Coherence of Supernova Neutrinos

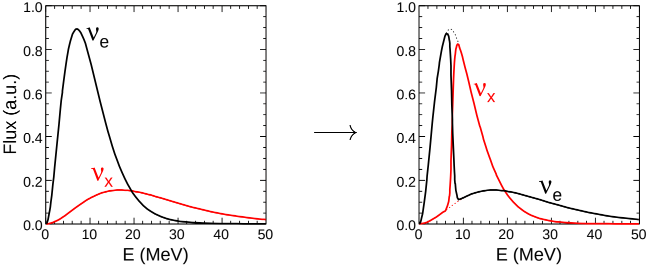
Jörn Kersten

University of Hamburg

Based on work done in collaboration with A.Yu. Smirnov

Interesting Effects with Supernova Neutrinos

- Oscillations inside the Earth
- MSW and collective effects inside the supernova



Is coherence preserved in these cases?

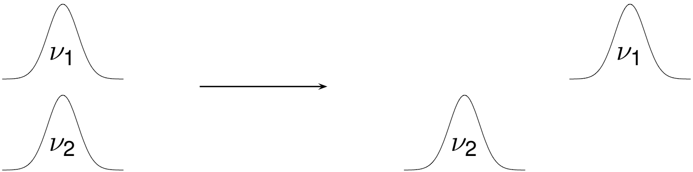
Decoherence

- Coherent 2-flavor oscillation probability

$$P(\nu_e \to \nu_e) = |\cos^2\theta + \sin^2\theta e^{i\phi}|^2 = 1 - \sin^2 2\theta \sin^2 \frac{\Delta m^2 L}{4E}$$

- Oscillation phase $\phi = -\frac{\Delta m^2 L}{2E}$
- Mass eigenstates have different velocities

  $\leadsto$ Wave packets cease to overlap



  $\leadsto$ Probability is incoherent sum

$$P(\nu_e \to \nu_e) = |\cos^2\theta|^2 + |\sin^2\theta e^{i\phi}|^2 = \cos^4\theta + \sin^4\theta$$

  $\leadsto$ No oscillations between supernova and Earth

Coherence Length of Supernova Neutrinos

- Wave packets overlap up to coherence length $L_{\text{coh}} \sim \dfrac{\sigma}{\Delta v}$
- Depends on
  - Size of wave packets σ
  - Velocity difference Δv of mass eigenstates

Size of Wave Packets

- Determined by length of production process
- Electron capture $pe^- \to n\nu_e$
- Time scale $\sim$ time electron needs to cross proton, $\tau \sim \sigma_e / v_e$
- Temperature $\sim 5\,\mathrm{MeV} \Rightarrow$ electron relativistic, $v_e \sim 1$
- Size of electron wave packet $\sigma_e \sim$ mean free path

Size of Wave Packets

- Determined by length of production process
- Electron capture $pe^- \to n\nu_e$
- Time scale $\sim$ time electron needs to cross proton, $\tau \sim \sigma_e / v_e$
- Temperature $\sim 5\,\mathrm{MeV} \Rightarrow$ electron relativistic, $v_e \sim 1$
- Size of electron wave packet $\sigma_e \sim$ mean free path
- Result: $\sigma \sim \sigma_e \sim \left(4\pi\alpha^2 n\right)^{-1/3} \sim 10^{-11}\,\mathrm{cm}$
- For comparison:
  - Atmospheric neutrinos: $\sigma \sim 1\,\mathrm{cm}$
  - Reactor neutrinos: $\sigma \sim 10^{-8}\,\mathrm{cm}$
- $\sigma_E \sim 1/\sigma \sim 1\,\mathrm{MeV}$ not much smaller than $E \sim 10\,\mathrm{MeV}$

Coherence Length of Supernova Neutrinos

- Wave packets overlap up to coherence length $L_{\text{coh}} \sim \dfrac{\sigma}{\Delta v}$
- Depends on
  - Size of wave packets $\sigma \sim 10^{-11}$ cm
  - Velocity difference Δv of mass eigenstates

Coherence Length of Supernova Neutrinos

- Wave packets overlap up to coherence length $L_{\text{coh}} \sim \dfrac{\sigma}{\Delta v}$
- Depends on
  - Size of wave packets $\sigma \sim 10^{-11}$ cm
  - Mass difference $\Delta m^2_{\text{atm}} \sim 10^{-3}\,\text{eV}^2$ or $\Delta m^2_{\text{sol}} \sim 10^{-5}\,\text{eV}^2$
  - Neutrino energy $E \sim 10\,\text{MeV}$

