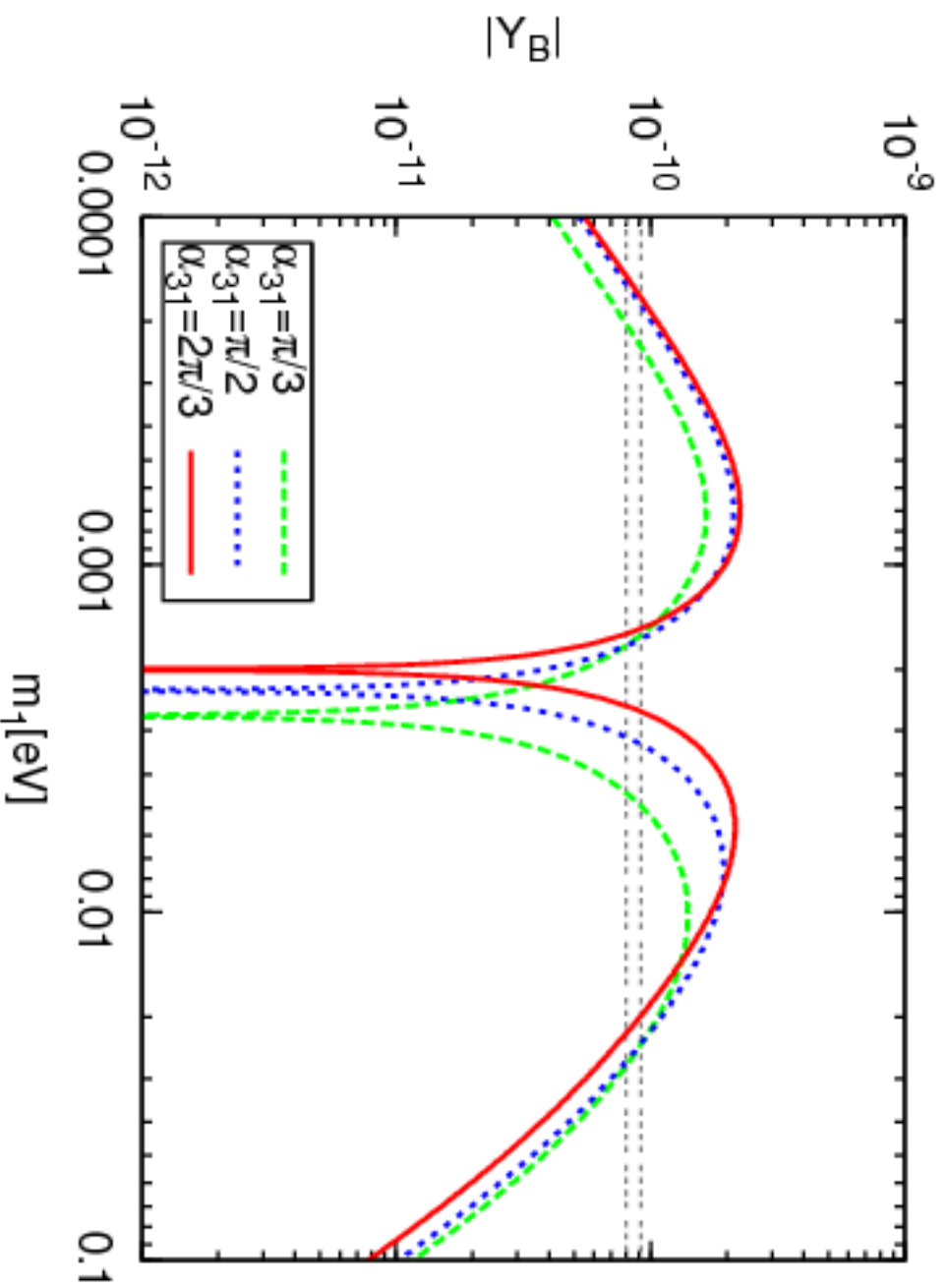


$m_3 \ll m_1 \ll m_2$ (IH), $R_{11} = 0$, real $R_{12}R_{13}$, Majorana CPV;

$\alpha_{32} = \pi/2$, $s_{13} = 0$, $M_1 = 10^{11}$ GeV; $R_{12}^2/R_{13}^2 = m_3/m_2$: maximises $|\epsilon_\tau|$;

i) $\text{sgn}(R_{12}R_{13}) = +1$; ii) $\text{sgn}(R_{12}R_{13}) = -1$.

