Earth Matter Effect on SN Neutrinos

in large-volume detectors

based on arXiv:1207.5049

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Werner-Heisenberg-Institut, München, Germany

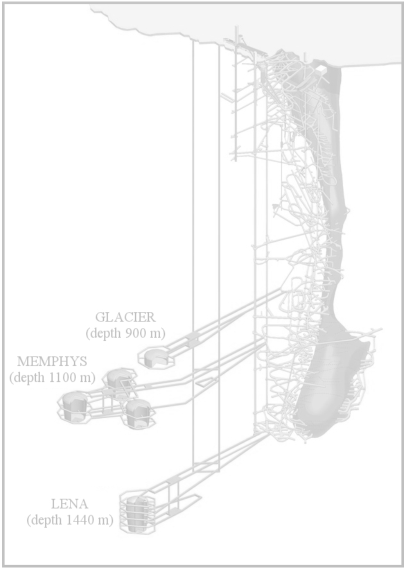
Rationale

During the last ten years the **Earth Matter Effect** (EME) on the observed spectrum of SN ν has been considered as a **guaranteed** signature to determine the ν mass hierarchy.

This is actually the case for **large differencies** among the fluxes of the different ν species.

According to the **state of the art of SN simulations** these differencies are **much smaller** that it was tought ten years ago.

We **reevaluate** the EME and the chances for **next-generation** underground detectors of observing it.



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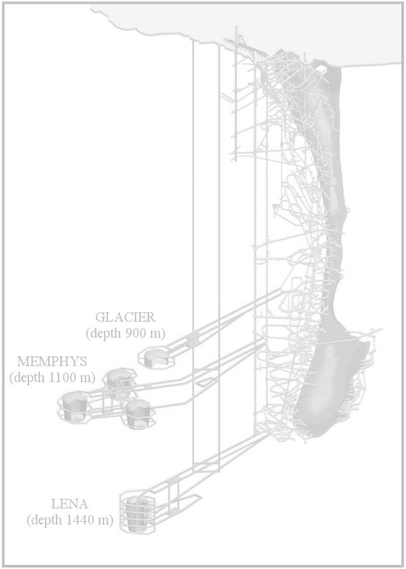

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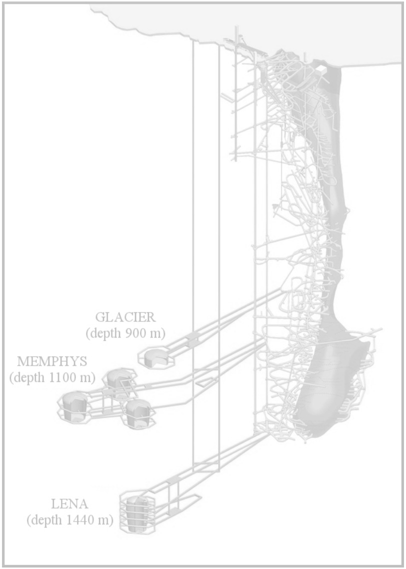

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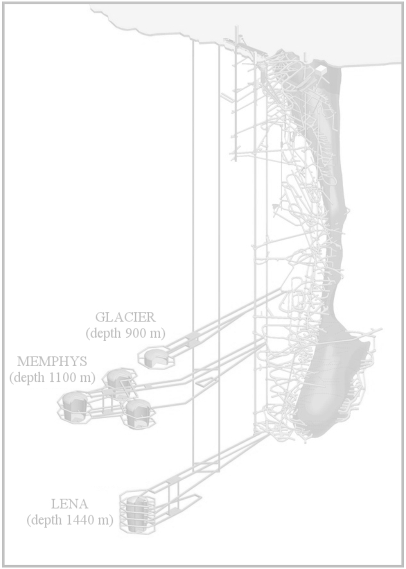

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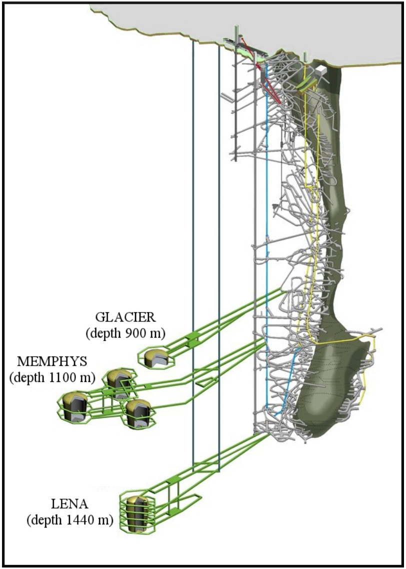

Supernova Neutrinos

SN Explosion $8 + M_\odot$

Core collapse SN: terminal phase of a massive star.
Shock wave driven explosion ejecting the outer mantle of the star.

Energy Scale $\sim 10^{53}$ erg

99% of the released energy ($\sim 10^{53}$ erg): $\sim \mathcal{O}(10\,\mathrm{MeV})$ **neutrinos**.

ν Flux $\sim 10^{57}$

$\sim 10^{57} \nu$ are emitted during a SN explosion.

Time Scale 10 s

Neutrino emission lasts ~ 10 s.

SN1604, the first observed galactic SN
–Kepler, 1604

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10. 8 M_{sun} progenitor mass
(spherically symmetric with Boltzmnann ν transport)

Neutronization burst

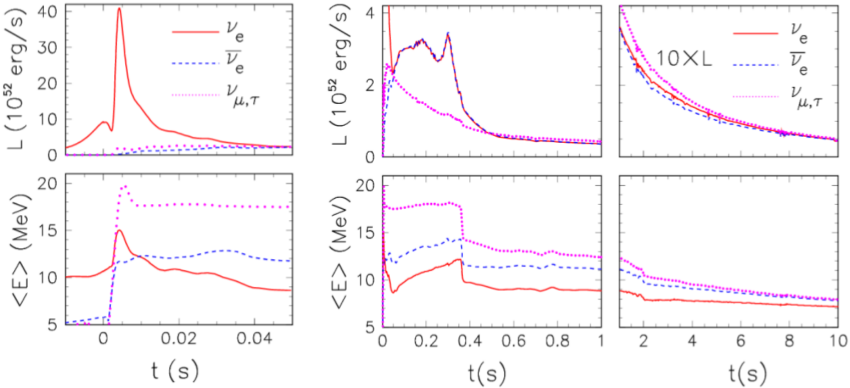
- Shock breakout
- De-leptonization of outer core layers