Coherence Length of Supernova Neutrinos

- Wave packets overlap up to coherence length $L_{\text{coh}} \sim \dfrac{\sigma}{\Delta v}$
- Depends on
  - Size of wave packets $\sigma \sim 10^{-11}$ cm
  - Mass difference $\Delta m^2_{\text{atm}} \sim 10^{-3}\,\text{eV}^2$ or $\Delta m^2_{\text{sol}} \sim 10^{-5}\,\text{eV}^2$
  - Neutrino energy $E \sim 10\,\text{MeV}$
  - Matter density
  - Mixing angle $\theta_{13} \sim 9^\circ$ or $\theta_{12} \sim 30^\circ$

Coherence Length of Supernova Neutrinos

- Wave packets overlap up to coherence length $L_{\text{coh}} \sim \dfrac{\sigma}{\Delta v}$
- Depends on
  - Size of wave packets $\sigma \sim 10^{-11}$ cm
  - Mass difference $\Delta m^2_{\text{atm}} \sim 10^{-3}\,\text{eV}^2$ or $\Delta m^2_{\text{sol}} \sim 10^{-5}\,\text{eV}^2$
  - Neutrino energy $E \sim 10$ MeV
  - Matter density
  - Mixing angle $\theta_{13} \sim 9^\circ$ or $\theta_{12} \sim 30^\circ$
- Supernova, inner region:
  $L_{\text{coh}} \sim 100$ km $\lesssim$ length scale of collective effects
- Earth: $L_{\text{coh}} \sim 1000$ km

Coherence Length of Supernova Neutrinos

- Wave packets overlap up to coherence length $L_{\text{coh}} \sim \dfrac{\sigma}{\Delta v}$
- Depends on
  - Size of wave packets $\sigma \sim 10^{-11}$ cm
  - Mass difference $\Delta m^2_{\text{atm}} \sim 10^{-3}\,\text{eV}^2$ or $\Delta m^2_{\text{sol}} \sim 10^{-5}\,\text{eV}^2$
  - Neutrino energy $E \sim 10$ MeV
  - Matter density
  - Mixing angle $\theta_{13} \sim 9^\circ$ or $\theta_{12} \sim 30^\circ$
- Supernova, inner region:
  $L_{\text{coh}} \sim 100$ km $\lesssim$ length scale of collective effects
- Earth: $L_{\text{coh}} \sim 1000$ km

Supernova neutrinos are special

Very short wave packets $\rightsquigarrow$ short coherence length

Sometimes Decoherence Is Irrelevant

$$\begin{aligned} P(\nu_e \to \nu_e) &= |\cos^2\theta + \sin^2\theta e^{i\phi}|^2 \\ &= \cos^4\theta + \sin^4\theta + 2\cos^2\theta\sin^2\theta\cos\phi \end{aligned}$$

- Measured probability: average over energy resolution ΔE of detector

Sometimes Decoherence Is Irrelevant

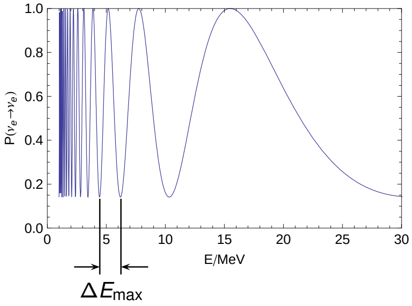
$$P(\nu_e \to \nu_e) = |\cos^2\theta + \sin^2\theta e^{i\phi}|^2$$
$$= \cos^4\theta + \sin^4\theta + 2\cancel{\cos^2\theta\sin^2\theta\cos}\,\phi$$

- Measured probability: average over energy resolution ΔE of detector
- ΔE too large $\rightsquigarrow$ interference term vanishes

Sometimes Decoherence Is Irrelevant

$$P(\nu_e \to \nu_e) = |\cos^2\theta + \sin^2\theta e^{i\phi}|^2$$
$$= \cos^4\theta + \sin^4\theta + 2\cos^2\theta\sin^2\theta\cos\phi$$

- Measured probability: average over energy resolution ΔE of detector
- ΔE too large $\rightsquigarrow$ interference term vanishes
  $\rightsquigarrow$ Need sufficient energy resolution to observe oscillations



Sometimes Decoherence Is Irrelevant

$$P(\nu_e \to \nu_e) = |\cos^2\theta + \sin^2\theta e^{i\phi}|^2$$
$$= \cos^4\theta + \sin^4\theta + 2\cos^2\theta\sin^2\theta\cos\phi$$