E. Molinaro, S.T.P., T. Shindou, Y. Takanishi, 2007



$m_1 < m_2 < m_3$ (NO(NH)), $R_{12} = 0$, real $R_{11}R_{13}$, Majorana CPV, $s_{13} = 0$;
 $\text{sgn}(R_{11}R_{13}) = -1$, $\sin^2\theta_{23} = 0.50$, $M_1 = 1.5 \times 10^{11}$ GeV;
 $\alpha_{32} = 2\pi/3; \pi/2; \pi/3$ (red, blue, green lines).

Low Energy Leptonic CPV and Leptogenesis: Summary

Leptogenesis: see-saw mechanism; N_j - heavy RH ν 's;

N_j, ν_k - Majorana particles

$$N_j: M_1 \ll M_2 \ll M_3$$

The observed value of the baryon asymmetry of the Universe can be generated

A. **CP-violation due to the Dirac phase δ in U_{PMNS}** , no other sources of CPV (Majorana phases in U_{PMNS} equal to 0, etc.); requires $M_1 \gtrsim 10^{11}$ GeV.

$$m_1 \ll m_2 \ll m_3 \text{ (NH):}$$

$$|\sin \theta_{13} \sin \delta| \gtrsim 0.09, \quad \sin \theta_{13} \gtrsim 0.09; \quad |J_{\text{CP}}| \gtrsim 2.0 \times 10^{-2}$$

$$m_3 \ll m_1 < m_2 \text{ (IH):}$$

$$|\sin \theta_{13} \sin \delta| \gtrsim 0.02, \quad \sin \theta_{13} \gtrsim 0.02; \quad |J_{\text{CP}}| \gtrsim 4.6 \times 10^{-3}$$

B. **CP-violation due to the Majorana phases in U_{PMNS}** , no other sources of CPV (Dirac phase in U_{PMNS} equal to 0, etc.); requires $M_1 \gtrsim 3.5 \times 10^{10}$ GeV.