Accretion

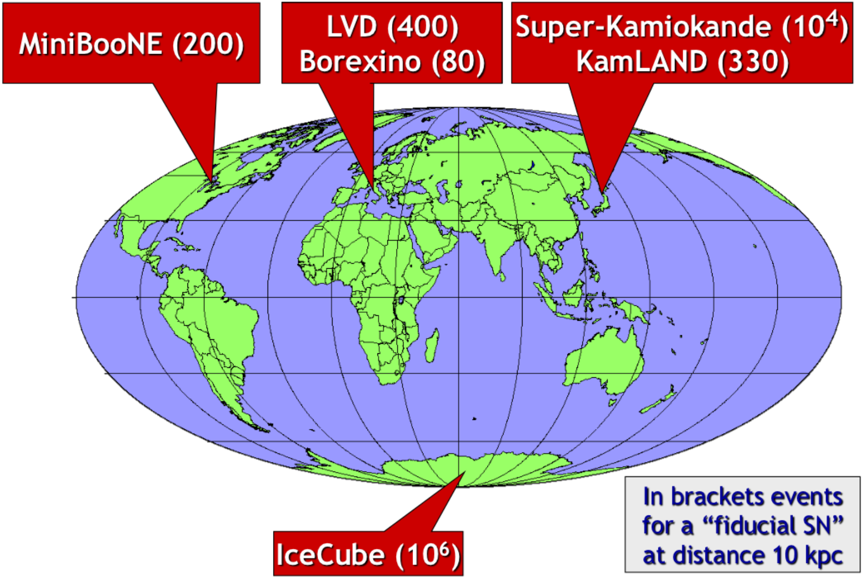
- Shock stalls ~ 150 km
- ν powered by infalling matter

Cooling

- Cooling on ν diffusion time scale



Large Detectors for Supernova Neutrinos



In brackets events for a "fiducial SN" at distance 10 kpc

LAGUNA Detectors for SN ν

Next-generation large-volume detectors might open a new era in SN neutrino detection:

MEMPHYS

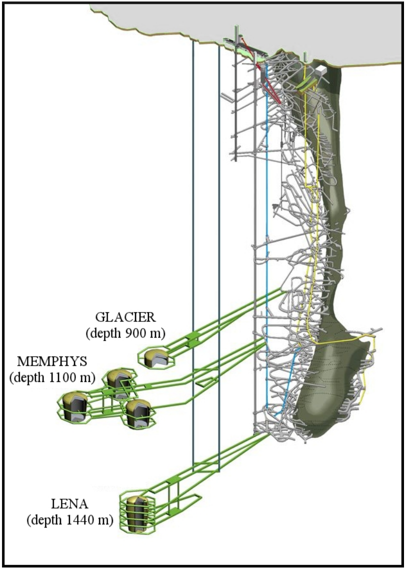
400 kton water Cherenkov detector ($\bar{\nu}$ flux)

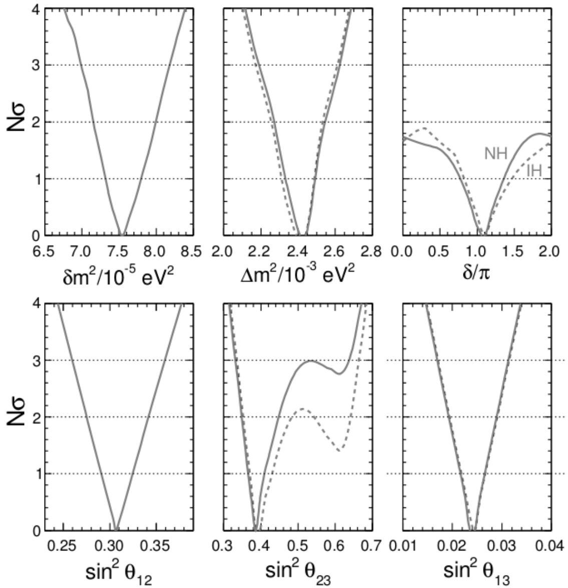
LENA

50 kton scintillator ($\bar{\nu}$ flux)

GLACIER

100 kton Liquid Ar TPC (ν flux)




Nσ
4
3
2
1
0
6.5 7.0 7.5 8.0 8.5
δm²/10⁻⁵ eV²
2.0 2.2 2.4 2.6 2.8
Δm²/10⁻³ eV²
0.0 0.5 1.0 1.5 2.0
δ/π
NH
IH
0.25 0.30 0.35
sin² θ12
0.3 0.4 0.5 0.6 0.7
sin² θ23
0.01 0.02 0.03 0.04
sin² θ13

Earth Matter Effect

Neutrino Oscillations in Earth

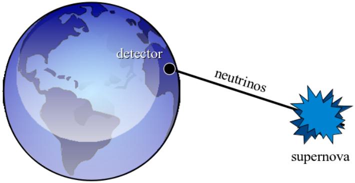
Modulations in the SN neutrino energy spectra.

Measurement of θ_{13}

$\theta_{13} \sim 8^\circ$ **large**

SN accretion Phase
Chakraborty *et al.* 2011

Only MSW matter effects.
No collective



Only **one possible scenario** compatible with a detection of the EME:

Normal Hierarchy

The effect appears in the spectrum of **antineutrinos**.

Inverted Hierarchy

The effect appears in the spectrum of **neutrinos**.

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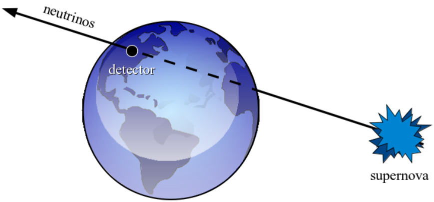

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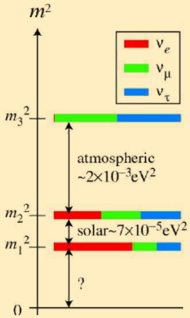

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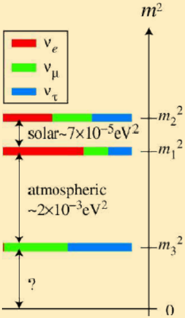


Detectors

- **Water Cherenkov** (MEMPHYS)
- **Liquid scintillators** (LENA)

$$\bar{\nu}_e + p \rightarrow n + e^+$$

Detectors

- **LAr time projection chambers** (GLACIER)

$$\nu_e +^{40} Ar \rightarrow^{40} K^* + e^-$$

Modulated (Anti)Neutrino Flux

$$F_{\bar{e}}^{D} = \sin^2\theta_{12} F_{\bar{x}}^{0} + \cos^2\theta_{12} F_{\bar{e}}^{0} + \Delta F^{0} \bar{A}_{\oplus} \sin^2(12.5\,\overline{\Delta m^2_{\oplus} L/E})$$

Flux Difference

$$\Delta F^0(E) = F_{\bar{e}}^{0}(E) - F_{\bar{x}}^{0}(E)$$

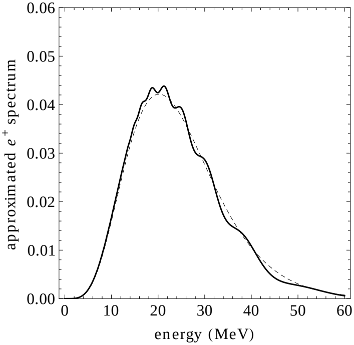
"Inverse-Energy"

$$y = \frac{12.5}{E}$$

Fourier Transform (FT)

$$k_{\oplus} = 2\overline{\Delta m^2_{\oplus}} L$$

does not depend on the incoming ν spectrum and has to contribute to the FT of the observed spectrum.