- Measured probability: average over energy resolution ΔE of detector
- ΔE too large $\rightsquigarrow$ interference term vanishes
  $\rightsquigarrow$ Need sufficient energy resolution to observe oscillations
- Corresponding uncertainty of arrival time $\Delta t \sim 1/\Delta E$
- Coherence preserved for wave packets arriving within Δt, even if they are spacially separated
  $\rightsquigarrow$ Detector restores coherence

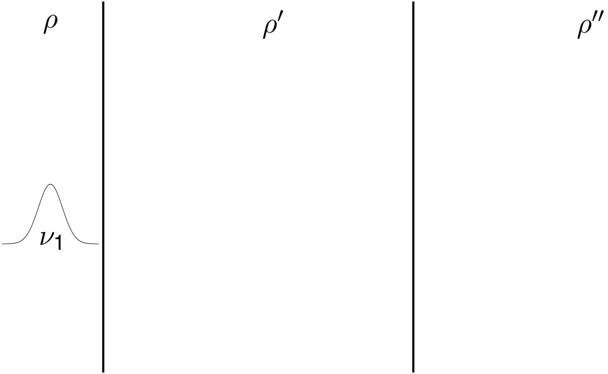
Sometimes Decoherence Is Irrelevant

$$P(\nu_e \to \nu_e) = |\cos^2\theta + \sin^2\theta e^{i\phi}|^2$$
$$= \cos^4\theta + \sin^4\theta + 2\cos^2\theta\sin^2\theta\cos\phi$$

- Measured probability: average over energy resolution ΔE of detector
- ΔE too large $\rightsquigarrow$ interference term vanishes
  $\rightsquigarrow$ Need sufficient energy resolution to observe oscillations
- Corresponding uncertainty of arrival time $\Delta t \sim 1/\Delta E$
- Coherence preserved for wave packets arriving within Δt, even if they are spacially separated
  $\rightsquigarrow$ Detector restores coherence
- Always the case in vacuum and matter with slowly changing density (adiabatic case)
- Does this change in non-adiabatic case?