C. **CP-violation due to both Dirac and Majorana phases in U_{PMNS}** .

D. Y_B can depend non-trivially on $\min(m_j) \sim (10^{-5} - 10^{-2})$ eV.

S. Pascoli, S.T.P., A. Riotto, 2006 (A-C);
E. Molinaro, S.T.P., T. Shindou, Y. Takanishi, 2007 (D).

Complex R : $\epsilon_{1l} \neq 0$, CPV from U and R

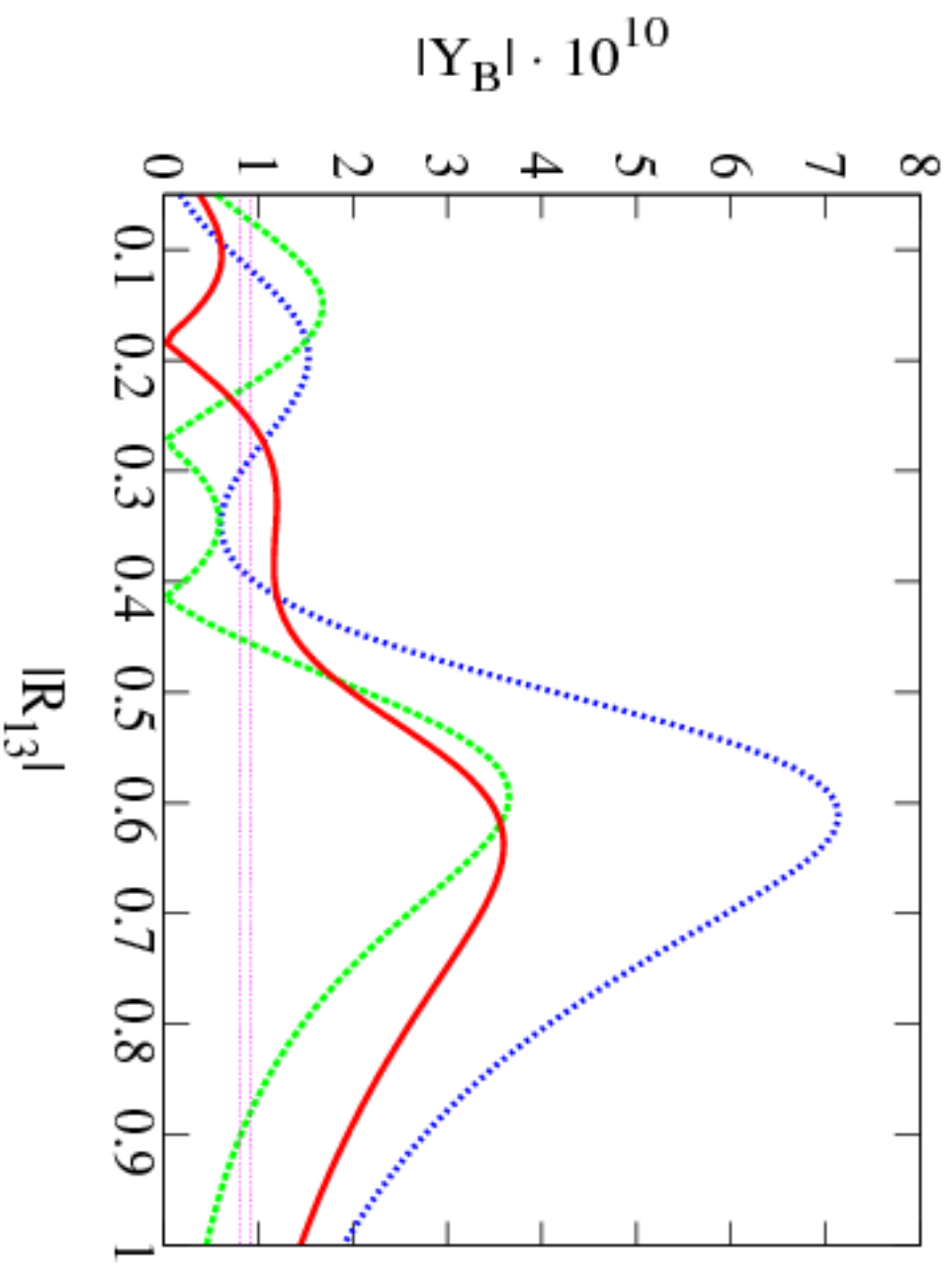
$m_1 \ll m_2 < m_3$ (NH), $M_1 \ll M_{2,3}$; $m_1 \cong 0$, $R_{11} \cong 0$ (N_3 decoupling)

$$R_{12}^2 + R_{13}^2 = |R_{12}|^2 e^{i2\varphi_{12}} + |R_{13}|^2 e^{i2\varphi_{13}} = 1,$$