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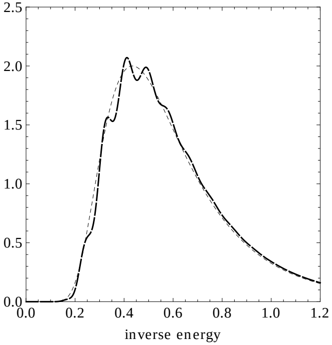

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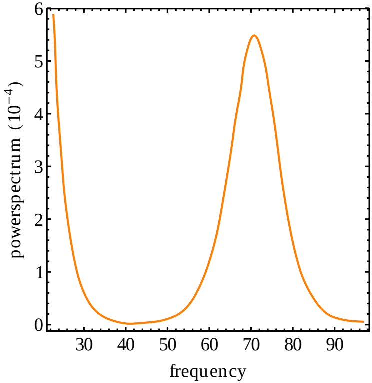
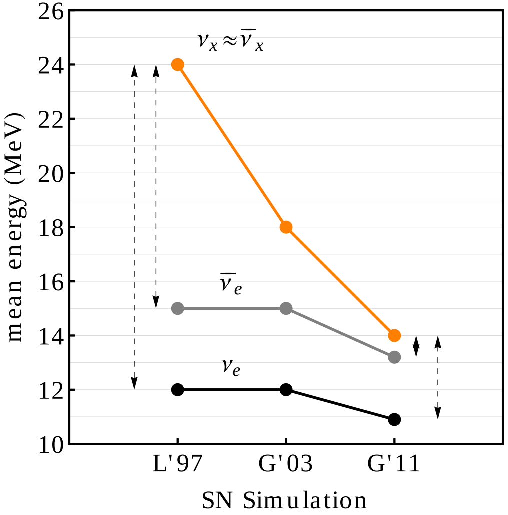
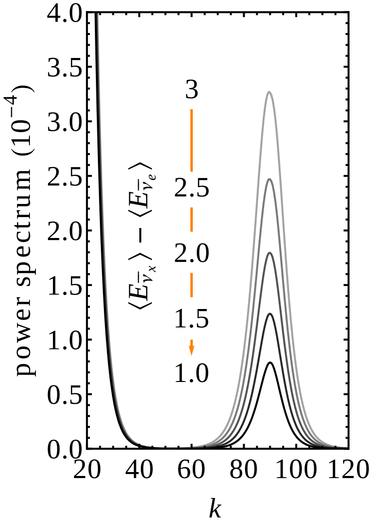


Trend of SN ν Average Energy

Totani *et al.* 1997
Raffelt *et al.* 2003
Serpico *et al.* 2011





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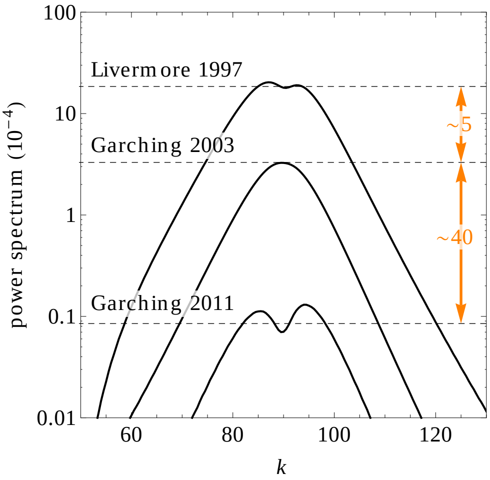

Livermore 1997 (*"Traditional" Simulation*)

The EME **IS** visible in the energy spectrum.

Garching 2003

The EME **IS** visible in the power spectrum of the "inverse-energy" spectrum.

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The EME **MAY BE** visible in the power spectrum of the "inverse-energy" spectrum.

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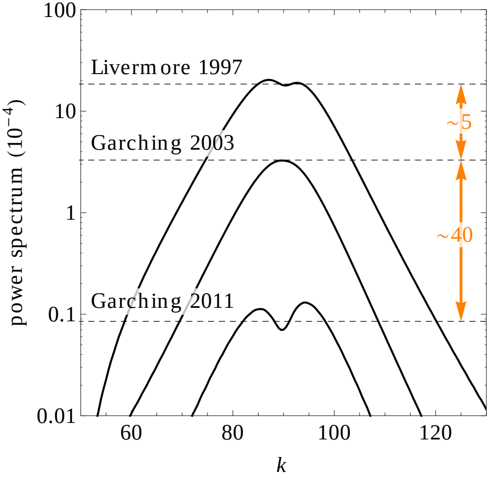

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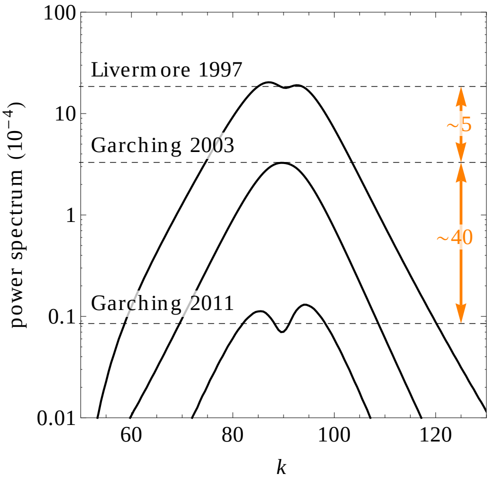

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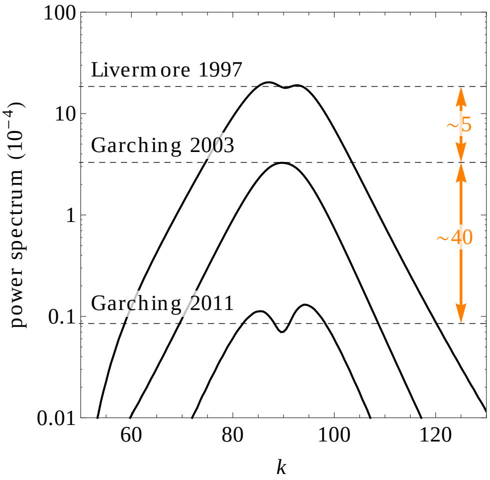