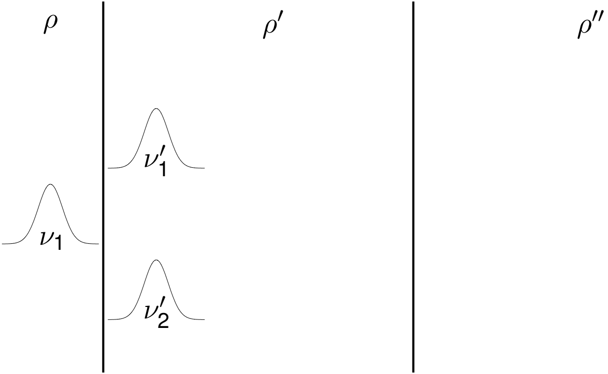
Adiabaticity Violation

- Simplest case: density step
- Each wave packet splits up into two



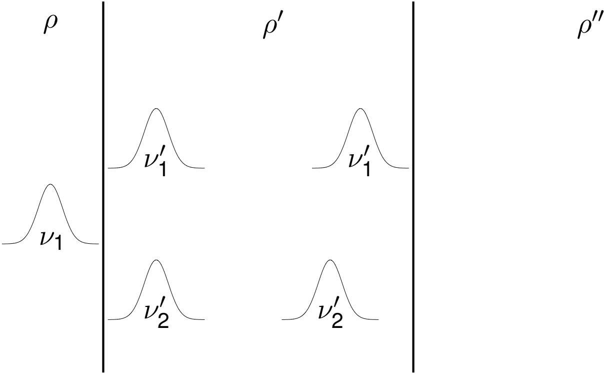
Adiabaticity Violation

- Simplest case: density step
- Each wave packet splits up into two



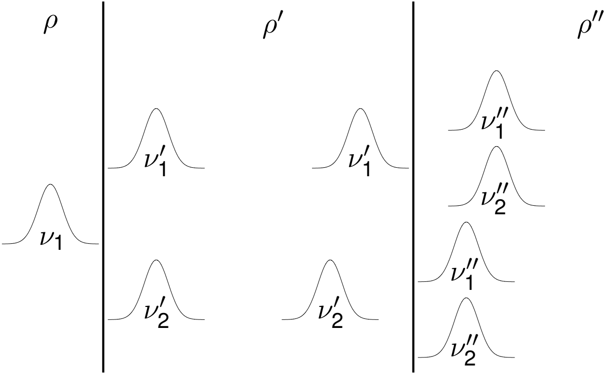
Adiabaticity Violation

- Simplest case: density step
- Each wave packet splits up into two



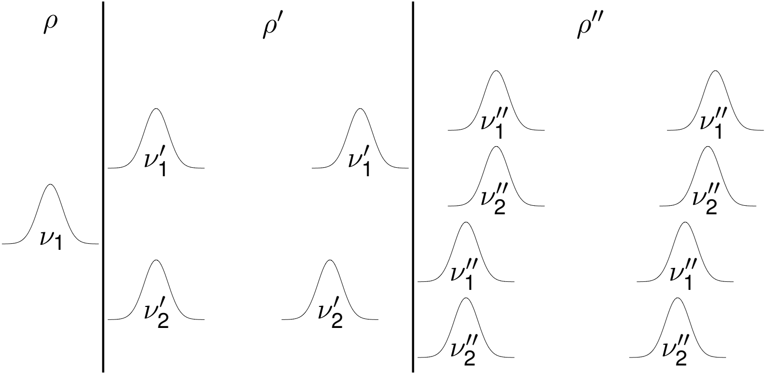
Adiabaticity Violation

- Simplest case: density step
- Each wave packet splits up into two



Adiabaticity Violation

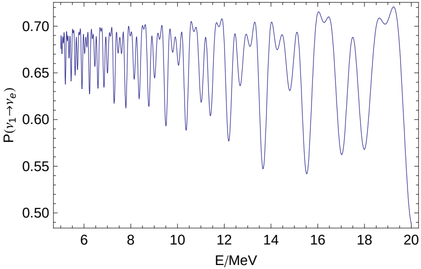
- Simplest case: density step
- Each wave packet splits up into two



Adiabaticity Violation

- Simplest case: density step
- Each wave packet splits up into two
- Different oscillation phase in each layer
- Complete coherence:

$$P = \left| a + b\, e^{i\phi_1} + c\, e^{i\phi_2} + \dots \right|^2$$



Adiabaticity Violation

- Simplest case: density step
- Each wave packet splits up into two
- Different oscillation phase in each layer
- Complete coherence:

$$P = \left| a + b\, e^{i\phi_1} + c\, e^{i\phi_2} + \dots \right|^2$$

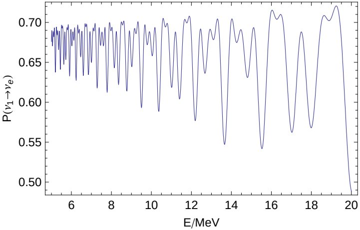
- Complete decoherence:

$$P = a^2 + b^2 + c^2 + \dots$$

- Energy resolution good enough to resolve all oscillation features
  $\Rightarrow$ detector restores coherence as before

Incomplete Averaging

$$P = \left| a + b\, e^{i\phi_1} + c\, e^{i\phi_2} + \ldots \right|^2$$
$$= a^2 + b^2 + c^2 + \cdots + 2ab \cos\phi_1 + 2bc \cos(\phi_2 - \phi_1)$$

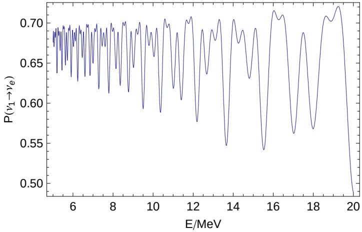


Energy resolution may be

- too bad to observe $\cos\phi_1$ term
- good enough to observe $\cos(\phi_2 - \phi_1)$ term if $\phi_1 \approx \phi_2$

Incomplete Averaging

$$P = \left| a + b\, e^{i\phi_1} + c\, e^{i\phi_2} + \ldots \right|^2$$
$$= a^2 + b^2 + c^2 + \cdots + 2ab \cos\phi_1 + 2bc \cos(\phi_2 - \phi_1)$$



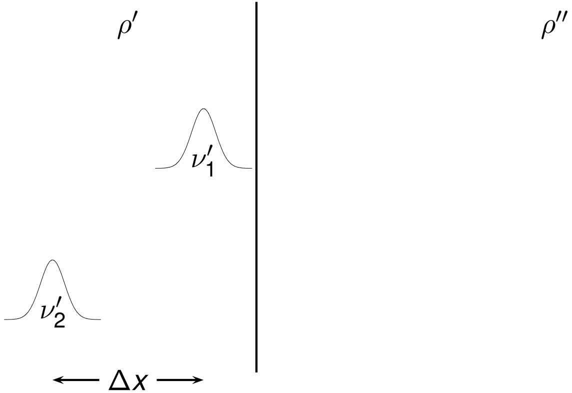
Energy resolution may be

- too bad to observe $\cos\phi_1$ term
- good enough to observe $\cos(\phi_2 - \phi_1)$ term if $\phi_1 \approx \phi_2$

But only if wave packets overlap (coherent case)?