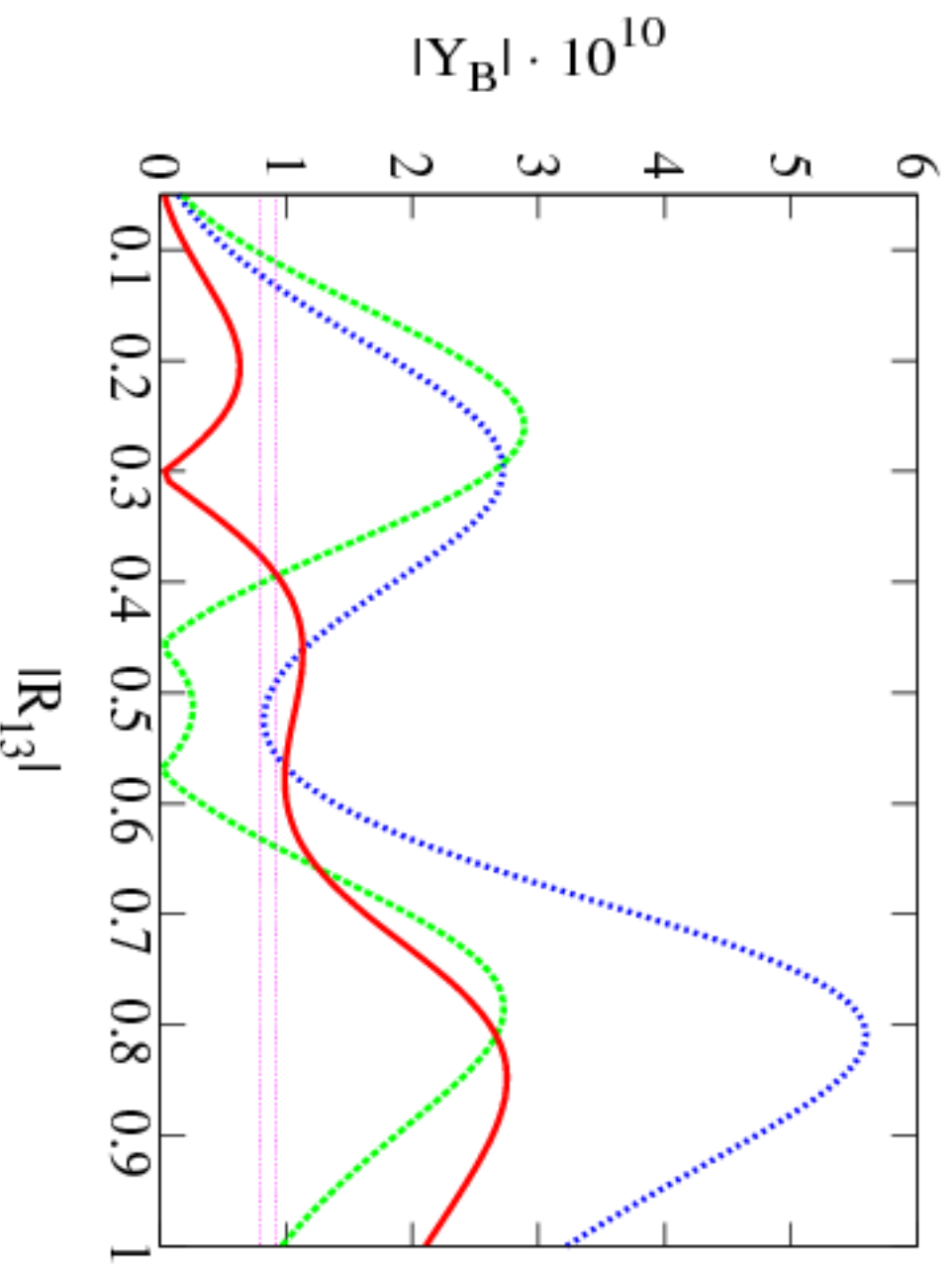
$$|R_{12}|^2 \sin 2\varphi_{12} + |R_{13}|^2 \sin 2\varphi_{13} = 0 : \text{sgn}(\sin 2\varphi_{12}) = -\text{sgn}(\sin 2\varphi_{13}).$$

$$\cos 2\varphi_{12} = \frac{1+|R_{12}|^4-|R_{13}|^4}{2|R_{12}|^2}, \quad \sin 2\varphi_{12} = \pm \sqrt{1 - \cos^2 2\varphi_{12}},$$

$$\cos 2\varphi_{13} = \frac{1-|R_{12}|^4+|R_{13}|^4}{2|R_{13}|^2}, \quad \sin 2\varphi_{13} = \mp \sqrt{1 - \cos^2 2\varphi_{13}}.$$



$m_1 < m_2 < m_3$ (NO(NH)), $R_{11} = 0$, CPV due to R and U ,
 $\alpha_{32} = \pi/2$, $s_{13} = 0$, $\sin^2 \theta_{23} = 0.50$, $|R_{12}| \cong 1$, $M_1 = 10^{11}$ GeV;
 $|Y_{B A_{HE}}^0|$ (R CPV, blue), $|Y_{B A_{Mix}}^0|$ (U CPV, green), total $|Y_B|$ (red line)



$m_1 < m_2 < m_3$ (NO(NH)), $R_{11} = 0$, CPV due to R and U ,
 $\alpha_{32} = \pi/2$, $s_{13} = 0$, $\sin^2 \theta_{23} = 0.64$, $|R_{12}| \cong 1$, $M_1 = 10^{11}$ GeV;
 $|Y_{B\text{AHE}}^0|$ (R CPV, blue), $|Y_{B\text{AHE}}^0|$ (U CPV, green), total $|Y_B|$ (red line)

Low Energy Leptonic CPV and Leptogenesis (contd.)