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Detector Characteristics

Water Cherenkov

$$\bar{\nu}_e + p \rightarrow n + e^+$$

- **e.g.:** MEMPHYS
- **mass:** 400 kton
- **energy resolution:**

$$\frac{\Delta}{\text{MeV}} = 0.47\sqrt{\frac{E_e}{\text{MeV}}}$$

- **# of events:**

10 kpc	2×10^4
1 kpc	2×10^6
0.2 kpc	4×10^7

Liquid scintillator

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- **e.g.:** LENA
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$$\nu_e +^{40} Ar \rightarrow^{40} K^* + e^-$$

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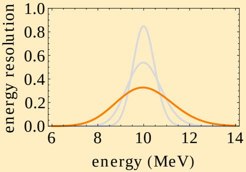

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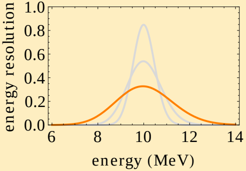

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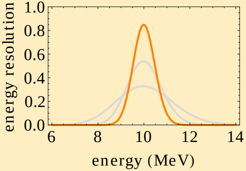

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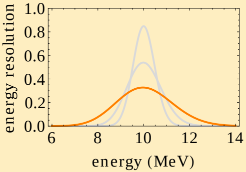

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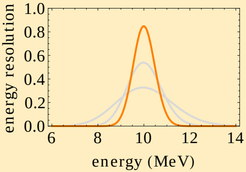

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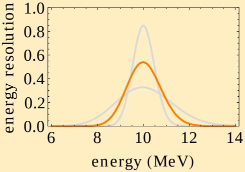

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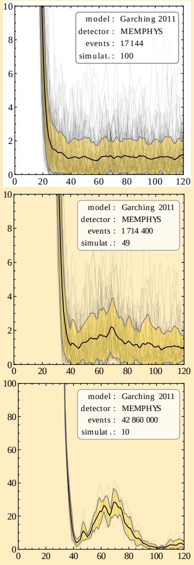
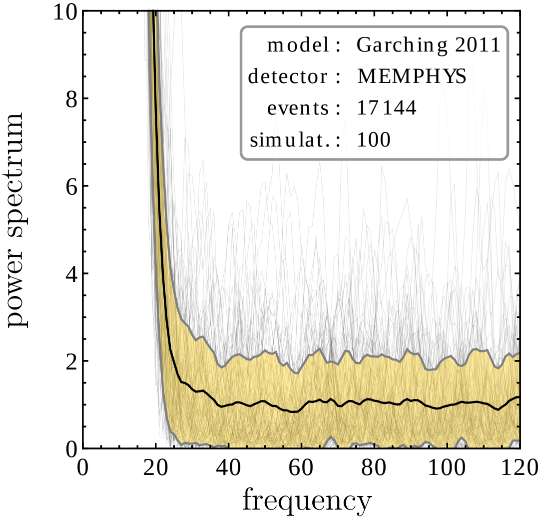
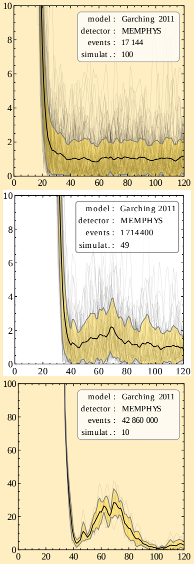
10 kpc

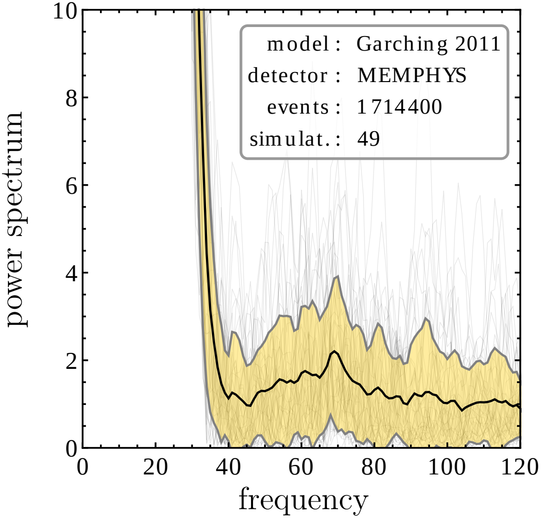




400 kton Water Cherenkov (MEMPHYS)

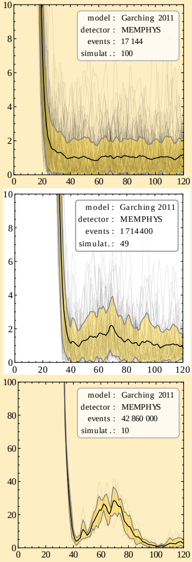
1 kpc

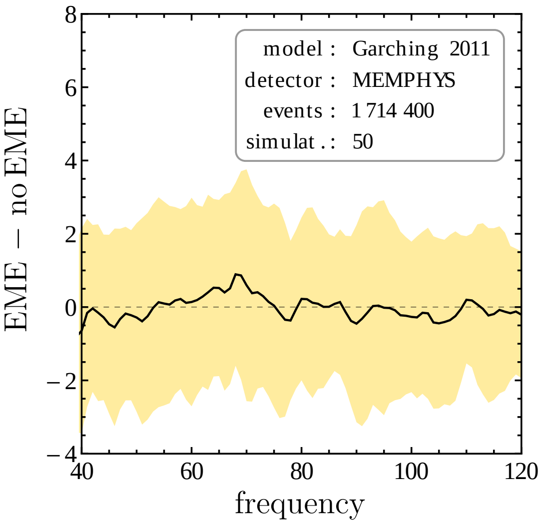




400 kton Water Cherenkov (MEMPHYS)