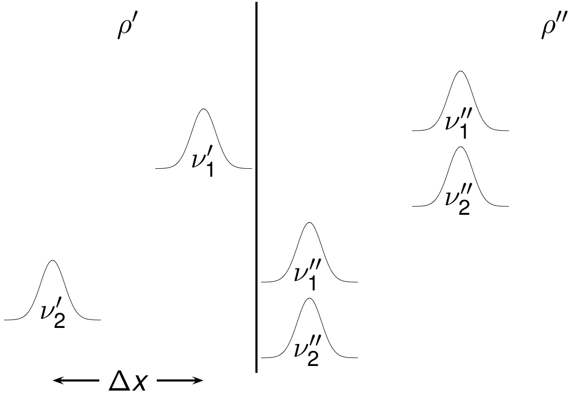
Wave Packet Catch-Up

- Wave packets don't need to separate forever:



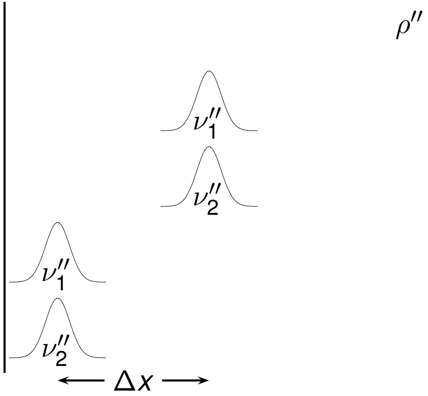
Wave Packet Catch-Up

- Wave packets don't need to separate forever:



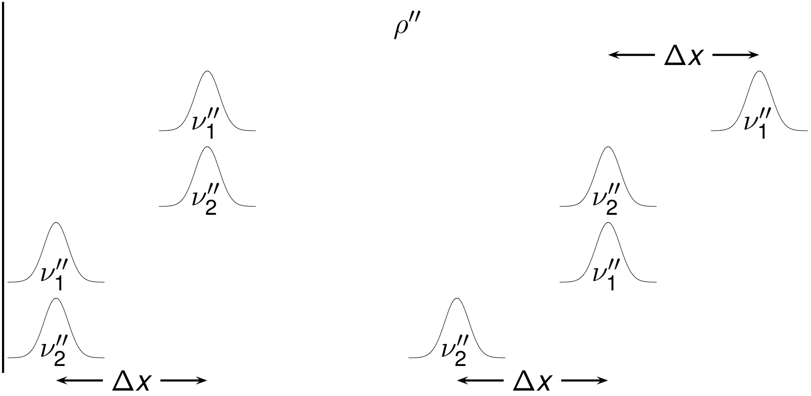
Wave Packet Catch-Up

- Wave packets don't need to separate forever:



Wave Packet Catch-Up

- Wave packets don't need to separate forever:



Wave Packet Catch-Up

- Wave packets don't need to separate forever:
- Wave packets catch up $\Longleftrightarrow$ ΔE good enough to observe $\cos(\phi_2 - \phi_1)$ term
- Decoherence and energy averaging are equivalent

Wave Packet Catch-Up

- Wave packets don't need to separate forever:
- Wave packets catch up $\Longleftrightarrow$ ΔE good enough to observe $\cos(\phi_2 - \phi_1)$ term
- Decoherence and energy averaging are equivalent
- Open questions:
  - How general is this?
  - Are collective effects in a supernova different?

Effect of Wave Packet Size on Spectrum

- Finite $\sigma_E \rightsquigarrow$ symmetric broadening of energy spectrum $f(E)$
- Change relevant for $\sigma_E^2 \gtrsim f/f''$

Effect of Wave Packet Size on Spectrum

- Finite $\sigma_E \rightsquigarrow$ symmetric broadening of energy spectrum $f(E)$
- Change relevant for $\sigma_E^2 \gtrsim f/f''$
- Example:
  - $f(E) \propto E^\alpha \exp[-(\alpha+1)\,E/\bar{E}]$
  - Mean energy $\bar{E} = 10\,\text{MeV}$
  - $\alpha = 3$
  - $\sigma_E = 2\,\text{MeV}$

  $\rightsquigarrow$ Potentially observable deformation of spectrum for $E \lesssim 3\,\text{MeV}$ and around $E \simeq 5\,\text{MeV}$

Conclusions

Supernova Neutrinos

- Extremely small size of wave packets $\sigma \sim 10^{-11}$ cm
- No experimental consequences in simple examples
- Possibly relevant for
  - collective effects in a supernova
  - energy spectrum