E. Interesting case: CPV due to the Majorana phases in U_{PMNS} and the R -phases

$$m_3 \ll m_1 < m_2 \text{ (IH)}, \quad M_1 \ll M_{2,3}; \quad m_3 \cong 0, \quad \text{Im}(R_{13}^2) = 0.$$

$$R_{11}^2 + R_{12}^2 + R_{13}^2 = 1:$$

$$|R_{11}|^2 e^{i2\varphi_{11}} + R_{12}^2 e^{i2\varphi_{12}} + R_{13}^2 = 1,$$

$$|R_{11}|^2 \sin 2\varphi_{11} + |R_{12}|^2 \sin 2\varphi_{12} = 0.$$

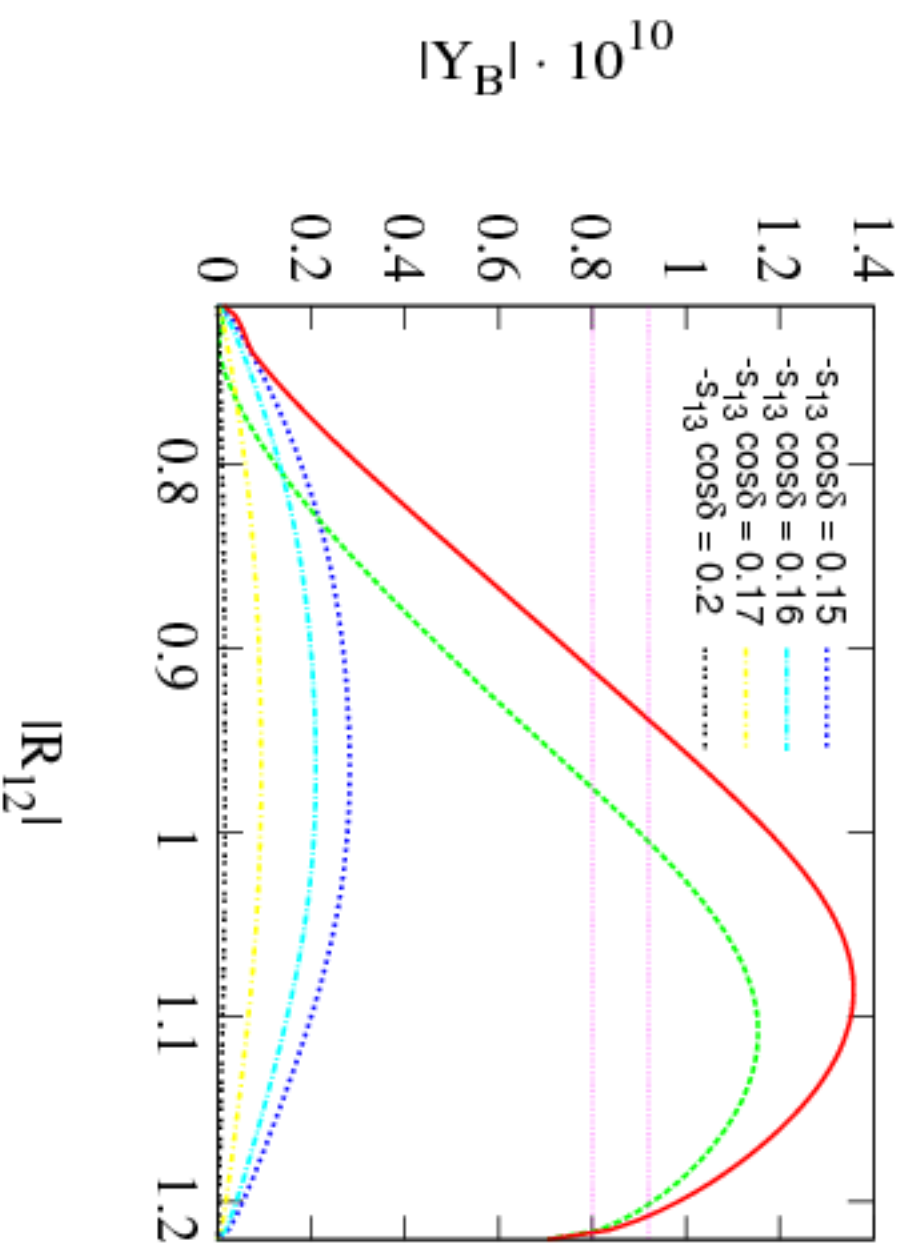
$$|Y_B^0 A_{\text{HE}}| \propto |R_{11}|^2 \sin(2\varphi_{11}) (|U_{\tau 1}|^2 - |U_{\tau 2}|^2) - \text{can be suppressed:}$$