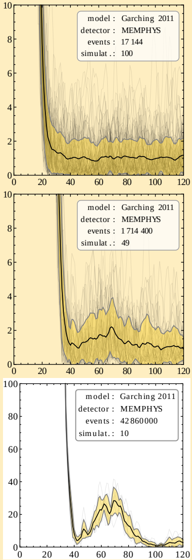
1 kpc

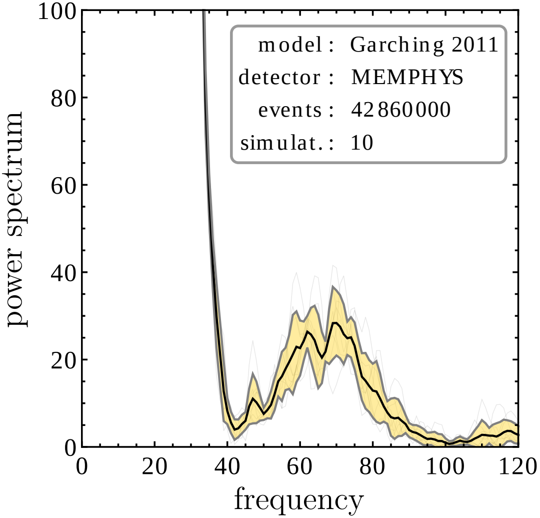




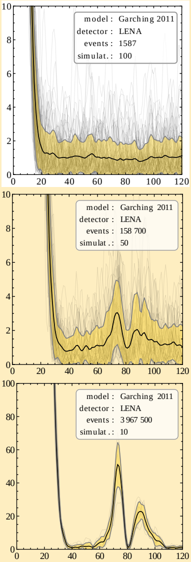
400 kton Water Cherenkov (MEMPHYS)

0.2 kpc

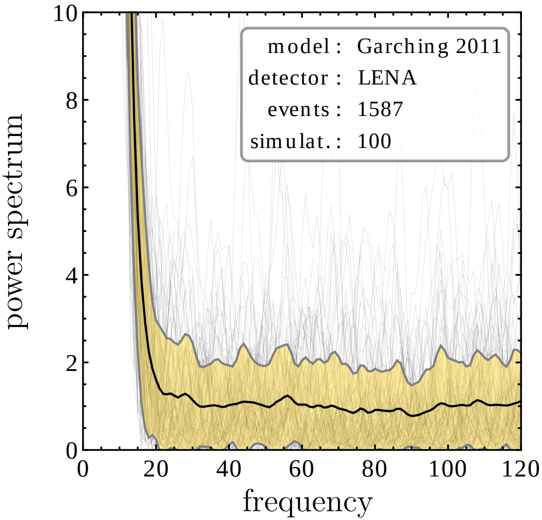




10 kpc



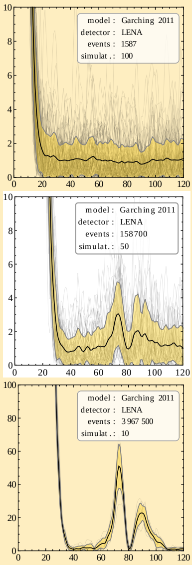
model : Garching 2011
detector : LENA
events : 1587
simulat. : 100
model : Garching 2011
detector : LENA
events : 158 700
simulat. : 50
model : Garching 2011
detector : LENA
events : 3 967 500
simulat. : 10

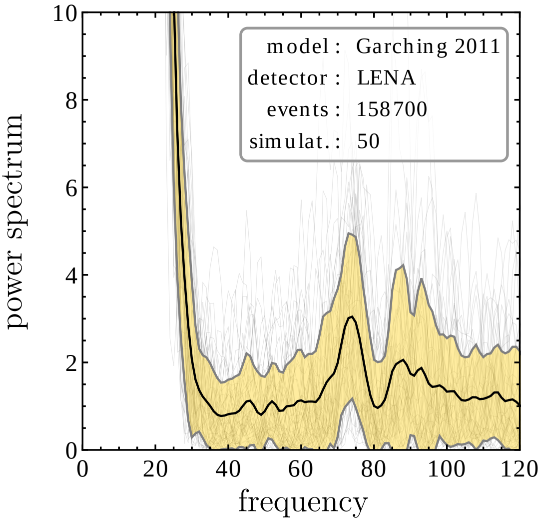


model : Garching 2011
detector : LENA
events : 1587
simulat. : 100
power spectrum
frequency

50 kton Liquid scintillator (LENA)

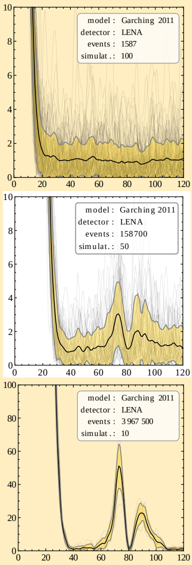
1 kpc

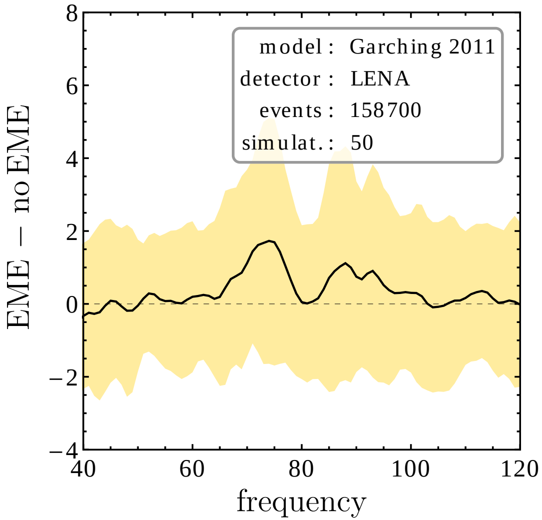




50 kton Liquid scintillator (LENA)

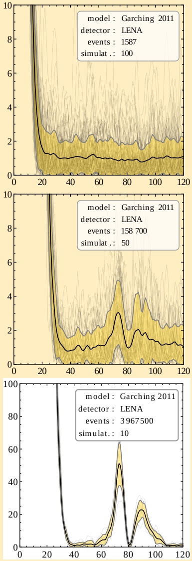
1 kpc

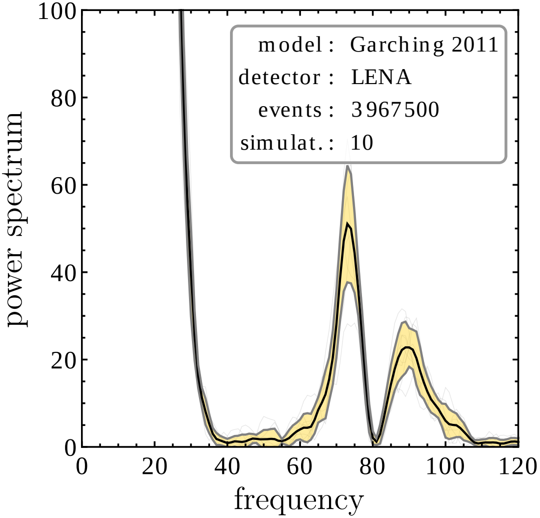




50 kton Liquid scintillator (LENA)