$$|U_{\tau 1}|^2 - |U_{\tau 2}|^2 \cong (s_{12}^2 - c_{12}^2) s_{23}^2 - 4 s_{12} c_{12} s_{23} c_{23} s_{13} \cos \delta \cong -0.20 - 0.92 s_{13} \cos \delta.$$

$$\sin^2 \theta_{12} = 0.3, \quad \sin^2 \theta_{23} = 0.5: \quad (-\sin \theta_{13} \cos \delta) \gtrsim 0.15$$

$$(\sin^2 \theta_{12} = 0.38, \quad \sin^2 \theta_{23} = 0.36: \quad 0.06 \lesssim (-\sin \theta_{13} \cos \delta) \lesssim 0.12)$$

E. Molinaro, S.T.P., 2008, 2010.



$m_3 \ll m_1 < m_2$ (IH), $R_{13} = 0$, Majorana and R -matrix CPV,
 $\alpha_{21} = \pi/2$, $(-s_{13} \cos \delta) = 0.15$, $|R_{11}| = 1.2$, $M_1 = 10^{11}$ GeV;
 $|Y_{B A_{\text{HE}}}^0|$ (R CPV, blue), $|Y_{B A_{\text{Mix}}}^0|$ (U CPV, green), total $|Y_B|$ (red line).

We have shown in the article that for a certain range of values of $\sin(\theta_{13})\cos(\delta)$ in the case of IH light neutrino mass spectrum and hierarchical heavy Majorana neutrino mass spectrum, the “high energy” contribution to the baryon asymmetry of the Universe will be strongly suppressed if the complex R_{13} element of the R -matrix satisfies the inequality: $|R_{13}|^2|\sin(2\tilde{\varphi}_{13})| < \min(|R_{11}|^2|\sin(2\tilde{\varphi}_{11})|, |R_{12}|^2|\sin(2\tilde{\varphi}_{12})|)$. In this Addendum we extend the analysis to the case of arbitrary complex R_{13} : we abandon the condition on $|R_{13}|^2|\sin(2\tilde{\varphi}_{13})|$ given above and determine the ranges of values of $|R_{13}|$ for which the indicated strong suppression of the “high energy” contribution to the baryon asymmetry still takes place even when the phase of R_{13} , $\tilde{\varphi}_{13}$, is allowed to assume large CP-violating values. In this case the contribution to the corresponding CP asymmetry due to the low energy Majorana CP violating phases is essential for producing baryon asymmetry compatible with the observations.

It follows from the numerical analysis performed in this Addendum that for, e.g., $M_1 = 10^{11}$ GeV, the scenario described above is realised and we have subdominant (strongly suppressed) “high energy” contribution to the observed value of the baryon asymmetry for arbitrary $\tilde{\varphi}_{13}$ if *i*) for $|R_{11}| < 0.5$, $|R_{13}|$ satisfies $|R_{13}| \lesssim |R_{11}|$; *ii*) for $0.5 \lesssim |R_{11}| < 1$ we have $|R_{13}| < 0.5$, and if *iii*) for $|R_{11}| > 1$ we have $|R_{13}| < |R_{11}|/2$. In each of the cases *i*) - *iii*) we can have successful leptogenesis, as our results show, due to the contribution to the baryon asymmetry associated with the Majorana CP violating phase(s) in the neutrino mixing matrix.

The results obtained in the present Addendum are illustrated in next figure where we show the total baryon asymmetry Y_B (red dots/line), the purely “high energy” contribution (blue dots/line) and the “mixed” term as function of $|R_{12}|$ for fixed values of $|R_{11}|$ and $|R_{13}|$. The lightest neutrino mass m_3 is set to zero and the lightest RH Majorana neutrino mass corresponds to $M_1 = 10^{11}$ GeV. The figure is obtained for $\sin(\theta_{13}) \cos(\delta) = -0.2$. In the upper left panel, we have set $|R_{11}| = 0.7$ and have taken a real R_{13} , with $R_{13} = 0.3$. The phase of R_{11} , $\tilde{\varphi}_{11}$, was varied in the interval $[3\pi/2, 2\pi]$, while the $R_{12} = |R_{12}|e^{i\tilde{\varphi}_{12}}$ is determined by the orthogonal condition: $R_{11}^2 + R_{12}^2 + R_{13}^2 = 1$. In all the other panels of Fig. 1 we consider a complex $R_{13} = |R_{13}|e^{i\tilde{\varphi}_{13}}$ and the “high energy” phase $\tilde{\varphi}_{13}$ was assumed to take random values in the interval $[0, 2\pi]$. We plot the behavior of each term contributing to the baryon asymmetry as a function of $|R_{12}|$ for fixed $|R_{11}|$ and $|R_{13}|$, following the procedure described above.