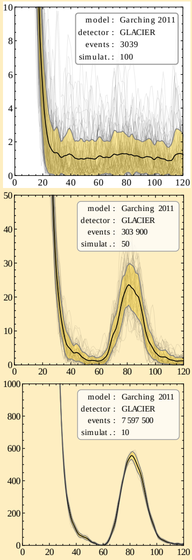
0.2 kpc

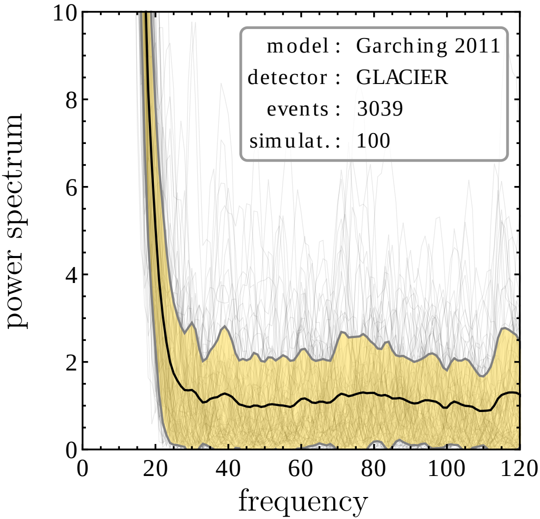




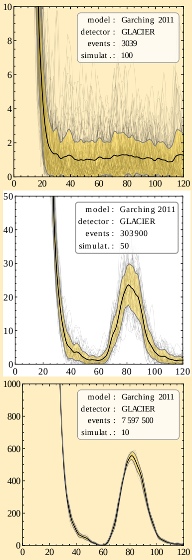
100 kton LAr TPC (GLACIER)

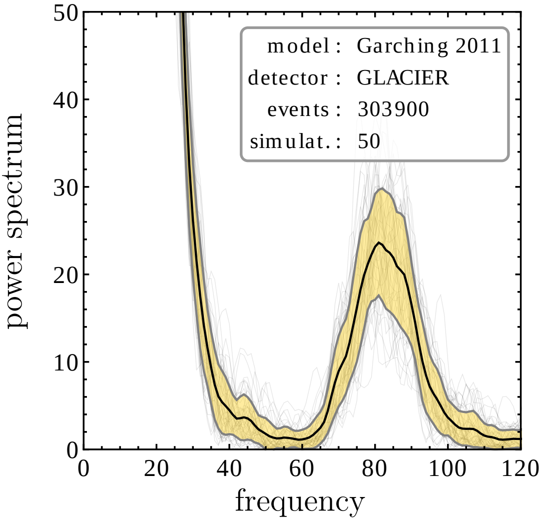
10 kpc





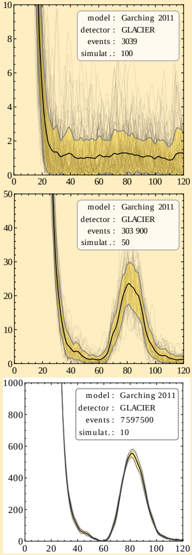
100 kton LAr TPC (GLACIER)

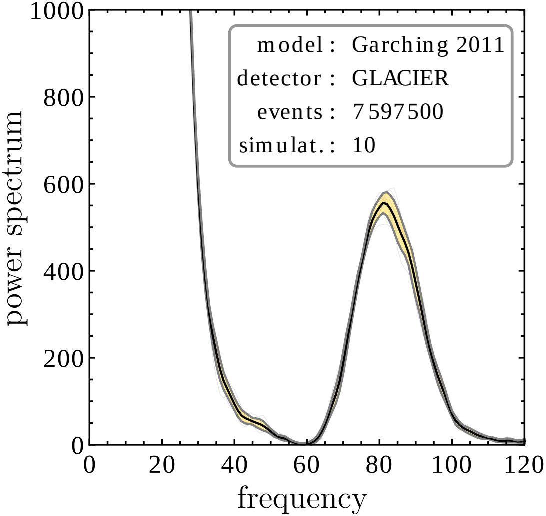


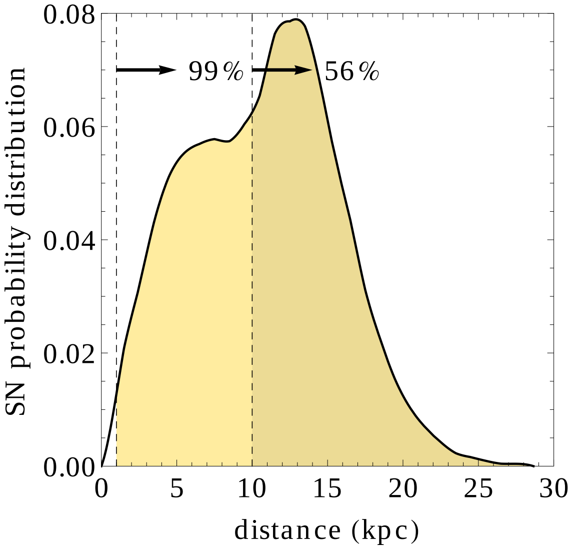


100 kton LAr TPC (GLACIER)

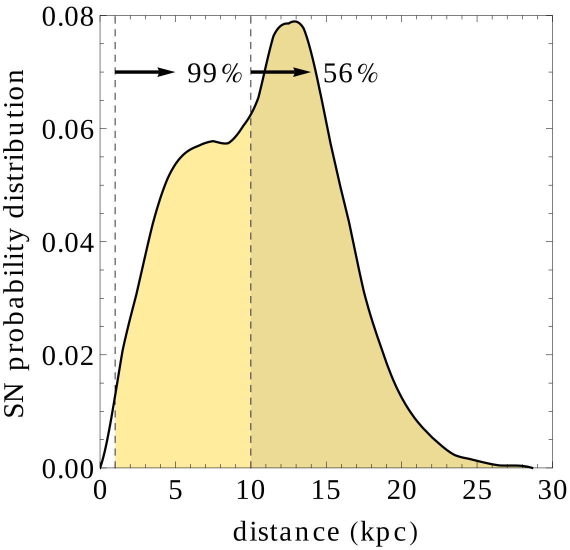
0.2 kpc






0.08
0.06
0.04
0.02
0.00
SN probability distribution
99 %
56 %
0
5
10
15
20
25
30
distance (kpc)

Galactic SN Distribution



According to **current** SN simulations:

100 kton WC

EME from few very close-by galactic SN

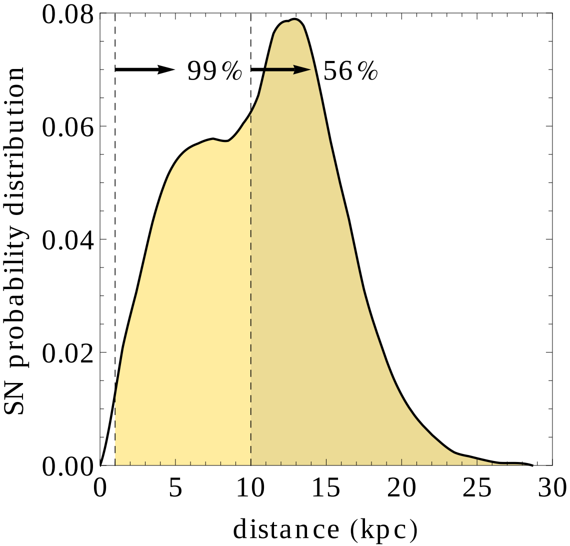
50 kton SC

EME from few very close-by galactic SN

100 kton LAr TPC

EME from $\mathcal{O}(10\%)$ galactic SN

Galactic SN Distribution



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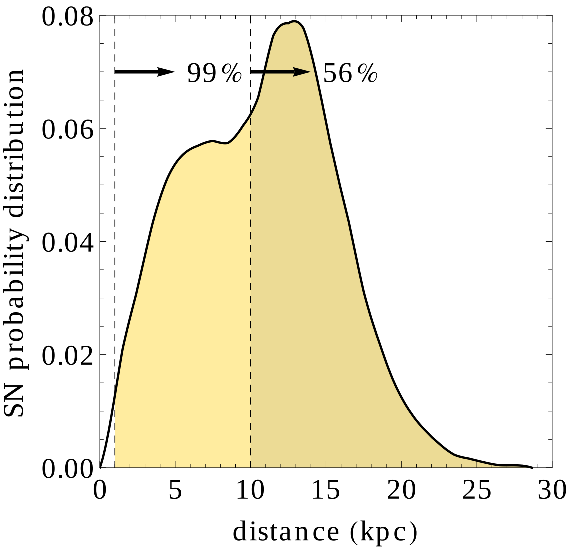

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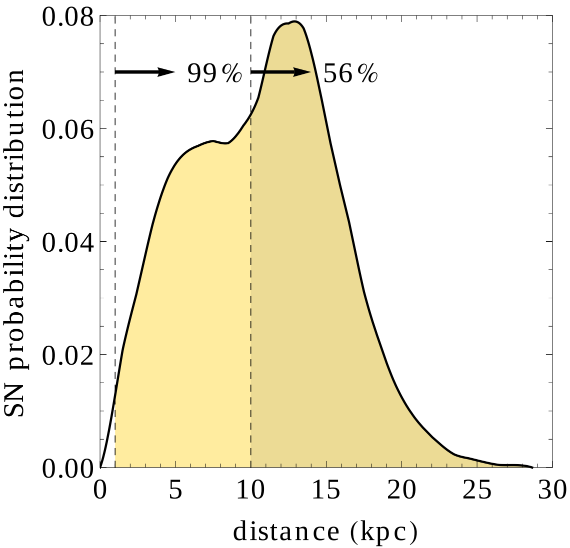

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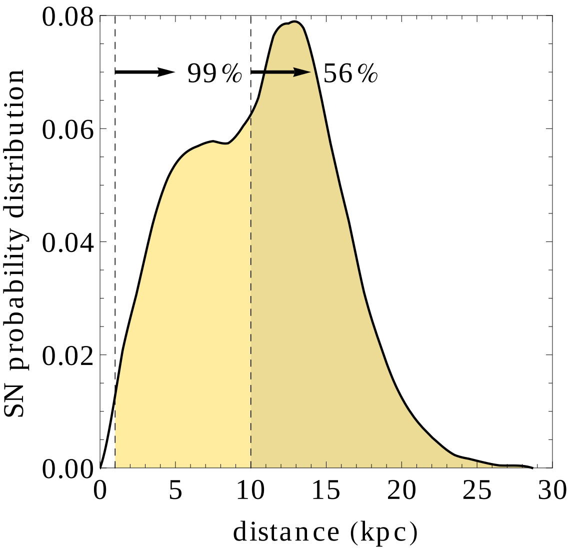

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Conclusions

EME seems to be **less promising** than what traditional SN simulations suggested.

However, SN ν can still be a **valuable tool** to get the ν mass hierarchy

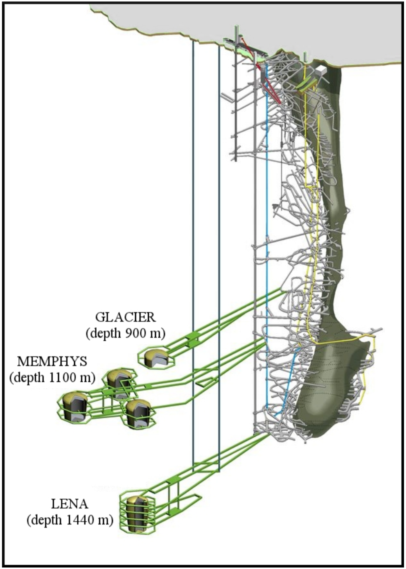
How?

Neutronization burst
Kalcheriess *et al.* 2006

Inverted hierarchy: **MEMPHIS** and **GLACIER** may see the peak.

Rise-time Serpico *et al.* 2012

MEMPHIS, **LENA**, **GLACIER**: Different Rise-times for ν species during the accretion phase.



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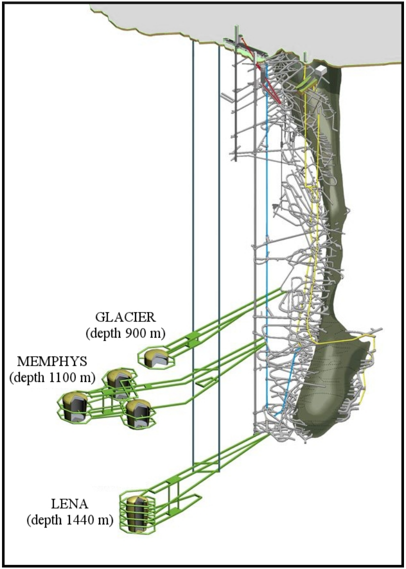

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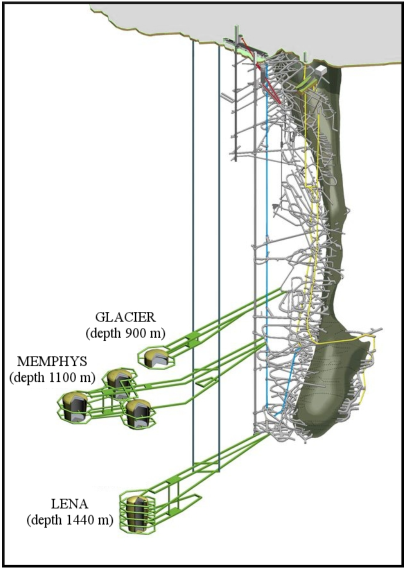