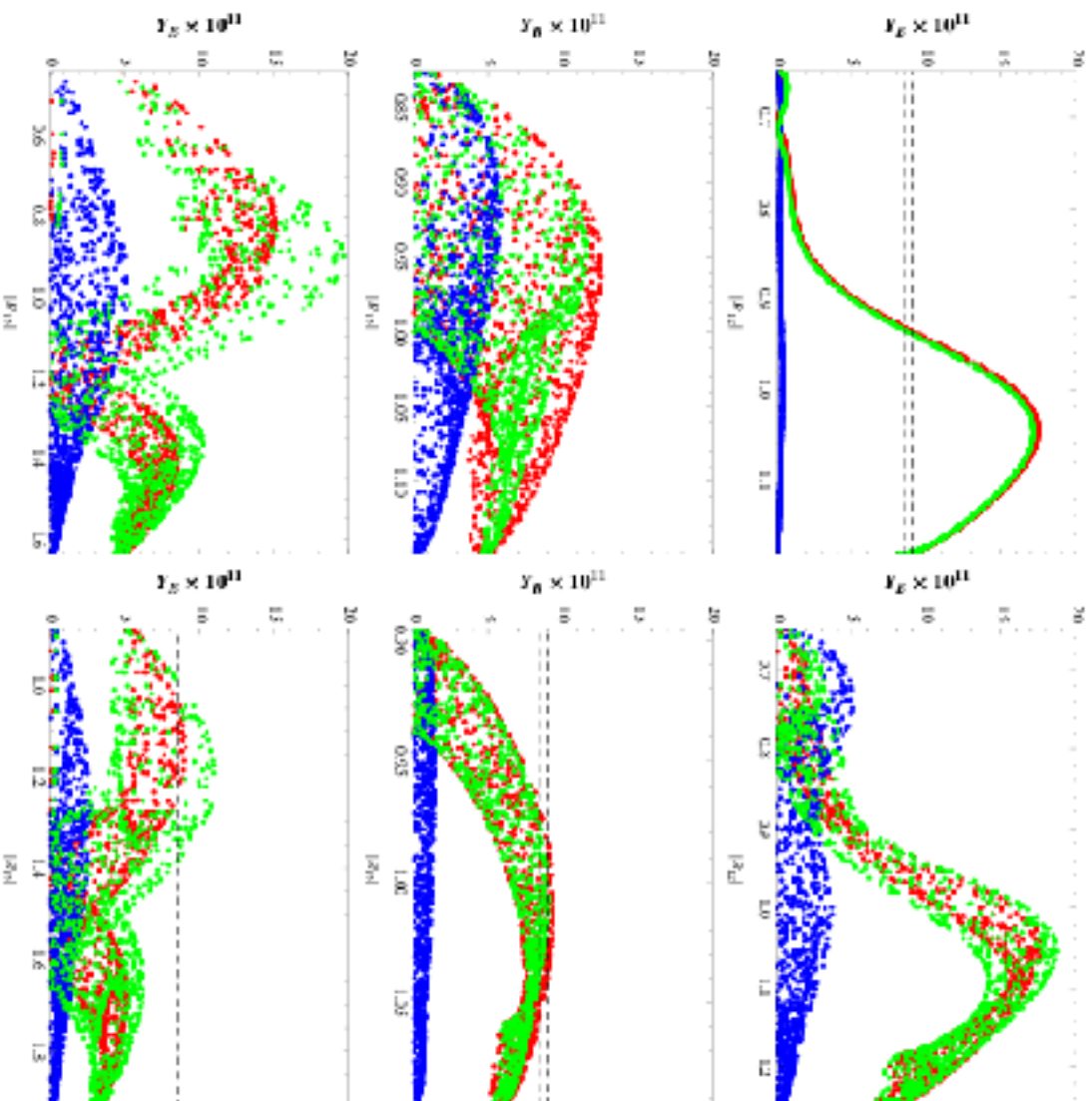


Figure 3: Total baryon asymmetry Y_B (red dots), “high energy” term (blue dots) and “midbox” term (green dots) vs $|R_{0-}|$ for $M_I = 10^{11}$ GeV: $m_2 = 0$, $\omega_2 = \pi/2$, $\omega_3 = 0.2$, $\delta = \pi$, complex M_{12} , M_{13} , and γ $|M_{11}| = 0.7$ and real $M_{12} = 0.8$ (upper left panel); $\delta\gamma$ $|R_{0-}| = 0.7$ and $|M_{11}| = 0.3$ (upper right panel); $\delta\gamma$ $|R_{0-}| = 0.6$ and $|M_{11}| = 0.4$ (middle left panel); $\delta\gamma$ $|R_{0-}| = 0.4$ (middle right panel); $\delta\gamma$ $|M_{11}| = 1.5$ and $|M_{12}| = 0.6$ (lower left panel); $\delta\gamma$ $|R_{0-}| = 1.5$ and $|M_{12}| = 0.7$ (lower right panel). The figures in the upper right panel, the middle and lower panels are obtained for complex M_{12} . The horizontal lines indicate the allowed range of the baryon asymmetry: $Y_B = (8.77 \pm 0.24) \times 10^{-11}$.

Total baryon asymmetry Y_B (red dots), “high energy” term (blue dots) and “mixed” term (green dots) vs $|R_{12}|$ for $M_1 = 10^{11}$ GeV, $m_3 = 0$, $\alpha_{21} = \pi/2$, $s_{13} = 0.2$, $\delta = \pi$, complex R_{11} , R_{12} , and $i)$ $|R_{11}| = 0.7$ and real $R_{13} = 0.3$ (upper left panel); $ii)$ $|R_{11}| = 0.7$ and $|R_{13}| = 0.3$ (upper right panel); $iii)$ $|R_{11}| = 0.4$ and $|R_{13}| = 0.4$ (middle left panel); $iv)$ $|R_{11}| = 0.4$ and $|R_{13}| = 0.2$ (middle right panel); $v)$ $|R_{11}| = 1.3$ and $|R_{13}| = 0.6$ (lower left panel); $vi)$ $|R_{11}| = 1.5$ and $|R_{13}| = 0.7$ (lower right panel). The figures in the upper right panel, the middle and lower panels are obtained for complex R_{13} . The horizontal lines indicate the allowed range of the baryon asymmetry: $Y_B = (8.77 \pm 0.24) \times 10^{-11}$.

Conclusions

The see-saw mechanism provides a link between the ν -mass generation and the baryon asymmetry of the Universe (BAU).

Any of the CPV phases in U_{PMNS} can be the leptogenesis CPV parameters.

Obtaining information on Dirac and Majorana CPV is a remarkably challenging problem.

Dirac and Majorana CPV may have the same source.

Low energy leptonic CPV can be directly related to the existence of BAU.

Understanding the status of the CP-symmetry in the lepton sector is of fundamental importance.

These results underline further the importance of the experimental searches for Dirac and/or Majorana leptonic CP-violation at low energies.