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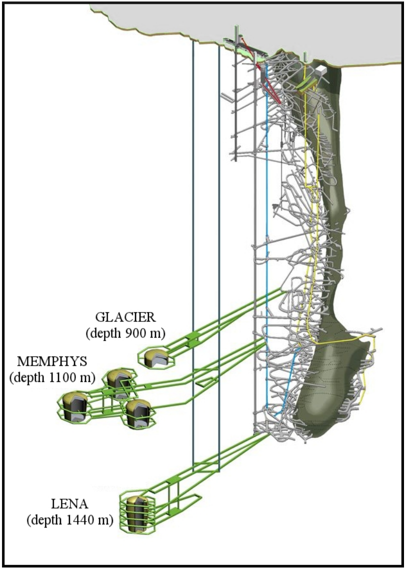

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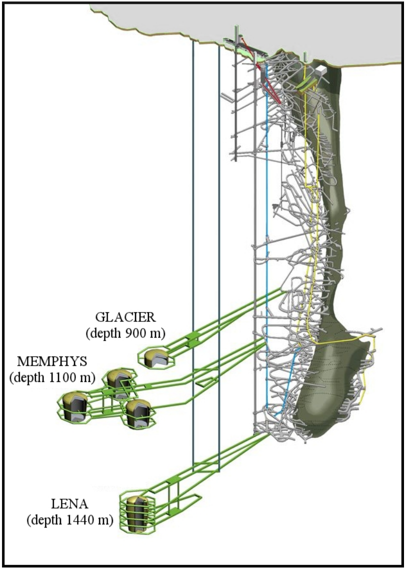

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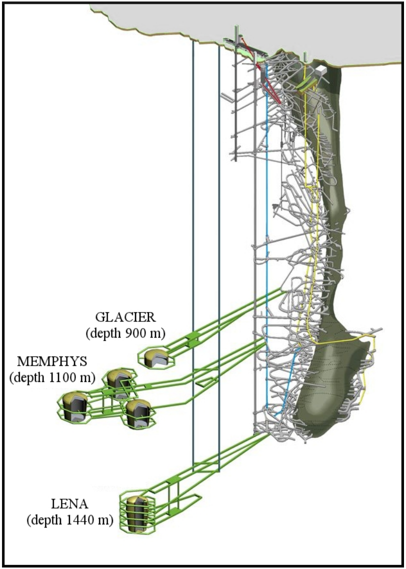
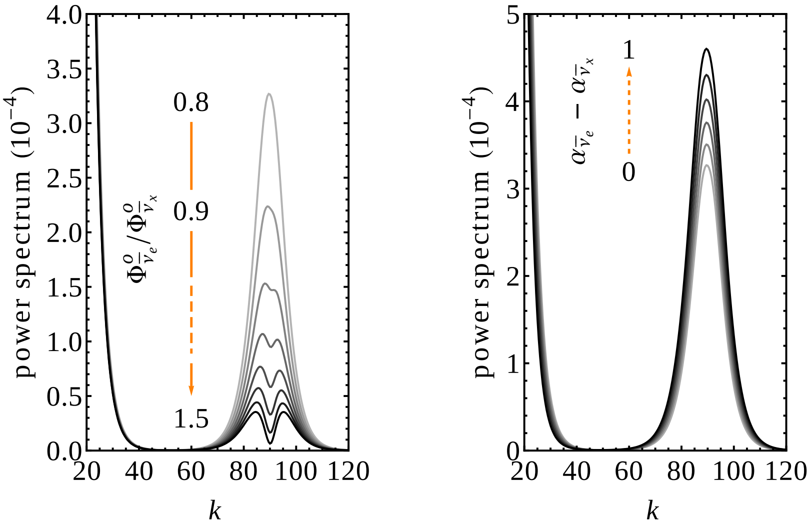
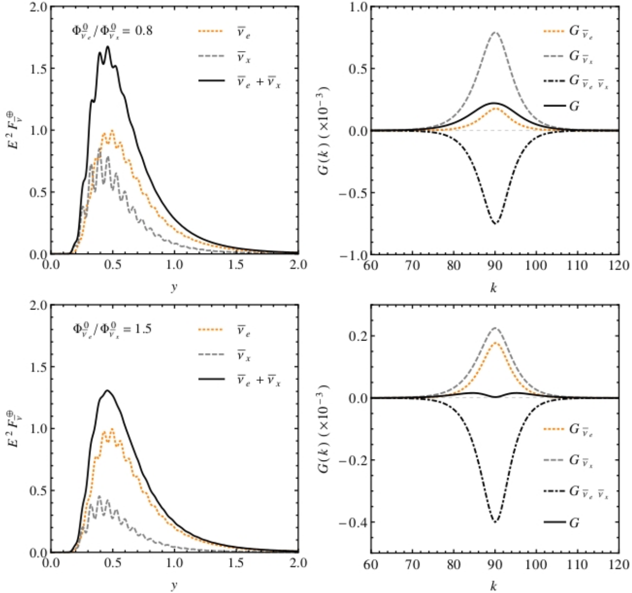


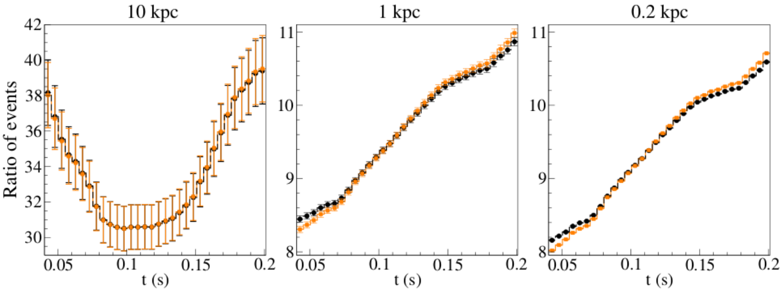
Appendix: Flux Ratio & Pinching Trends



Appendix: Double Peak



Appendix: Two Detectors



$$\text{discrete FT} \qquad g_N(k) = \frac{1}{\sqrt{N}} \sum_i e^{iky_i} = \sqrt{N} \, \langle e^{iky} \rangle$$

$$\langle e^{iky} \rangle \simeq \int f(y) \, e^{iky} \, \mathrm{d}y = g(k) \qquad \text{continuous FT}$$

$$g_N(k) \simeq \sqrt{N} \, g(k)$$

$$P_N(k) = |g_N(k)|^2 \simeq N \, |g(k)|^2 = N P(k)$$